NAVIGATION-GUIDANCE-BASED ROBOT TRAJECTORY PLANNING FOR INTERCEPTION OF MOVING OBJECTS

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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0-612-41242-3
To Mahbobeh,

my trusted friend and beloved wife
ABSTRACT


Motivated by current trends in automation of industrial applications, the general problem area addressed in this thesis is “on-line robot-motion planning for intercepting randomly moving objects.” The specific objective of the thesis is “the use of navigation-guidance techniques in the development of a generalized scheme that can intercept a randomly moving object with time-optimality.

Two novel methods are presented for on-line-robotic-interception of fast-maneuvering objects which do not require a priori information on the moving-object’s motion. Both techniques combine a navigation-guidance-based method with a conventional object-tracking technique. Thus, they are classified as hybrid interception schemes with two phases: Phase I, during which the robot is under the control of a navigation-guidance-based technique, and Phase II, during which the robot’s control is switched to a conventional tracking method.

For the first proposed method, an Ideal Proportional Navigation Guidance (IPNG) technique is used during Phase I. This technique moves the interceptor rapidly toward the rendezvous point. For the second proposed method, the augmented form of the IPNG technique is suggested for Phase I, when a reliable estimation of the target’s acceleration can be provided to the interceptor. Both techniques are modified in this thesis to reflect the greater mobility of a robotic manipulator over an airborne missile for robotic interception of fast-maneuvering targets.

Since IPNG techniques have been originally designed for missile guidance, they do not attempt to match the target’s velocity at the interception point. In this thesis, for smooth interception, a tracking method is proposed to be switched on to at an optimal time in order to bring the robot to the interception point matching both target’s position and velocity. On-line selection of this time-optimal switching point is discussed in the dissertation.
The convergence of the proposed interception methods under ideal conditions is addressed. The effect of noise in target’s position readings on the on-line estimation of the interception time, and, subsequently, on the overall performance of the proposed technique is also discussed.

Extensive computer simulations illustrate the effectiveness of the IPNG-based interception methods developed in this thesis over pure tracking-based techniques proposed in the cited literature.
ACKNOWLEDGEMENTS

I wish to thank my co-supervisor, Professor B. Benhabib for his guidance, encouragement, and support during the course of this research. His exemplary personality and technical insight have made my graduate research fruitful and enjoyable. I have been extremely fortunate to have him as an advisor.

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My parents and my sisters have never wavered in their support of me. Their constant love, encouragement, and prayer provided me with emotional support. My wife, Mahbobeh, has been truly patient throughout this time. It is to her that I dedicate this thesis.

I have been blessed with two lovely sons, Erfan and Ali. They have been the joy of my every day’s life. Thank you guys for everything.

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Finally, I thank God for his silent support in all the moments of accomplishing this research.
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NOMENCLATURE

**LATIN LETTERS**

- $a_{AIPNG}$: Acceleration command of the AIPNG.
- $a_C$: Acceleration command.
- $a_{IPNG}$: Acceleration command of the IPNG.
- $a_{PNG}$: Acceleration command of the PNG.
- $a_T$: Target's acceleration.
- $C(q, \dot{q})$: Coriolis acceleration in robot's dynamic model.
- $e$: Relative-position vector between the target and the robot.
- $e_r, e_\theta$: Unit vectors in polar coordinate system.
- $e_\alpha, e_\gamma$: Noise in target's position.
- $F$: The vector representing robot's actuator's effort in task-space.
- $F, H$: Transition, and output matrices in target's model represented in state-space.
- $f'(s), f''(s)$: Tangential and normal unit vectors to the target's path.
- $G(q)$: Gravitational force vector.
- $G(t)$: Target's trajectory.
- $h$: Angular momentum defined based on the relative motion between the target and the interceptor.
- $I$: Symbol used for the objective function, and identity matrix.
- $J(q)$: Jacobian of the robot.
- $K_p, K_d$: Proportional and derivative gains.
- $K_{min}, K_{max}$: Minimum and maximum values of the factor used to limit the acceleration command of the IPNG/AIPNG.
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<td>$L$</td>
<td>Direction normal to the acceleration command in the PNG-based navigation laws.</td>
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<td>$m$</td>
<td>The ratio between the sampling times of the vision system and the controller.</td>
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<td>$miss$</td>
<td>Final miss-distance between the target and the robot.</td>
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<td>$M(q)$</td>
<td>Inertia matrix of the robotic manipulator.</td>
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<td>$M_x, C_x, g_x$</td>
<td>Terms related to the robot’s dynamics calculated in the task-space.</td>
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<td>Estimation of the robot’s dynamic terms.</td>
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<td>$n$</td>
<td>Number of degrees of freedom of the robot.</td>
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<td>$P$</td>
<td>Covariance matrix of the expected values of the state in the Kalman filter.</td>
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<td>$P(q)$</td>
<td>Vectorial expression for the forward kinematics of the robotic manipulator.</td>
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<td>$Q(k)$</td>
<td>Covariance matrix of the process noise.</td>
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<td>$q$</td>
<td>Robot’s joint state.</td>
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<tr>
<td>$q_d$</td>
<td>Desired robot’s joint state.</td>
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<td>$r_E, \dot{r}_E$</td>
<td>Coordinates of the transition point from AIPNG to a PD-type CT-method in the phase-plane.</td>
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<td>$r_0, r_f$</td>
<td>Initial and final relative position between the target and the robot.</td>
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<td>$s$</td>
<td>Laplace transform variable.</td>
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<td>$\dddot{s}<em>{\text{min}}, \dddot{s}</em>{\text{max}}$</td>
<td>Minimum and maximum acceleration of the robot along a path.</td>
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<td>$T_c$</td>
<td>Robot’s controller sampling time.</td>
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<td>$T_f$</td>
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<td>$T_p$</td>
<td>The time taken to predict target’s motion.</td>
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<td>$Tol_p, Tol_v$</td>
<td>Interception tolerances in position and velocity.</td>
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<td>$T_{\text{rc}}$</td>
<td>Sampling time of the robot’s motion execution.</td>
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<tr>
<td>$T_v$</td>
<td>Sampling time of the vision system.</td>
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\( t_0, t_f \)  
The time at which the robot is at the initial and final states, respectively.

\( t_{AIPNG} \)  
The time during which the robot is under the control of the AIPNG.

\( t_{CT}, t_{PD-CT} \)  
The time during which the robot is under the control of a PD-type CT-method.

\( t_{go} \)  
Time to go until intercept.

\( t_{IPNG} \)  
Time during which the robot is under the control of an IPNG.

\( t_{int} \)  
Rendezvous time.

\( t_{mod\_CT} \)  
Time during which the robot is under the control of a modified CT-method.

\( \bar{t}_{CT} \)  
Estimation of the time during which the robot is under the control of the CT-method.

\( \bar{t}_{int} \)  
Estimation of the interception time.

\( U \)  
Energy expended by the interceptor until intercept.

\( U_{LOS} \)  
Unit vector in the LOS direction.

\( U_{opt} \)  
Interceptor's optimal velocity along the LOS direction obtained via OPNG.

\( V_C \)  
Closing velocity between the target and the robot.

\( V_{h}, V_T \)  
Interceptor's and target's velocity.

\( V_{opt} \)  
Interceptor's optimal velocity normal to the LOS direction obtained via OPNG.

\( V_r, V_\theta \)  
Relative velocities between the target and the interceptor in the LOS and a direction normal to the LOS, respectively.

\( V_{rel} \)  
Relative velocity between the target and the interceptor.

\( v(k) \)  
Sequence of zero-mean white Gaussian measurement noise.

\( w(k) \)  
Sequence of zero-mean white Gaussian process noise.

\( X_t, Y_t \)  
Target's position in X and Y directions, respectively.

\( x_T \)  
Target's position vector.
\( x_R, X_r \)  

Robot's position vector.

**GREEK LETTERS**

\( \alpha \)  
Target's acceleration bound, and the percentage of the maximum available torque used in the IPNG and AIPNG.

\( \beta \)  
Magnitude of the acceleration component added to the \( a_{IPNG} \) and \( a_{AIPNG} \) in the LOS direction.

\( \delta \tau_{int} \)  
Uncertainty in the interception time estimation.

\( \delta \tau_{CT} \)  
Uncertainty in the estimation of the time during which the robot is under the control of a CT-method.

\( \eta \)  
The ratio between the interceptor's and the target's velocities.

\( \kappa \)  
Radius of curvature of the robot's path.

\( \lambda \)  
Navigation gain.

\( \varepsilon \)  
The position-difference between the target and the interceptor.

\( \theta_{LOS} \)  
The angle between the LOS and a reference line.

\( \rho \)  
Weighting factor.

\( \sigma \)  
Weighting factor.

\( \sigma_t, \sigma_T \)  
The angle between the Interceptor's and target's velocities and the LOS, respectively.

\( \sigma_x, \sigma_y \)  
Target's motion variances in X and Y directions, respectively.

\( \tau \)  
The joint actuators torque.

\( \tau_d \)  
The joint actuators disturbance torque.

\( \omega_l \)  
Turning rate of the interceptor's velocity.

\( \omega_n \)  
Natural frequency of the 2nd-order systems.

\( \xi \)  
Damping ratio.
<table>
<thead>
<tr>
<th>ACRONYMS</th>
<th>Description</th>
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<tbody>
<tr>
<td>3D</td>
<td>Three Dimensional.</td>
</tr>
<tr>
<td>AIPNG</td>
<td>Augmented Ideal Proportional Navigation Guidance.</td>
</tr>
<tr>
<td>AL</td>
<td>Acceleration Lines.</td>
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<tr>
<td>APNG</td>
<td>Augmented Proportional Navigation Guidance.</td>
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<tr>
<td>APPE</td>
<td>Adaptive Prediction, Planning and Execution.</td>
</tr>
<tr>
<td>ARMAX</td>
<td>Auto Regressive Moving Average with Auxiliary Inputs.</td>
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<td>CLOS</td>
<td>Command to the Line of Sight.</td>
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<td>CT</td>
<td>Computed Torque.</td>
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<tr>
<td>diag</td>
<td>Diagonal.</td>
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<tr>
<td>DOF</td>
<td>Degrees of Freedom.</td>
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<td>EKF</td>
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<td>KF</td>
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<td>LOS</td>
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<td>OGL</td>
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<td>OSP</td>
<td>Optimal Switching Point.</td>
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<td>PD</td>
<td>Proportional and Derivative.</td>
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<td>PNG</td>
<td>Proportional Navigation Guidance.</td>
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<td>PPNG</td>
<td>Pure Proportional Navigation Guidance.</td>
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<td>PTP</td>
<td>Point to point.</td>
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<tr>
<td>VSC</td>
<td>Variable Structure Control.</td>
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CHAPTER ONE

INTRODUCTION

1.1 PROBLEM STATEMENT AND MOTIVATION

The general problem area addressed in this thesis is “robot-motion planning for intercepting randomly moving objects.” Recent research in this area has resulted in a variety of interception methods for grasping objects off conveyors, which adapt to the slow-maneuvering motion of the object. One must note, however, that when the target is fast-maneuvering, such interception planning methods lose their time-efficiency feature due to the lack of reliable long-term predictability of the object’s motion. In this context, the objective of this thesis can be stated as “the development of a generalized scheme that can intercept a randomly moving object with time-optimality.”

The thesis’ work has been motivated by current trends in automation of industrial applications. Today’s industrial robot, however, can only function satisfactorily in rigidly-structured environments and does not tolerate uncertainties or variations in its work space. Better environment perception, more effective adaptation to tasks, improved dynamic performance and mobility are the main issues in developing intelligent robot technologies, [1]. Performing a multitude of tasks without complete a priori information is a key priority in that respect.

Many researchers have utilized a variety of sensors, in particular computer vision, to obtain intelligent robotic systems, [2]. Exemplary research areas for achieving this high-level objective include: development of methodologies for on-line path planning for mobile robots in dynamic environments [3], recognition and robotic grasping of known and unknown stationary objects [4], and robotic tracking and interception of known moving objects [5].

In this context, this thesis’ primary objective is: Development of a generalized navigation-guidance-based technique for the robotic interception of randomly moving objects. A survey of the pertinent literature is provided below.
1.2 Robotic Interception of Moving Objects – A Review of Literature

Robotic interception of moving targets has been widely addressed in the literature. Robot-motion planning is a primary sub-problem in this context and highly dependent on the target's motion type. The target can be considered as either fast- or slow-maneuvering. A slow-maneuvering target moves on a continuous path with constant velocity or acceleration. In such a case, accurate long-term prediction of the target's motion is possible and time-optimal interception methods can be employed. For a fast-maneuvering-type motion, on the other hand, the target varies its motion randomly and quickly, making time-optimal interception a difficult task. Variety of interception methods have been reported for the grasping of slow-maneuvering objects [e.g., 6,7]. However, one must note that, when the target is fast maneuvering, many of such interception-planning methods lose their time-efficiency feature due to the lack of reliable long-term prediction of the object's motion.

1.2.1 Interception of Slow-Maneuvering Objects

Three common methods utilized for intercepting slow-maneuvering objects are discussed herein.

1.2.1.1 Point-to-Point (PTP) Robot Trajectory Planning

Point-to-point robot trajectory planning techniques render optimal solution to the robotic interception of moving objects whose motion is completely known a priori. The time-optimal solution is in an open-loop form (i.e., the optimal control input is derived as function of time only rather than as function of the instantaneous state of the moving object), [8].

The most common algorithm addressed in the literature is based on variational calculus. The non-linear two-point-boundary-value problem is solved numerically, [9,10].

Near-time-optimal PTP robot trajectory planning techniques have also been proposed in the literature. Rana et al, [11] propose a technique for open-loop minimum-time trajectory planning for robotic manipulators subject to constraints on the robot's actuator torques using evolutionary algorithm. A repeated path modification is carried out by this method to search for a time-optimal trajectory.
Another near-time-optimal solution to the aforementioned interception problem falls under the category of time-optimal control of robotic manipulators when moving along a pre-specified path. Bobrow et al., [12] showed that the optimal solution is a bang-bang control. They showed that a robotic manipulator must move with its either maximum permissible acceleration or maximum permissible deceleration along the pre-specified path. This, subsequently, suggests that at least one actuator must be saturated all the time. This can jeopardize the stability of the motion in the presence of feedback, [13].

Shiller et al., [14] introduced Acceleration Lines (AL) derived at the boundary points taking the robot’s dynamics into account. In their method, a time-optimal path between two boundary points is one starting and ending in the direction of an AL computed at the start and end points. This technique yields a time-efficient algorithm for computing a near-global-time-optimal solution to the PTP robot trajectory planning.

1.2.1.2 Prediction, Planning and Execution (PPE) Technique

Prediction, Planning and Execution (PPE) methods are well suited for intercepting slow-maneuvering objects traveling along predictable trajectories. Since the object’s motion can be reliably predicted, there exists no need for continuous feedback of the object’s state to the robot-motion-planning module. Figure 1.1 shows a conceptual diagram of the PPE technique.

![Figure 1.1: Schematic diagram of a PPE system.](image-url)
Kimura et al. [6], reported a real-time ball-catching application using a PPE technique. Similarly, the catching of a toy hovercraft bouncing off the walls of a square enclosure was reported in [7]. In both applications, the target's motion type was known (i.e., parabolic in the case of the ball thrown in the air and linear in the case of the toy hovercraft) and also predictable on-line.

When using a PPE technique, the robot is directly sent to an anticipated rendezvous point on the target's predicted trajectory. In some PPE-application cases, a *global-minimum time* is achievable, [15]. However, such optimizations are computationally cumbersome and not usually suitable for on-line implementation.

PPE strategies, therefore, involve three general on-line stages: predicting the object trajectory, planning a robot trajectory for rendezvous with the object (using a gross-motion strategy), and then, executing the planned trajectory.

### 1.2.1.3 Adaptive Prediction Planning and Execution (APPE) Technique

In PPE techniques, the motion of the moving object is predicted and robot motion to intercept it is then planned and executed. This approach can be used in an *adaptive* mode (APPE), where the three stages of the PPE technique are repeated as necessary to guarantee the successful completion of the interception task, [16,17].

The cornerstone of the APPE technique is the selection, evaluation, and update of the time-optimal rendezvous (intercept) point, which could be chosen anywhere on the target's predicted path. In the APPE scheme proposed in [17], the first step of the robot-motion planning stage is the generation of a robot travel-time curve, which describes robot motion times from the robot's initial location to potential rendezvous locations on target's predicted path, $G(t)$. The intersection of this curve with a corresponding target travel time line to the same locations on $G(t)$ yields the (near) time-optimal interception point, Figure 1.2. The target's trajectory is continuously re-predicted and updated in runtime. Subsequently, the optimal interception point is also updated and the robot motion to this point is re-planned in runtime.

A typical APPE system is shown in Figure 1.3.
1.2.2 Interception of Fast-Maneuvering Objects

Visual servoing techniques have been widely used for intercepting fast-maneuvering objects whose motion is not reliably predictable. Another widely used technique for intercepting fast-maneuvering objects involves navigation-guidance methods. These two techniques are discussed below.
1.2.2.1 Interception via Visual Servoing Techniques

An APPE trajectory has to be radically changed when the object changes its motion randomly. Updating APPE robot trajectory (i.e., selection of a new time-optimal rendezvous point) is computationally cumbersome and would generate significant delay in the system. Furthermore, since the input torque obtained from the time-optimal solution to the interception problem leaves little or no room for the extra torque of the feedback control action, it is difficult to combine a minimum-time open-loop solution with an additional feedback controller. In APPE techniques, this problem has been solved by using arbitrary reduced torque bounds, at the cost of losing time-optimality, so that a torque head room is created for the feedback control, [13].

For fast-maneuvering objects, no reliable long-term target motion prediction is available. Thus, the minimum-time interception in an absolute sense is not a critical issue. Reliable interception, rather than fast interception, is the main issue in intercepting fast-maneuvering objects.

In visual servoing techniques, the object’s position is obtained from computer images taken by a camera. The position and velocity of the object are predicted to compensate for the computational delay which is inherent in obtaining the object position from the computer images. Different methods have been suggested for the target’s velocity prediction: Recursive filters/estimators such as Kalman filter and ARMAX models are the most commonly used ones, [18,19]. Kalman filters are usually deployed when the target’s dynamics is partly known in advance, while ARMAX models assume no a priori knowledge on target’s motion.

In visual servoing techniques the difference between the state (location and velocity) of the end-effector and the state of the object is the objective function to be minimized. The robot trajectory is generated via a controller, which minimizes this difference (or a function related to this difference) over each control period. For interception to occur, this difference must be reduced to zero prior to the object leaving the robot’s work-space. The robot trajectory planner determines the desired trajectory (subgoal) one control sampling period ahead of the controller. The nominal desired trajectory points for the manipulator motion are generated on-line based on the current position and velocity of the end-effector relative to the current position and velocity of the moving object, [18,19].
In order to account for torque bounds of the manipulator, however, an additional term has to be added into the objective function reflecting the manipulator’s expended energy, [19,20]. The objective function is normally defined as follows:

\[ I = f(\epsilon^2, \dot{\epsilon}^2, U^2), \]  

(1.1)

where \( \epsilon \) and \( \dot{\epsilon} \) denote the difference in position and velocity between the end-effector and the object, respectively. \( U \) in Equation (1.1) denotes the control input (i.e., energy expended by the manipulator). Despite the fact that Equation (1.1) takes the control input bounds into consideration, it does not guarantee that the torques at joint levels will not exceed their limits, [19].

The adaptive version of visual servoing techniques have shown robustness in the face of uncertainties, such as noise in target position readings and unmodeled dynamics of the robot, [5,19]. However, since these techniques have to go through a learning process they loose their time efficiency significantly (in regard to time to intercept), [19].

The controller of the visual servoing system tries to match the current position and velocity of the object, as a pseudo-final desired trajectory, all the time. This slows down the robot when the end-effector is initially at a far distance from the moving object. Namely, matching the object’s velocity when the target is far from the robot current location increases the interception time. Lei et al, [21] addressed this problem by combining a PPE-like approach with a visual servoing technique. Since the initial position error was large in their application, they utilized an on-line polynomial trajectory planning to drive the robot to the vicinity of the moving object. A position-based controller was then switched on for fast and stable tracking. This method is heuristic and a fine definition of target’s vicinity has to be rationalized. Lin et al, [22] used a heuristic coarse tuning method to minimize the objective function given in Equation (1.1). They used a weighting factor applied to each term in Equation (1.1). Therefore, their tracker does not have to match object’s velocity when the robot is significantly far from the object. The weighting factors are computed by minimizing the predicted interception time at each time step. In order to predict the interception time, a constant-velocity model is assumed for the object’s motion. The robot’s dynamics was disregarded in their paper.

Improving the interception time of visual servoing techniques is still an open research area.
1.2.2.2 Interception via Navigation Guidance Techniques

Another widely used method for tracking moving objects reported in the literature falls under the category of navigation and guidance theory. Such techniques, however, have normally been used for tracking fast-maneuvering, free-flying targets (e.g., missiles tracking evasive aircrafts).

In these techniques, the current state of the target is normally provided to the interceptor control system through radar tracking. Well known $\alpha$-$\beta$-$\gamma$ and Kalman filters have been widely used for this purpose.

Guidance laws typically fit within the following five categories, Command to the Line-of-Sight (CLOS), Pursuit, Proportional Navigation Guidance (PNG), Optimal Control Guidance, and other guidance laws dominated by differential game methods. PNG is the most commonly used guidance method and it has become a well-researched topic over the past 50 years with numerous applications of analytical treatment and implementation of missile guidance, [23,24].

In navigation guidance techniques, interception normally means closing the distance between the interceptor and the target by bringing the interceptor into the collision course with the target at the intercept. However, some rare attempts to promote this guidance technique to the rendezvous guidance (i.e., matching target’s velocity at intercept as well as the position) for intercepting satellites orbiting the earth or colliding with the target on a pre-specified course have also been reported in the literature, [25-27].

The PNG-based techniques seek to close the distance between the target and the interceptor while nullifying the angular rate of this distance at the same time. In a series of papers, a generalization of PNG was obtained by solving a quadratic optimization problem, [28-30]. The objective function to be minimized is normally a combination of the time to intercept and the energy expenditure by the interceptor, given as:

$$I = \int_0^{T_f}(1+\rho U^2)dt,$$

where $T_f$ denotes the interception time, $U$ denotes the expended energy in each controller sampling time and $\rho$ is a weighting factor. By comparing the objective functions presented in
Equations (1.1) and (1.2), one can note that the time to intercept is not attempted to be optimized in Equation (1.1), while it is taken into consideration in Equation (1.2).

It has been frequently shown in the literature that PNG yields time-optimal solution to the interception problem for *cruising targets* (i.e., targets moving with constant velocity), [28-30]. For accelerating targets, however, the time-optimality of the PNG-based guidance techniques fails. Bryson et al, [31] showed that PNG can cope with a maneuvering target (i.e., target moving with non-zero acceleration) by simply varying the guidance gain. However, there exists no reliable tuning procedure available.

An alternative to PNG law for fast-maneuvering targets has been developed as Augmented Proportional Navigation Guidance (APNG) [e.g., 31]. This technique yields time-optimal solution for intercepting fast-maneuvering targets under the following conditions:

- Both the interceptor and the target can only maneuver in the direction normal to their instantaneous velocity (i.e., they are assumed to be aerodynamically-controlled objects); and,
- The average acceleration of the target is known and available to the guidance control system.

Many have augmented the PNG law to account for deviations from the constant-velocity or acceleration of the target [e.g., 32,33]. Yuan et al, [33], reported a new guidance scheme of Ideal Proportional Navigation Guidance (IPNG). This technique shows better mathematical tractability (i.e., less sensitivity to the initial conditions of the interception) over the PNG-based navigation techniques. Its capture criterion is solely defined by the navigation gain and does not depend on the initial conditions. It has been shown that this technique is similar to the PNG when the interceptor has a clear speed superiority over the target, [34]. Although IPNG has not been well received for missile guidance, since it is not suitable for an aerodynamically-controlled environment when the interceptor's speed is not clearly superior to the target's, it is a robust tracker in terms of robotic interception as will be shown in this thesis.

As mentioned earlier, navigation techniques have been used in the past for on- or off-line generation of paths in non-robotic environments and only recently in mobile robotics, [36]. A rare attempt of a robotic manipulator use, [37], yielded a method for on-line path
planning in a dynamic environment. The simulation results, based on PNG, however, were restricted to tracking non-maneuvering targets moving on a straight line.

Navigation techniques are advantageous over pure robotic tracking methods, which utilize visual servoing, due to their simplicity (i.e., only a few parameters are involved in navigation laws). Moreover, PNG-based techniques bring the interceptor close to the target as fast as possible without attempting to match its velocity. This feature especially makes them fast interceptors in terms of closing the distance between the interceptor and the target. However, for stable tracking and smooth grasping, when the interceptor gets close to the target, a tracker must be utilized before intercept occurs.

1.3 RESEARCH OBJECTIVES AND DISSERTATION REVIEW

The primary objectives of this thesis are set as follows:

(1) Development of a generalized navigation guidance-based technique for the robotic interception of randomly moving objects;

(2) Development of an overall hybrid interception method that integrates a conventional tracking method with a navigation-guidance-based technique; and,

(3) Development of an on-line decision making mechanism for achieving minimum interception-time for the proposed hybrid interception scheme.

Chapter 2 provides a brief mathematical discussion of robotic interception of randomly moving objects. This is followed by the introduction of the tracking (i.e., visual servoing) and navigation-based techniques addressed in this thesis and a general description of the proposed hybrid solution.

Chapter 3 presents a solution for robotic interception when utilizing an IPNG technique as Phase I and a PD-type Computed Torque (CT) method as Phase II of the hybrid interception method.

Chapter 4 presents a solution for robotic interception when utilizing an AIPNG technique as Phase I and a modified PD-type Computed Torque (CT) method as Phase II of the hybrid interception method for intercepting targets whose acceleration can be reliably estimated on-line.

The convergence issues of these techniques are addressed in Chapter 5.
Conclusions of the work presented in this dissertation are reported in Chapter 6 along with recommendations for future work.
CHAPTER TWO

PROBLEM DEFINITION

In this chapter the on-line interception of randomly-moving objects is discussed, first in terms of the general definition of robotic interception, and then in terms of the specific issues addressed in this thesis. For the former, visual servoing techniques and interception methods based on navigation guidance for fast-maneuvering targets are discussed. For the latter, developing a new hybrid interception scheme combining a navigation-guidance-based technique with a conventional tracking method is addressed.

As mentioned in Chapter 1, navigation guidance methods can provide faster interception than do conventional trackers. However, since navigation techniques are designed to bring the intercepter into a collision course with the target, rather than attempting to accomplish a smooth grasp, they must be modified for robotic interception. They must be complemented with a tracker for allowing the robot to match the target’s state at the last stage of the interception. Thus, our proposed interception method includes two phases. In Phase I, a navigation guidance technique is in control, then, at the final stage of the interception, namely Phase II, a tracking method is switched on for smooth grasping of the moving object, Figure 2.1.

![Figure 2.1: A schematic diagram of robotic interception via the proposed technique.](image-url)
2.1 **On-Line Interception of Randomly Moving Objects: Problem Formulation**

A randomly-moving object interception is a particular form of dynamic robot motion planning. In this problem, instead of moving toward a static goal position, the robot must intercept a moving object (target). In this context, the problem addressed in this thesis encompasses the issues of (i) reliable interception of moving objects, whose motion is not known a priori, and (ii) time-optimal interception.

The autonomous manufacturing environment considered primarily comprises a 6-DOF robot and a "conveyor" device transporting different parts. The motion of the conveyor is not known in advance, and random variations in its motion are expected. The state of the object as a function of time is identified through a vision system. Visual recognition and tracking of the motion of the object is assumed to be provided to the robot's motion-planning module, and thus, they are not addressed herein. The randomly-moving object is assumed to stay within the robot's workspace for a limited time. The current state of the robot is obtained from its controller.

Interception is defined herein as (positionally) approaching a moving object and its velocity matching:

\[
|x_T - x_R| \leq TOL_p \quad \text{and} \quad |\dot{x}_T - \dot{x}_R| \leq TOL_v, \tag{2.1}
\]

where \(x_T\) and \(x_R\) denote the Cartesian pose of the target and the robot, respectively, and \(TOL_p\) and \(TOL_v\) are user defined convergence thresholds.

The governing equation of motion for the robot is given as, [37]:

\[
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{2.2}
\]

where, \(q, \dot{q}, \ddot{q}, \tau \in \mathbb{R}^n\) are the joint position, velocity, acceleration vectors, and robot's actuators' torque, respectively; \(M(q) \in \mathbb{R}^{n \times n}\) is the inertia matrix; \(C(q, \dot{q}) \in \mathbb{R}^{n \times n}\) represents the matrix of centripetal and Coriolis terms; and, \(G(q) \in \mathbb{R}^n\) represents the gravitational terms. (No external forces are applied on the robot during its approach to the rendezvous position).

The robot is subject to the following joint constraints:

\[
q_{i,\min} \leq q_i \leq q_{i,\max}; \quad |\dot{q}_i| \leq \dot{q}_{i,\\text{limit}}; \quad |\ddot{q}_i| \leq \ddot{q}_{i,\\text{limit}}; \quad \text{and}, \quad |\tau_i| \leq \tau_{i,\\text{limit}}, \quad i = 1,2,3,...,n, \tag{2.3}
\]

where \(n\) denotes the degrees of freedom of the robotic manipulator.
2.2 **INTERCEPTION VIA VISUAL SERVOING**

The primary task in interception via visual servoing is to control the *pose* of the robot's end-effector, $x_R$, using visual information extracted from the image. The camera may be fixed, or mounted on the robot's end-effector.

In 1983 Sanderson and Weiss [38] introduced an important classification of visual servoing methods as *position-* and *image-based* servoing techniques. In position-based control, features are extracted from the images and used in conjunction with a geometric model of the target to determine the pose of the target with respect to the camera. In image-based servoing, the servoing is carried out on the basis of image features directly. However, the term visual servoing is widely used for any system that uses a machine vision to close a position-control loop.

Visually-guided machines have been built to emulate human skills [e.g., 39,40]. They have been also proposed for catching free-flying objects on earth or in space. Bukowski et al. [41] report on the use of a Puma 560 to catch a ball with an end-effector-mounted net. The robot is guided by a fixed stereo-vision camera system. Skofterland et al. [42] discuss the capture of a free-flying polyhedron in space also with a vision-guided robotic manipulator.

In all visual-servoing techniques used for moving-object interception, the nominal (desired) trajectory points for the manipulator motion are taken as the current state of the moving object, generated on-line based on the data obtained through the vision system. The on-line robot-motion planner normally determines the desired trajectory point for the robot controller (*subgoal* or *pseudo target*) one control sampling period ahead [e.g., 19], Figure 2.2. The dynamics of the robot is not explicitly needed to generate the control action when utilizing a *self-tuning* controller. However, for controllers based on the Computed Torque (CT) control method, the dynamics of the robot has to be known.

Two different sampling times normally exist in a visual-servoing system. One is due to image processing of the images of the moving object, $T_v$, and one related to the control sampling period, $T_C$. In most cases, the vision sampling time is much larger than the control sampling period ($m>>1$, where $m = T_v / T_C$). Since processing the image taken by a camera and extracting a needed object's feature is time consuming and introduces a delay into the system, a motion predictor is needed in order to predict target's motion one vision sampling time ahead to compensate for such inherent delays.
Figure 2.2: Conceptual block diagram of a typical interception scheme via visual servoing, [19].

The objective function to be minimized in a visual-servoing system is normally given as, [19-21];

$$\min_U \left\{ \sum_i (\epsilon_i^T Q \epsilon_i + \dot{\epsilon}_i^T \Gamma \dot{\epsilon}_i) \right\}, \quad (2.4)$$

where $\epsilon_i \in \mathbb{R}^{n \times 1}$ denotes the difference between the target and the robot’s pose, $Q$ and $\Gamma$ are symmetrically-positive-definite weighting matrices, and $U$ denotes the control effort, $U \in \mathbb{R}^{n \times 1}$. As can be seen from Equation (2.4), the difference in pose and velocity between the target and the robot is continuously minimized. In order to take the bounds on the robot’s actuator torques into account, the objective function (2.4) has to be modified in the following manner, [19,20];

$$\min_U \left\{ \sum_i \left( \epsilon_i^T Q \epsilon_i + \dot{\epsilon}_i^T \Gamma \dot{\epsilon}_i + U_i^T \gamma U_i \right) \right\}, \quad (2.5)$$

where $\gamma$ is a symmetrically-positive-definite weighting matrix. Although this modified objective function moderates the control effort needed to intercept a moving object, it does not guarantee that the robot’s torque limits will not be exceeded.
As can be seen from either Equation (2.4) or Equation (2.5), the objective function to be minimized does not include the time to intercept. Namely, no action is normally taken in current visual servoing techniques to optimize the time of intercept. Moreover, the gross motion of the manipulator is not distinguished from its fine motion. For instance, even when $\varepsilon$ is large (i.e., the robot is far from the target) the robot controller attempts to closely match the target’s velocity.

In response to the above drawback, several researchers proposed separate strategies for the gross-motion and the fine-motion planning stages, [21,22]. Lin et al. [22], for example, propose a heuristic approach for predicting an interception pose on-line and then estimating the manipulator velocity based on this prediction. Their objective function is given as follows:

$$\min_{T_f, e} \left\{ T_f - \rho \left( \dot{x}_k \cdot \dot{x}_T \right) + \sigma \varepsilon^T I_{nnn} \varepsilon \right\}, \tag{2.6}$$

where $T_f$ is a rough estimation of the travelling time from the current end-effector position to its intersection with the predicted moving object’s trajectory and $\rho$ and $\sigma$ are weighting factors. The objective function is minimized until the robot reaches the vicinity of the target, defined by a specified tolerance applied to $\varepsilon$ and $\dot{\varepsilon}$. At this point a fine motion planning technique is switched on for stable tracking. No tuning procedure was suggested in this work. The robot’s dynamics was also disregarded.

In conclusion, visual servoing techniques are not designed for time-optimal interception of moving objects. However, when an object is fast-maneuvering (and therefore its motion is not reliably long-term predictable) the time-optimality of the interception time is not a critical issue in an absolute sense. In this context, improving on the interception time of visual-servoing techniques is an on-going research area.

### 2.3 Visual Servoing via Computed Torque (CT) Method

One of the control techniques widely used in visual-servoing applications falls under the category of Computed Torque (CT) control method [e.g., 43-46]. This is a special application of feedback linearization of non-linear systems. By this technique, a global linearization, as opposed to a local linearization, of the system dynamics is obtained by means of a non-linear state feedback.
Since the CT technique depends on the inversion of the robot's dynamics, it is also referred to as inverse-dynamics control. Its feedback equation is derived by using the robot's dynamic model given as:

\[ M(q)\ddot{q} + N(q,\dot{q}) + \tau_d = \tau, \tag{2.7} \]

where \( N(q,\dot{q}) = C(q,\dot{q})\ddot{q} + G(q) \) and \( \tau_d \) denotes the disturbance torque. As noted, Equation (2.7) is a simplified version of Equation (2.2) with an additional term related to the disturbance torque, \( \tau_d \). The tracking error (i.e., the error between the desired pose and current pose of the robot) is given, in joint-space, as:

\[ e(t) = q_d(t) - q(t). \tag{2.8} \]

One can conclude that:

\[ \ddot{e} = \dddot{q}_d(t) - \dot{q}(t) = \ddot{q}_d + M^{-1}(N + \tau_d - \tau). \tag{2.9} \]

Let \( U = \ddot{q}_d + M^{-1}(N - \tau) \) and \( W = M^{-1}\tau_d \). One can then write:

\[ \frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}U + \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}W. \tag{2.10} \]

Equation (2.10) renders a linear system, where \( W \) denotes the disturbance in the system. Based on this linearization, the CT control law is written as:

\[ \tau = M(\ddot{q}_d - U) + N. \tag{2.11} \]

Variety of controllers have been suggested for the linear system given in Equation (2.10). PD controllers are the most common ones [43]. The linearized feedback control law is: \( U = -K_d\dot{e} - K_pe \). Thus, from (2.11),

\[ \tau = M(\ddot{q}_d + K_d\dot{e} + K_pe) + N. \tag{2.12} \]

By comparing Equation (2.12) with Equation (2.7), the closed-loop error equation is derived as:

\[ \ddot{e} + K_d\dot{e} + K_pe = W, \tag{2.13} \]

where,

\[ K_p = \text{diag}\{\omega_{n1}^2, \ldots, \omega_{nn}^2\}, \quad K_d = \text{diag}\{2\xi_1\omega_{n1}, \ldots, 2\xi_n\omega_{nn}\}, \tag{2.14} \]

and the characteristic equation of the closed-loop system is:

\[ H = \prod_{i=1}^{n} \left( s^2 + \omega_{ni}^{-2} s + 2\xi_i\omega_{ni} \right). \tag{2.15} \]
One should note that as long as the disturbance $W$ is bounded so is the error, $e(t)$. Since $M^{-1}$ is bounded, [46], the boundedness of $W$ implies boundedness of $\tau_d$, $W = M^{-1}\tau_d$.

In general, the error equation of the PD-type CT-method in the presence of torque disturbance and unmodeled dynamics is written as, [46]:

$$\ddot{e} + K_d \dot{e} + K_p e = M^{-1}\left(M - \bar{M}\right)(K_d \dot{e} + K_pe) + M^{-1}\tau_d + M^{-1}\left(M - \bar{M}\right)\dot{\bar{r}}_d + M^{-1}(N - \bar{N}),$$  \hspace{1cm} (2.16)

where $\bar{M}$ and $\bar{N}$ are the estimates of the robot's dynamics model. For $\tau_d = 0$, $\bar{M} = M$, and $\bar{N} = N$, the error equation given in (2.16) is simplified as:

$$\ddot{e} + K_d \dot{e} + K_pe = 0.$$ \hspace{1cm} (2.17)

Equation (2.17) is important to the moving-object interception problem. It can be used to estimate the interception time on-line (i.e., the time at which $e$ and $\dot{e}$ are within pre-specified tolerances). This interception-time estimation depends only on the initial condition of the errors, namely $e(t)$ and $\dot{e}(t)$, and does not depend on the target's motion type.

2.4 Navigation-Guidance-Based Interception

Navigation guidance techniques have been widely used for intercepting free-flying objects. In contrast to the conventional tracking methods based on visual servoing, they yield both time- and energy-optimal solution to the interception problem. Proportional navigation guidance (PNG) has been the most utilized navigation technique for the past 5 decades. PNG yields time-optimal solution for intercepting cruising targets [29]. Its augmented version, namely APNG, yields time-optimal solution for intercepting targets with non-constant acceleration [23]. However, in APNG, it is assumed that the target's acceleration is available to the intercept control system.

In navigation-guidance schemes, interception is normally defined as closing the distance between the interceptor and the target in a finite time. These schemes bring the interceptor into a collision course with the target as fast as possible. This is normally achieved by minimizing an objective function that is a combination of the weighted time-to-intercept and expended energy [28].

The optimality of the PNG law can be better understood by considering the interception scenario in which the target is cruising on a straight line. The interceptor control
system turns the interceptor’s velocity until it is on a collision course and locked on the target (i.e., the interceptor comes on to the collision course and keeps its heading constant). The quickness of the interceptor’s reaching the collision course depends on the weighting factors acting on the expended energy and time-to-intercept in the objective function. Without any weighting factor acting on the expended energy, the solution of the optimal intercept problem would be an infinite normal acceleration rotating the interceptor’s velocity vector on to the homing-triangle (i.e., collision-course) direction followed by a zero normal acceleration flight until intercept.

There are important issues to be addressed when utilizing a navigation-guidance-based technique for robotic interception. For instance, as was mentioned earlier, these techniques normally bring the interceptor on to a collision course with the target rather than attempting a smooth grasp. This suggests that at the final stage of the robotic interception via navigation guidance techniques, a conventional tracking method has to take over for stable and smooth tracking (i.e., matching target’s pose and velocity at intercept). Furthermore, in order to reflect a robot’s capabilities over missiles in regard to their maneuverability, the control command, which is normally in a form of acceleration command in navigation guidance techniques, can be upgraded for robots for faster interception. These issues will be addressed in detail in Chapters 3 and 4.

PNG and APNG laws are discussed below in Section 2.4.1. In Section 2.4.2, IPNG and Augmented IPNG (AIPNG) are addressed.

### 2.4.1 PNG and APNG Techniques

Due to its simplicity of onboard implementation, PNG has attracted a considerable amount of interest in the missile-guidance-related literature since its inception in 1940s, [34]. Navigational-guidance schemes can be categorized into two major classes: Interceptor velocity referenced class, and the Line-Of-Sight (LOS) referenced class. The velocity-referenced schemes, (e.g., PNG) are more practically implementable in aerodynamically-controlled environments with no requirement on forward/ backward acceleration, [34].

The conventional PNG law generates an acceleration command, \( a_{PNG} \), normal to the interceptor’s velocity vector according to:

\[
a_{PNG} = \lambda \ V_i \times \hat{\theta}_{LOS},
\]  

(2.18)
where \( \lambda \) is the navigation gain, \( V_I \) denotes the interceptor's velocity vector, and \( \dot{\theta}_{LOS} \) denotes the angular rate of the LOS, Figure 2.3. The interceptor control system should be designed so as to achieve this acceleration command accurately and quickly. The LOS angular rate, \( \dot{\theta}_{LOS} \), is measured by an on-board seeker, which tracks the target.

![Image of interception via PNG](image)

**Figure 2.3:** Interception via PNG.

It has been shown that, the acceleration command given in Equation (2.18) yields a solution to the interception problem where the following objective function is minimized, [e.g., 29]:

\[
J = k T_f + \frac{1}{2} \int_0^{T_f} \omega_f^2(t) dt ,
\]

where \( T_f \) is the interception time, \( k \) is a weighting factor, and \( \omega_f \) denotes the flight direction turning rate, \( a_f = \omega_f \times V_f \). Guelman et al. [47] show that the kinematic interception is guaranteed when using PNG for cruising targets.

The stability of the PNG law, when tracking maneuvering targets, has also been recently discussed in the literature. Ha et al. [48] show that when the target’s acceleration is a piecewise continuous, unknown function of \( t \), but is upper bounded with a known constant, \( \alpha \), as:

\[
|a_f(t)| \leq \alpha ,
\]

then, the PNG law given in Equation (2.18) always intercepts the target under the following conditions:
The navigation gain is chosen so that:

\[ p \sin \sigma_r(0) - \sin \sigma_i(0) \leq \beta \text{ for some constant } \beta \in [0, 1 - \rho] ; \]

(2) \ \ |\sigma_r(0)| < \frac{\pi}{2} ; \text{ and}

(3) The navigation gain is chosen so that:

\[
\lambda > 1 + \frac{p + \alpha r(0) / (\beta |V_i|^2)}{\sqrt{1 - |p + \beta|^2}},
\]

where \( p \) is defined as \( p = \frac{|V_r|}{|V_i|} < 1 \), and \( \sigma_r \) and \( \sigma_i \) are the angles between the target’s and the interceptor’s velocity and the LOS, respectively. By utilizing Lyaponuv method, they also show that there exists a finite time, \( T_f \), such that \( r(T_f) = 0 \), \( r(t) \leq r(0) \), and \( T_f < \frac{r(0)}{|V_i| \sqrt{1 - p^2 + \beta^2}} \), under the conditions mentioned above. It should be noted that, if Condition 2 is not satisfied, then, \( r(t) \leq r(0) \) is not guaranteed. The following interception conditions must be satisfied when utilizing the Lyaponuv method:

- The interceptor and the target are considered as geometric points,
- Compared with the overall guidance loop, the interceptor and seeker dynamics are fast enough to be neglected, and
- \( |V_i| > |V_r| \).

Although the PNG always intercepts maneuvering targets, it does not yield an optimal solution. Many researchers have reported on the Augmented PNG (APNG) law in which case time- and energy-optimal solution has been achieved [e.g., 23, 31]. In APNG, the target’s acceleration is utilized as:

\[
a_{APNG} = \lambda |V_i| \hat{\theta}_{LOS} + \frac{\lambda}{2} |a_T|.
\]  \hspace{1cm} (2.21)

The above acceleration command is applied in a direction defined by \( V_i \times \hat{\theta}_{LOS} \).

### 2.4.2 IPNG and AIPNG Techniques

Ideal Proportional Navigation Guidance (IPNG) is an improvement over the conventional PNG with respect to its mathematical tractability, [33, 34]. In IPNG, the
command acceleration is applied in a direction normal to the relative velocity between the interceptor and the target, rather than normal to the interceptor’s velocity as in the PNG law, and its magnitude is proportional to the product of LOS angular rate and the relative velocity. 

IPNG tries to turn the relative velocity on to the direction of LOS with utmost effort:

\[ a_{IPNG} = \lambda (V_T - V_r) \times \dot{\theta}_{LOS}. \]  

(2.22)

Although the IPNG technique seems to be less viable for being implemented in an aerodynamically-controlled environment than the PNG, some of its characteristics, such as being less sensitive to the interception initial conditions (i.e., better mathematical tractability) and having more relaxed capture criterion make it suitable for robotic interception. Another advantage of the IPNG for robotic interception is its capability of bringing the LOS angular rate to zero significantly faster than that in the conventional PNG technique [33, 34]. This is achievable by choosing high navigation gains. The higher the navigation gain is the faster the LOS angular rate approaches zero [33, 34]. Yuan et al. [33] show that interception can always be achieved successfully when \( \lambda > 1 \), and LOS angular rate approaches zero when \( \lambda > 2 \), for cruising targets.

When the interceptor has an absolute speed superiority over the target, then, PNG and IPNG become similar [34]. The performance of PNG depends on the value of \( \rho = \|V_T\|/\|V_r\| \). If one lets \( \rho \rightarrow 0 \), then, the PNG formulae converge to those of the IPNG: 

\[ \lim_{\rho \rightarrow 0} PNG(\rho) = IPNG. \]  

Stability of the IPNG is assured if \( \rho \rightarrow 0 \). The condition under which the IPNG is stable when tracking maneuvering targets can be also derived directly from those discussed in Section 2.4.1. However, in those conditions \( \rho \) has to be chosen as a small value.

It can be shown that when LOS angular rate approaches zero, then, the relative velocity between the target and the robot has to lie on the LOS [34]. This suggests that after this condition has been reached, the dimensionality of the interception is reduced to one. This characteristic of the IPNG, especially, makes it attractive for robotic interception. It will be shown in Chapter 4 that this reduction in dimensionality can improve the interception time. This reduction in dimensionality is valid only for cruising targets when utilizing the IPNG.

IPNG has been augmented by the target’s acceleration. This Augmented IPNG (AIPNG) guarantees occurrence of the reduction in dimensionality regardless of target’s maneuver type. This technique, when utilized for maneuvering targets, is analogous to the
IPNG for cruising targets, therefore, its stability is assured. Capability of the AIPNG combined with a tracking method in reducing the dimensionality of the system and improving the interception time will be also discussed in detail in Chapter 4.

2.5 OVERVIEW OF THE PROPOSED SOLUTION

A navigation-guidance-based technique is proposed in this thesis for robotic interception. Since such techniques normally bring the interceptor into a collision course with the target, rather than a smooth grasp, the control of the robot must be switched to a tracking method before intercept occurs. A methodology for on-line search of the optimal switching point (i.e., the point that yields minimum interception time) must be developed. By utilizing this scheme, minimum interception time is achievable.

Figure 2.4 shows a schematic diagram of the robot trajectory towards the interception point. The typical variation of the interception time versus switching time (the time at which the control of the robot is switched from the navigation guidance to a conventional tracking method) is also shown in this figure.

Figure 2.4: Optimal interception via our proposed hybrid interception technique.

The use of IPNG is proposed herein for robotic interception when an accurate estimation of target’s acceleration is not available. In this case, the robot trajectory planning includes two phases: Phase I, during which the robot is under the control of the IPNG technique, and Phase II, during which a tracking method, namely a PD-type CT-control
method, takes over, bringing the robot to an interception with the target matching its pose and velocity.

The overall interception time, $t_{\text{int}}$, is calculated as:

$$t_{\text{int}} = t_{\text{IPNG}} + t_{\text{PD-CT}} ,$$

(2.23)

where $t_{\text{IPNG}}$ denotes the switching time, and $t_{\text{PD-CT}}$ denotes the time during which the robot is under the control of a CT-method. The interception time can be on-line estimated as follows:

$$\tilde{t}_{\text{int}} = t_{\text{IPNG}} + \tilde{t}_{\text{PD-CT}} .$$

(2.24)

It will be shown that (1) $\tilde{t}_{\text{PD-CT}}$ can be calculated accurately on-line, and (2) it does not depend on the target’s motion type. By utilizing Equation (2.24), the on-line estimation of the minimum interception time is achievable.

As an alternative method, AIPNG is proposed to be utilized during Phase I of our hybrid interception scheme. In this case, the robot trajectory planning includes two phases: Phase I, during which the robot is under the AIPNG control, Phase II, during which the robot closes its distance with the target, moving with its maximum permissible deceleration, then, at a certain time a PD-type CT-method takes over for final-stage stable tracking. Phase II has been developed based on the fact that the AIPNG technique reduces the dimensionality of the interception scheme to one. One can then write:

$$t_{\text{int}} = t_{\text{phase I}} + t_{\text{phase II}} .$$

(2.25)

In this case, there is no need for on-line interception time estimation. However, the important issue in implementing this technique, herein, is to develop a methodology for estimating $t_{\text{phase II}}$ based on the maximum executable permissible deceleration of the robotic manipulator.

In order to reflect the greater mobility of robotic manipulators over missiles, both navigation guidance techniques have to be modified. In an aerodynamically-controlled environment, the interceptor has only a velocity-turning maneuvering capability, in which case the magnitude of the interceptor’s velocity is kept constant and its direction changes. However, robots can maneuver in any direction at any time, as long as the control-effort limits, namely joint torques, of the robot are not exceeded. This suggests that the navigation-guidance-based techniques, utilized for robotic interception for faster interception, can be improved, as discussed in Chapters 3 and 4 for IPNG and AIPNG, respectively.
2.6 SUMMARY

On-line interception of randomly-moving objects is developed. Current research trend utilizing visual servoing techniques is discussed in detail. As a particular case, the PD-type CT-method is explained. The navigation-guidance-based techniques, namely PNG, APNG, IPNG, and AIPNG, are discussed as alternatives to conventional tracking methods. The navigation guidance techniques normally yield faster interception times than those by the conventional tracking methods. However, they have to be complemented by a tracking method at the final stage of the interception for stable tracking, and matching the target’s pose and velocity. In this context, the hybrid interception scheme proposed in this thesis, as a solution to the fast and reliable interception of fast-maneuvering objects, is discussed. In this new hybrid interception method, the robot is initially under the control of a navigation guidance technique, then at some proper moment, namely optimal switching time, the control of the robot is switched to a conventional tracking method based on a PD-type CT-method.
CHAPTER THREE

ROBOTIC INTERCEPTION VIA IDEAL PROPORTIONAL NAVIGATION GUIDANCE

IPNG is developed in this chapter for the robotic interception of fast-maneuvering objects. First, the advantages of the IPNG over other navigation guidance techniques is addressed in term of robotic interception. The modification of the IPNG for smooth grasp of the moving object is discussed next. In this context, a tracking method based on a CT-control technique is proposed for the final stage of the robot’s trajectory towards the intercept point.

During the first phase of the interception, the IPNG control input (i.e., IPNG acceleration command) can be upgraded for fast robotic interception. In general, a navigation-based interception scheme renders an acceleration command based on the current states of the target and the interceptor. This acceleration command produces a slow motion if the interceptor is initially stationary. By introducing an effective motion-initialization technique, the overall interception time can be significantly shortened.

Furthermore, unlike a missile, the end-effector of a robotic arm can maneuver in any direction. By taking the robot’s dynamics into account, the acceleration command found through the navigation-based interception scheme can be augmented during the robot motion in a way that the manipulator moves toward the interception point with a higher acceleration.

3.1 ROBOTIC INTERCEPTION VIA AN IDEAL PROPORTIONAL NAVIGATION GUIDANCE

In this section, the Ideal Proportional Navigation Guidance (IPNG) will first be reviewed and compared with other PNG-based navigation guidance techniques. Thereafter, the potential shortcomings of an un-modified IPNG-based robotic interception scheme will be addressed in detail. Methods employed in this Chapter to address these shortcomings for achieving a faster interception will also be discussed.
3.1.1 Ideal Proportional Navigation Guidance

As addressed in Chapter 2, Proportional Navigation Guidance (PNG) has attracted a considerable amount of interest in the missile-guidance field, since its inception in the 1940s, due to its simplicity of onboard implementation. In all forms of PNG-based interception schemes, a command acceleration, \( a_c \), which is calculated based on the current states of the target and the interceptor, can be expressed in the following general form:

\[
a_c = \lambda L \times \dot{\theta}_{LOS} \tag{3.1}
\]

where \( L \) is the normal direction to the command acceleration, \( \dot{\theta}_{LOS} \) is the LOS angular velocity, and \( \lambda \) is the navigation gain.

Let us first consider a planar relative motion described by the polar coordinates \((r, \theta)\) with the coordinate frame's origin being located on the interceptor, Figure 3.1. In this figure, \( r \) is the vector of the position-difference between the interceptor and the target, \( \theta_{LOS} \) is the angle of LOS with respect to a reference line, \((e_r, e_\theta)\) are unit vectors, \( V_I \) is the interceptor's velocity, and \( V_T \) is the target's velocity. The different \( L \) directions used by the three well-known guidance laws are defined as, [34]:

1) Ideal Proportional Navigation Guidance (IPNG): \( L = \dot{r}e_r + r\dot{\theta}e_\theta \equiv \dot{r} \) \( (3.2a) \)
2) Proportional Navigation Guidance (PNG): \( L = -V_I \) (interceptor velocity) \( (3.2b) \)
3) Optimal Proportional Navigation Guidance (OPNG): \( L = U_{opt}(\theta)e_r + V_{opt}(\theta)e_\theta \). \( (3.2c) \)

where \( U_{opt} \) and \( V_{opt} \) are optimal velocities of the interceptor in the radial and tangential directions, respectively (the subscript LOS is dropped for sake of simplicity).\(^1\)

In [34], it was reported that toward the end of interception, the IPNG and PNG yield solutions similar to those given by the OPNG. Although, the IPNG may be of less practical

\(^1\) OPNG is derived based on optimization of the following objective function:

\[
l = T_f + \gamma \int_0^{T_f} |a_I|^2 \, dt,
\]

where \( T_f \) denotes the interception time, \( \gamma \) is a weighting factor, and \( a_I \) is interceptor's acceleration. This objective function takes into account the time of intercept and the energy expended during the interception.
use in missile guidance compared to PNG, its mathematical tractability and robustness to the initial conditions of the interceptor makes it especially suitable for robotic interception.

In general, the relative motion between an interceptor guided by (3.2a) and a non-maneuvering target (i.e., $a_T = 0$) is described by the following non-linear equations:

\begin{align*}
\ddot{r} - r\dot{\theta}^2 &= -\lambda r\dot{\theta}^2, \\
\dot{r}\dot{\theta} + 2\dot{r}\dot{\theta} &= \lambda r\dot{\theta}.
\end{align*}

With a change of the independent variable from $t$ to $\theta$, (3.3) can be rewritten as follows:

\begin{align*}
V_r' - V_\theta &= -\lambda V_\theta, \\
V_\theta' + V_r &= \lambda V_r,
\end{align*}

where primes denote the differentiation with respect to $\theta$, $V_r = \dot{r}e_r$, and $V_\theta = \dot{r}\dot{\theta}e_\theta$. By solving Equations (3.4a) and (3.4b) simultaneously, one obtains:

\begin{align*}
V_r &= -\sin(\alpha\theta + \beta), \\
V_\theta &= \cos(\alpha\theta + \beta),
\end{align*}

where $\alpha$ and $\beta$ are constants related to the initial condition of the interception.

![Diagram](image)

**Figure 3.1:** Interception via a PNG-based technique in a plane.

Yang et al. [34] showed that PNG given by (3.2b) yields the following results:

\begin{align*}
V_r &= -\frac{1}{\eta-1}\sin(\theta - \beta) - \frac{\eta}{\eta-1}\sin(\alpha\theta + \beta), \\
V_\theta &= \frac{1}{\eta-1}\cos(\theta - \beta) + \frac{\eta}{\eta-1}\cos(\alpha\theta + \beta),
\end{align*}
where \( \eta = \frac{|V_t|}{|V_r|} \). By comparing (3.5) and (3.6), one can see that when the interceptor has an absolute speed superiority over the target (i.e., \( \eta > 1 \)), then PNG and IPNG become indistinguishable. Yang et al. [34] also showed that for \( \eta >> 1 \), IPNG and PNG become similar to OPNG toward the end of interception (i.e., \( \lim_{\theta_{\text{LOS}} \to 0} \text{OPNG} = \text{PNG} = \text{IPNG} \)). However, one should note that OPNG is not practically implementable if the target’s motion is not known a priori.

Three issues are usually addressed in navigational guidance techniques: (1) capture criterion, (2) accumulative velocity increment\(^2\) of the interceptor, and (3) interception time. The IPNG guarantees interception for \( \lambda > 1 \), [33], while the PNG yields the same results for \( \lambda > 2 \), [34]. The IPNG yields a shorter interception time than that for the PNG, however, for \( \eta >> 1 \) the difference is indistinguishable. This is achieved at the cost of yielding a higher accumulative velocity increment. Since the latter is not a critical issue for a robotic manipulator, an IPNG technique seems more suitable for robotic interception.

Another advantage of the IPNG, however, is that for \( \lambda > 2 \), the LOS angular rate approaches zero before intercept occurs, [33]. The higher the \( \lambda \) is, the faster this happens. By bringing the angular rate of the LOS to zero, the dimensionality of the interception is reduced to one. This reduction in dimensionality, especially, makes the IPNG technique very attractive for robotic interception. This issue will be discussed in further detail in Chapter 4.

The acceleration command of the IPNG is computed as:

\[
a_{\text{IPNG}} = \lambda \mathbf{r} \times \dot{\theta}_{\text{LOS}},
\]

where \( \mathbf{r} \) represents the position-difference vector between the target and the robot. In Equation (3.7), \( \dot{\theta}_{\text{LOS}} \) can also be expressed as a function of \( \mathbf{r} \) and \( \dot{\mathbf{r}} \) as:

\[
\dot{\theta}_{\text{LOS}} = \left\{ \frac{\mathbf{r} \times \dot{\mathbf{r}}}{|\mathbf{r}|^2} \right\}.
\]

By substituting Equation (3.8) into Equation (3.7), one obtains:

---

\(^2\) The cumulative velocity increment is calculated as:

\[
\Delta V = \int_0^T |a| \, dt.
\]
\[ a_{IPNG} = \frac{\lambda}{|r|^2} \{ \hat{r} \times (r \times \dot{r}) \}. \]  

(3.9)

Since \( \dot{r} \times (r \times \dot{r}) = r(\dot{r} \times \dot{r}) - \dot{r}(r \times \dot{r}) \), Equation (3.9) can be rewritten as:

\[ a_{IPNG} = K_p(t, \lambda) r + K_d(t, \lambda) \dot{r}, \]

(3.10)

where \( t \) denotes time and the coefficients \( K_p \) and \( K_d \) are defined as follows:

\[ K_p(t, \lambda) = \frac{\lambda}{|r|^2} \{ |r|^2 \}, \quad K_d(t, \lambda) = \frac{-\lambda (r \cdot \dot{r})}{|r|^2}. \]

(3.11)

Equation (3.11) represents a geometric interception scheme similar to a PD-tracker with time-variant gains. \( K_d \) can be rewritten as follows:

\[ K_d(t, \lambda) = \frac{-\lambda |\dot{r}| \cos(r, \dot{r})}{|r|}. \]

(3.12)

Ha et al. [48] showed that when \( \eta >> 1 \) and \( \dot{r}(0) < 0 \), then \( r(t) < 0 \) for \( t \in [0, T_f] \), where \( T_f \) denotes the interception time. This suggests that \( \cos(r, \dot{r}) < 0 \), therefore, \( K_d(t, \lambda) > 0 \).

Since \( a_{IPNG} \) is normal to the relative velocity between the target and the interceptor, \( \dot{r} \), \( |\dot{r}| = \text{const} \). Subsequently, one can conclude:

\[ K_d(t, \lambda) = \frac{C_1}{|r|}, \quad \text{and} \quad K_p(t, \lambda) = \frac{C_2}{|r|^2}, \]

(3.13)

where \( C_2 \) is a constant, and \( C_i \) is a bounded coefficient, \( |C_i| \leq \lambda |\dot{r}| \). Equation (3.13) suggests that: (1) \( K_p \) and \( K_d \) are positive coefficients and (2) they are monotonically increasing over time when \( r \) approaches zero. This supports the idea that in navigational guidance techniques, the interceptor’s velocity increases toward the end of the intercept while closing its distance with the target. This stipulates the advantage of a navigation-guidance-based tracking method over other tracking methods in which the interceptor’s velocity decreases even when the interceptor is far from the target.

In [33], it was reported that during the interception period, \( \dot{\theta}_{LOS} \) will approach infinity when \( \lambda < 2 \), and will approach zero when \( \lambda > 2 \). In other words, no matter what the initial condition of \( \dot{r} \) is, interception can always be achieved successfully when \( \lambda > 2 \). One important property of the IPNG is that the interception solution for a slow-maneuvering target yields a
constant relative velocity between the interceptor and the target (while its direction is turned toward the target during the interception period). This behavior of the IPNG technique will be fully utilized when combining this technique with a conventional tracking method for robotic interception.

3.1.2 Robot Dynamic Model

A rigid robotic manipulator with $n$ degrees of freedom in joint space is governed by Equation (2.7). Mappings between the joint coordinates $q$ and the robot end-effector coordinates $X_r$ are given as:

$$X_r = P(q),$$

$$\dot{X}_r = J(q)\dot{q},$$

$$\ddot{X}_r = \dot{J}(q)\dot{q} + J(q)\ddot{q},$$

where $P(q)$ represents the forward kinematic relation for the end-effector and $J(q)$ is the end-effector Jacobian matrix. By substituting Equations (3.14a-3.14c) into Equation (2.7), one can obtain the robot’s dynamic equation in task space:

$$MJ^{-1}\{\dddot{X}_r - jJ^{-1}\dddot{X}_r\} + CJ^{-1}\dddot{X}_r + g = T.$$  (3.15)

The arguments of the $M$, $C$, and $g$ have been dropped for simplicity. The $\dddot{X}_r$ in Equation (3.15) can be replaced with $a_{\text{IPNG}}$. Re-arranging the remaining terms, one can obtain the dynamic equation of motion, when utilizing the IPNG technique, as:

$$MJ^{-1}a_{\text{IPNG}} + \{C - MJ^{-1}J\}J^{-1}\dddot{X}_r + g = T.$$  (3.16)

In Equation (3.16), the torque vector, $T$, is subject to dynamic constraints as:

$$|T_i| \leq |T_{i\text{max}}|, \quad i = 1, 2, ..., n$$  (3.17)

where $T_{i\text{max}}$ is the maximum torque available in the $i$th actuator. The relationship between the acceleration vector, $a_{\text{IPNG}}$, and the torque needed to produce this acceleration, $T$, is linear for the current robot configuration.
3.1.3 Initialization of Robot Motion

From Equation (3.1), it can be seen that when the interceptor is initially at rest \((V_I = 0)\), the acceleration command of IPNG would be perpendicular to the target's velocity. This may render an initial interceptor's velocity with a non-optimal direction away from the interception point. In order to achieve a better interception time, the robot motion can be initiated using the maximum permissible acceleration of the robot toward the interception point. However, since the interception point is not known a priori and also since the maximum permissible velocity of the robot depends on the robot's configuration, a more realistic way of robot-motion initialization would be sending the robot toward the currently known location of the target, Figure 3.2. This path can be defined in parametric form in task space as:

\[
\begin{align}
X_r &= f(s) \quad (3.18a) \\
\dot{X}_r &= f'(s)\dot{s} \quad (3.18b) \\
\ddot{X}_r &= f''(s)\dot{s}^2, \quad (3.18c)
\end{align}
\]

where \(f\) is a vectorial function and \(s\) is the path length (i.e., a scalar parameter).

![Figure 3.2: Robot-motion initialization.](image)
In Equations (3.18a-3.18c), \( f'(s) = \frac{\partial f(s)}{\partial s} \) is a unit vector tangent to the path, and \( f''(s) = \frac{\partial^2 f(s)}{\partial s^2} \) is a vector of magnitude \( 1/\kappa \) normal to the path, \( \kappa \) being the radius of curvature of the path. By substituting Equations (3.18a-3.18c) into Equation (3.15), the following relation is obtained:

\[
T = a_1 \ddot{s} + a_2 \dot{s}^2 + a_3 \dot{s} + a_4,
\]

where:

\[
a_1 = MJ^{-1} f'(s), \\
a_2 = MJ^{-1} f''(s), \\
a_3 = CJ^{-1} f'(s) - MJ^{-1} J^{-1} f'(s), \\
a_4 = g.
\]

Equation (3.19) can also be expressed as:

\[
\ddot{s}_i = \frac{a_1 \ddot{s}^2 + a_2 \dot{s}^2 + a_3 \dot{s} + a_4}{a_1}, \quad i = 1, 2, ..., n.
\]

The upper and lower bounds for each \( \ddot{s}_i \) are obtained by substituting the upper and lower bounds of \( T \), from Equation (3.17):

\[
\ddot{s}_{\text{max}} = \min_i \left\{ \frac{T_{\text{max}} - a_2 \dot{s}_i^2 - a_3 \dot{s}_i + a_4}{a_1} \right\}, \quad i = 1, 2, ..., n
\]

\[
\ddot{s}_{\text{min}} = \max_i \left\{ \frac{T_{\text{min}} - a_2 \dot{s}_i^2 - a_3 \dot{s}_i + a_4}{a_1} \right\}, \quad i = 1, 2, ..., n.
\]

It should be noted that \( \ddot{s} \) must lie in the range:

\[
\ddot{s}_{\text{min}} < \ddot{s} < \ddot{s}_{\text{max}}.
\]

Otherwise, the maximum permissible velocity of the robot is considered to be violated, and the robot has to depart from its current path, [49].

By replacing \( \ddot{s} \) in Equation (3.18c) with \( \ddot{s}_{\text{max}} \) defined in Equation (3.22a), one can obtain the maximum permissible acceleration of the robot end-effector along the path toward
the target. The torque needed to produce this acceleration can also be computed by utilizing $\ddot{s}_{\text{max}}$ in Equation (3.19). The robot is, therefore, accelerated from its current state using this acceleration for a period of time. In the proposed technique, this robot-motion initialization stage is terminated when either the robot reaches its maximum permissible velocity, namely the point at which $\ddot{s}_{\text{min}} = \ddot{s}_{\text{max}}$, or a pre-defined time threshold has already been reached, Figure 3.3.

3.1.4 Modifying the IPNG Acceleration Command

Two issues will be discussed in this subsection: (i) limiting the $a_{\text{IPNG}}$, when joint torques are violated; and, (ii) upgrading the $a_{\text{IPNG}}$, to achieve faster interception.

(i) Limiting the $a_{\text{IPNG}}$: The control-torque computed in Equation (3.16) might violate the dynamic constraints of the robot given in Equation (3.17). In this case, the acceleration command found via the IPNG technique must be modified. Since the relationship between the robot's tip acceleration and the actuator torques is linear in task space, [49], the new acceleration, and subsequently, the actuator torques to produce this acceleration, can be found through a simple linear search. In this search, the direction of the acceleration vector is maintained constant and only its magnitude is changed:

$$a_c = Ka_{\text{IPNG}},$$

where $K$ is a constant that is continuously updated.

For each actuator, a set of permissible values for the coefficient $K$ is defined as:

$$S_i = \{K | T_i < |T_{\text{max}}|\}, \quad i = 1, 2, ..., n,$$  \hspace{1cm} (3.25)

where $T_i$ denotes the torque needed for producing the acceleration given in Equation (3.24) for the ith actuator. The minimum and maximum of the intersection of all the sets computed in Equation (3.25) yield the corresponding minimum and maximum permissible values of $K$ that satisfy the dynamic constraints:

$$K_{\text{min}} = \min \left( \bigcap_{i=1}^{n} S_i \right),$$  \hspace{1cm} (3.26a)

$$K_{\text{max}} = \max \left( \bigcap_{i=1}^{n} S_i \right).$$  \hspace{1cm} (3.26b)

$K_{\text{max}}$ is selected as the desired coefficient to be used in Equation (3.24), Figure 3.4.
Figure 3.3: Algorithm for robot motion initialization
(ii) **Upgrading the \(a_{IPNG}\):** IPNG renders an acceleration command which continuously turns the interceptor toward the interception point while maintaining it at an almost-constant speed. Since robots, unlike missiles, are capable of performing maneuvers in any desired direction, the acceleration command of the IPNG can be modified. This modification is carried out by boosting the \(a_{IPNG}\), while maintaining the torques within a certain percentage of the maximum available level as:

\[
|T_i| < \alpha |T_{i \text{max}}|, \quad i = 1, 2, ..., n
\]

where \(\alpha\) is a pre-defined constant between 0 and 1.

The LOS is selected as the direction in which the added acceleration component is computed. This increases the closing-velocity (i.e., the time rate of the change of the instantaneous distance between the target and the robot), making the interceptor move toward the interception point faster, Figure 3.5. This figure shows the linear mapping between the robot's joint torques and its tip acceleration in 2D space. The square zone representing the torque limits is mapped to a parallelogram whose vertices represent the maximum permissible acceleration/ deceleration of the robot. In 3D, this mapping is from a cube in the torque space to a parallelepiped in the robot's tip acceleration space, [50,51]. One should note that the relative velocity between the target and the robot lies on the LOS direction when \(\dot{\theta}_{LOS}\) approaches zero. This can be verified through Equation (3.8).
Figure 3.5: Boosting the IPNG acceleration command.

The resultant acceleration produced by combining the two acceleration vectors, as shown in Figure 3.5, must lie within the pre-defined robot’s tip allowable-acceleration region as:

\[ a_c = a_{IPNG} + \beta \frac{r}{|r|}, \]

(3.28)

where \( \beta \) is a constant:

\[ \beta = \max \left( \bigcap_{i=1}^{n} H_i \right), \]

(3.29)

and, \( H_i \) is defined as:

\[ H_i = \left\{ \beta |T_i| < \alpha |T_{i,max}| \right\}, \quad i = 1, 2, \ldots, n. \]

(3.30)

Figure 3.6 represents the overall procedure for modifying \( a_{IPNG} \), when needed, according to (i) and (ii) above.

3.2 A HYBRID METHOD FOR ROBOTIC INTERCEPTION

A hybrid IPNG-tracking method, which quickly brings the interceptor to a smooth grasp with the target, is discussed in this section. In this method, the robot is under the IPNG control first, then, at a proper time a tracking method takes over bringing the robot to the interception. Below, we will first discuss robotic interception via a traditional tracking method. Following that, a methodology for finding a proper time at which this tracking
method takes over the control of the robot motion from the IPNG. Figure 3.7, (hereinafter. called the switching point), will be addressed. It has been shown that in a friendly environment and under ideal conditions (i.e., no noise in moving object's position readings), the curve representing the interception time versus the switching time, Figure 3.7, is a single-minimum curve [17]. For cases when noise is present in target's position measurements, however, the curve might have several local minima. In this case, the tracking method takes control of the robot at the first minimum (Chapter 5).

![Figure 3.6: The algorithm for modifying the IPNG acceleration command.](image)

![Figure 3.7: Utilization of the proposed hybrid interception scheme.](image)

### 3.2.1 Robotic Interception via a Traditional Tracking Method

The literature on trajectory tracking strategies for control of robotic manipulators is extensive. The proposed techniques mainly present solutions to computing the torques needed to achieve accurate tracking. A commonly used method is the Computed-Torque (CT)
scheme, [e.g., 43-46]. In this controller, the torque is computed by the inverse dynamic equation of the manipulator.

Since the IPNG is utilized in robot’s task-space, as relevant to the discussion of the CT scheme, the robot’s dynamic equation in task space is derived as:

\[ \mathbf{M}_x \ddot{\mathbf{x}}_r + \mathbf{C}_x \dot{\mathbf{x}}_r + \mathbf{g}_x = \mathbf{F}, \]  

where

\[ \mathbf{M}_x = J^{-T} M J^{-1}, \]  
\[ \mathbf{C}_x = J^{-T} \left[ \mathbf{C} - M J^{-1} \dot{\mathbf{J}} \right] J^{-1}, \]  
\[ \mathbf{g}_x = J^{-T} \mathbf{g}, \]  
\[ \mathbf{F} = J^{-T} \mathbf{T}. \]  

The vector \( \mathbf{F} \) in Equation (3.32d) is the equivalent force/torque vector acting on the end-effector of the robot. The CT control scheme, using a PD-type controller, can now be expressed in task-space as follows:

\[ \mathbf{F} = \tilde{\mathbf{M}}_x \left[ \mathbf{K}_p (\mathbf{X}_d - \mathbf{X}_r) + \mathbf{K}_d (\dot{\mathbf{X}}_d - \dot{\mathbf{X}}_r) + \ddot{\mathbf{X}}_d \right] + \tilde{\mathbf{C}}_x \dot{\mathbf{x}}_r + \tilde{\mathbf{g}}_x, \]  

where (\( \cdot \)) indicates that the estimated values of the dynamic parameters were used in the computations; the subscript \( d \) denotes the desired pose of the robot’s tip, which corresponds to the target’s pose; and, the coefficients \( \mathbf{K}_p \) and \( \mathbf{K}_d \) are the proportional and derivative gain matrices, respectively. Arimoto [52] proves that this control technique yields a global asymptotic stable tracking solution.

The basic concept employed by the CT scheme is to achieve dynamic decoupling of all joints using nonlinear feedback. The closed-loop system is obtained utilizing Equation (3.31) and the applied control force according to Equation (3.33):

\[ \dot{\mathbf{x}}_r = \dot{\mathbf{U}}_i - \left[ \tilde{\mathbf{M}}_x J^{-1} \right] \left[ \mathbf{M}_x - \tilde{\mathbf{M}}_x \right] \ddot{\mathbf{x}}_r + \left( \mathbf{C}_x - \tilde{\mathbf{C}}_x \right) \dot{\mathbf{x}}_r + \left( \mathbf{g}_x - \tilde{\mathbf{g}}_x \right), \]  

where \( \dot{\mathbf{U}}_i \) is given as:

\[ \dot{\mathbf{U}}_i = \left[ \mathbf{K}_p (\mathbf{X}_d - \mathbf{X}_r) + \mathbf{K}_d (\dot{\mathbf{X}}_d - \dot{\mathbf{X}}_r) + \ddot{\mathbf{X}}_d \right]. \]

If the robot’s model is known exactly, that is, \( \tilde{\mathbf{M}}_x = \mathbf{M}_x \), \( \tilde{\mathbf{C}}_x = \mathbf{C}_x \) and \( \tilde{\mathbf{g}}_x = \mathbf{g}_x \), the decoupled closed-loop system is expressed as:

\[ (\mathbf{X}_d - \dot{\mathbf{X}}_r) + \mathbf{K}_d (\dot{\mathbf{X}}_d - \dot{\mathbf{X}}_r) + \mathbf{K}_p (\mathbf{X}_d - \mathbf{X}_r) = 0. \]
Equation (3.36) can also be written as:
\[ \ddot{r} + K_d \dot{r} + K_p r = 0, \quad i = 1,2,...,n, \] (3.37)
where \( r = X_d - X_r \). Equation (3.37) presents a set of well known second-order differential equations with constant coefficients. The Laplace transform for Equation (3.37) is given as:
\[ s^2 + K_d s + K_p = 0, \quad i = 1,2,...,n, \] (3.38)
where \( K_{di} \) and \( K_{pi} \) are the velocity and position gains for the \( i \)th joint. For critical damping of the system one must have:
\[ K_{di} = 2\sqrt{K_{pi}}, \quad i = 1,2,...,n. \] (3.39)
Therefore, our choice of the matrices \( K_p \) and \( K_d \) must be such that Equation (3.39) is always satisfied.

Fast and non-oscillatory response of the robot are two desired factors in planning the robot’s trajectory for tracking moving objects. In order to achieve fast response, one must choose the position gain \( K_p \) as large as possible, which yields a large \( K_d \). In practice, however, the choice of the velocity gain, \( K_{di} \) is limited by the noise present in the velocity measurement. In order to avoid oscillatory-response, the proportional and derivative gains have been selected as in Equation (3.39) representing a critically-damped system. However, a critically-damped system may overshoot. The transient response of such a tracking-system depends on the initial relative position and velocity between the target and the interceptor. For time-optimal interception, they both must lie outside the overshoot zone, which is described below.

By solving Equation (3.37), while using Equation (3.39), for \( r \) and \( \dot{r} \), one obtains:
\[ r(t) = \left\{ r(0) + \left[ \dot{r}(0) + \frac{K_{di} r(0)}{2} \right] t \right\} e^{-\frac{K_{di} t}{2}} i = 1,2,...,n \] (3.40)
\[ \dot{r}(t) = \left\{ -\frac{K_{di}}{2} + \frac{\dot{r}(0) + \frac{K_{di} r(0)}{2}}{r(0) + \left[ \dot{r}(0) + \frac{K_{di} r(0)}{2} \right] t} \right\} r(t), \quad i = 1,2,...,n, \] (3.41)
where \( r(0) \) and \( \dot{r}(0) \) denote initial errors in position and velocity between the target and the robot’s tip, respectively.
The no-overshoot design goal can now be formulated by studying Equation (3.40) in an error phase plane, Figure 3.8. Bringing any initial point on the error phase plane to the origin as fast as possible without crossing the \( \dot{r} \)-axis causes no-overshoot in robot’s trajectory when intercepting a moving object.

From Equation (3.40), it can be seen that \( r(t) = 0 \) when \( \left( \dot{r}(0) + \frac{K_d}{2} r(0) \right) + r(0) = 0 \). This yields,

\[
 t = \frac{-r(0)}{\left( \dot{r}(0) + \frac{K_d}{2} r(0) \right)} .
\]  

(3.42)

From Equation (3.42), an overshoot zone in \( r - \dot{r} \) plane, as shown in Figure 3.8, can be defined, where:

\[
\dot{r}(0) < -\frac{K_d}{2} r(0) \quad \text{if} \quad r(0) > 0 , \quad (3.43a)
\]

\[
\dot{r}(0) > -\frac{K_d}{2} r(0) \quad \text{if} \quad r(0) < 0 . \quad (3.43b)
\]

One should note that Equations (3.43a) and (3.43b) determine the overshoot zone in three x-y-z directions. A time-optimal solution to the interception problem is one that does
not cause the robot to overshoot in $x$, $y$, or $z$ direction. As shown in Figure 3.8, one of the trajectories, represented in the error-phase plane, crosses the $\dot{r}$-axis, thereby indicating an overshoot situation. Equations (3.43a) and (3.43b) indicate that any phase trajectory that starts from the overshoot zone is bound to cross the $\dot{r}$-axis, hence causing an overshoot. One should note that, when $r(0)$ and $\dot{r}(0)$ have the same sign (i.e., the phase trajectory starts from the 1st or the 3rd quadrant of the phase-trajectory plane) no overshoot could occur.

3.2.2 Time-Optimal Interception Combining IPNG and CT Methods

Combining an IPNG-based interception scheme with the method explained in Section 3.2.1 would yield a shorter interception time than that obtained by a pure CT-based tracker. Time-optimal interception, for the method proposed in this thesis, is achievable, however, only if the IPNG switches to the CT-based tracking method at an optimal instant.

The overall interception time, $t_{int}$, is defined as:

$$t_{int} = t_{IPNG} + t_{CT},$$

(3.44)

where $t_{IPNG}$ and $t_{CT}$ are the robot motion times under the control of the IPNG and tracking methods, respectively. The latter brings the robotic manipulator to a pre-grasp situation (i.e., matching the robot’s location and velocity with those of the target’s).

The interception time calculated via Equation (3.44) is naturally a function of the switching instant. Normally, $t_{int}$ is a single-minimum function, Figure 3.7. The problem at hand, therefore, is to determine on-line when the CT tracker should take over the control of the robot motion for minimum-time interception. This can be achieved by predicting $t_{int}$ continuously during the IPNG-based stage of the motion:

$$t_{int} = t_{IPNG} + t_{CT},$$

(3.45)

where $t_{int}$ denotes the estimated overall interception time and $t_{CT}$ denotes the estimated remaining robot motion-time at switching time.

The time $t_{CT}$ can be calculated at any instant using the current states of the robot and the target in Equation (3.37), rewritten here in a matrix form as:

$$\dot{X} = AX, \quad X(0) = X_0, \quad X = \begin{bmatrix} \{r \} \\ \{\dot{r} \} \end{bmatrix}, \quad A = \begin{bmatrix} \{0\}_{3x3} & \{f\}_{3x3} \\ -K_p \{f\}_{3x3} & -K_d \{f\}_{3x3} \end{bmatrix}, \quad i = 1,2,\ldots,n. \quad (3.46)$$
In (3.46), only the first three joints of the manipulator are taken into consideration since these would normally be the bottleneck joints for the calculation of the robot-motion time. The first-order differential equation (3.46) can be solved in real-time for $X$ using numerical or symbolic routines.

The estimated interception time, $\tilde{t}_m$, matches closely the real interception time, $t_m$, when the robot actuators' torque limits are not reached while the robot motion is under the CT control. Otherwise, the estimated interception time might deviate from the real interception time, yielding an erroneous decision on the switching time, $t_{IPNG}$. In order to minimize this potential discrepancy, a simple quadratic search method is proposed herein. At each sampling time, if at least one of the actuators is saturated, a new set of gains, $K_p$ and $K_d$, is calculated through a curve-fitting method. The resultant set of gains keep the robot actuator torques within the limits given in Equation (3.17), and thus also yields a no-overshoot robot trajectory toward the interception point$^3$.

Interception is assumed to be achieved when the following user-defined permissible tolerances, for relative position and velocity between the target and the robot, respectively, are satisfied for $N, N>1$, consecutive sampling intervals:

$$|\mathbf{r}| < Tol_p , \text{ and } |\dot{\mathbf{r}}| < Tol_v .$$

(3.47)

3.3 IMPLEMENTATION OF THE PROPOSED TECHNIQUE

In this section, the potential real-time implementation of the proposed interception technique is discussed. The working environment is assumed to include a robotic manipulator and an object conveyor. The state of the moving object as a function of time is identifiable through a vision system. Since the visual recognition and tracking of the motion of the object introduces a time delay in the system, the present and future positions of the object have to be predicted, which itself introduces another time-delay, [17,19]. This elapsed time due to the target-motion prediction, $T_p$, naturally depends on the model used for this purpose.

$^3$ As an alternative to this search method, Kelly et al. [53] suggested a CT-based control law that takes into account the torque saturation in robot's actuators. Their controller yields an error equation as follows:

$$\ddot{\mathbf{r}} + K_d \tanh (\Lambda \dot{\mathbf{r}}) + K_p \tanh (\Omega \mathbf{r}) = 0 ,$$

where $K_p, K_d, \Lambda$, and $\Omega$ are diagonal positive definite matrices.
The calculation time needed for computing robot-motion commands based on the proposed interception scheme, and the time needed to predict the time-optimal switching point also represent another combined elapsed time, $T_c$, in the system. Thus, the overall time-delay is $T_v + T_p + T_c$. If one denotes the sampling period for updating robot-motion commands by $T_{re}$, the following relations can be assumed:

\[ \frac{T_{re}}{T_v} = S_1, \]  
\[ \frac{(T_p + T_c)}{T_v} = S_2. \]  

(3.48a)  
(3.48b)

It is assumed that $S_1 > S_2$.

Figure 3.9 shows the time-history of computational events for the integrated system. The constant $k$ in the figure denotes the number of vision sampling periods that the target-motion-prediction module has to wait in order to collect enough data on the target's motion (e.g., $k$ must be at least 3 in order to estimate target's acceleration). Velocity of the target is simply calculated by numerically differentiating the target's position, identified through the vision system. The prediction module must predict the target's motion $(S_2 + 1)T_v$ seconds ahead.

![Diagram showing time history of events' occurrences in the proposed interception scheme.](image)

**Figure 3.9**: Time history of events' occurrences in the proposed interception scheme.
The steps needed for computing the robot-motion command are as follows: (1) The \( a_{IPNG} \) is determined based on the target's predicted state at \((S_2 + 1)T_v\) seconds ahead and the robot's simulated state at the same instant; (2) The actuator torques needed to produce \( a_{IPNG} \) are determined via the inverse dynamics of the robot; (3) The \( a_{IPNG} \) is upgraded or modified by using the method explained in Section 3.4; and, (4) It is checked whether switching from the IPNG to the CT-tracking technique at the current time would yield the minimum-time interception; If yes, (5a) the control is switched to the tracking method and new torques are computed; Otherwise, (5b) the current actuator-torque commands are applied.

3.4 Simulation Results and Discussion

In this section, computer simulations of the proposed interception scheme are presented. All time-delays were modeled in our simulations. For simplicity a SCARA-type two-link planar robot (Figure 3.10) was utilized. The physical parameters of the manipulator are given in Table 3.1, [14]. The object to be grasped is assumed to be a point mass moving in the X-Y plane. The X-Y coordinates of the object are assumed to be available to the interception system via a vision system. The dynamic simulation module, SIMULINK, and a MATLAB robotic toolbox were used, [54].

![Figure 3.10: A schematic diagram of a planar 2-DOF manipulator with revolute joints.](image-url)
Table 3.1: Manipulator’s Physical Parameters.

<table>
<thead>
<tr>
<th>$l_1$</th>
<th>$a_1$</th>
<th>$m_1$</th>
<th>$I_1$</th>
<th>$T_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>0.5 m</td>
<td>1 kg</td>
<td>0.2 kg-m$^2$</td>
<td>10 Nm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$l_2$</th>
<th>$a_2$</th>
<th>$m_2$</th>
<th>$I_2$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>0.5 m</td>
<td>1 kg</td>
<td>0.2 kg-m$^2$</td>
<td>10 Nm</td>
</tr>
</tbody>
</table>

The sampling times used in the simulation were: $T_{rc} = 40$ ms, $T_v = 20$ ms, $T_p = 5$ ms and $T_c = 15$ ms. The sampling time of the robot controller at joint-level was chosen as 5 ms. The manipulator is initially at rest with its base at (0.0,0.0) m and its tip at (0.0,1.0) m in elbow-down configuration. The grasping tolerances used were $Tol_p = 20$ mm (2% of the maximum distance between the robot and the target), and $Tol_v = 20$ mm/s (5% of the maximum target’s speed). The coefficient $\alpha$ in Equation (3.30) was chosen as 0.5.

The proposed hybrid interception scheme was applied to a variety of object trajectories. Some of them are given herein to illustrate worst-case scenarios. In all the simulations a navigation constant of $\lambda = 3.0$, and proportional and derivative gains of $K_p = 1.0I_{2x2}$ and $K_d = 2.0I_{2x2}$ were employed.

Figure 3.11a shows the simulation results for the case where the object is moving on a sinusoidal curve toward the robot:

$$X_t = 1.0 - 0.4\sin\left(\frac{\pi t}{4}\right), \quad \text{and} \quad Y_t = 0.05t.$$  \hspace{1cm} (3.49)
The X-Y interception trajectory via the CT-based tracking method is also shown in Figure 3.11a. The proposed technique yields an interception-time of $t = 3.44$ seconds, while the pure tracking method yields $t = 6.24$ seconds. In Figure 3.11b, the variation of the interception time versus the switching time is presented. There is a very close match between the real and estimated interception times up to the optimal switching point. Thereafter, due to robot actuator torque-limit violation, the estimated interception time deviates from the real interception time. As seen in Figure 3.11b, there is a sudden large increase in $t_{int}$ at the minimum switching time. This is due to the overshoot in robot's trajectory.

![Figure 3.11b: Real versus estimated interception times.](image1.png)

Figure 3.11b: Real versus estimated interception times.

Figure 3.12a shows the simulation results for the case where the object is moving on a sinusoidal curve away from the robot:

$$X_t = 1.0 + 0.05t, \quad \text{and} \quad Y_t = 0.4\sin\left(\frac{\pi t}{4}\right). \quad (3.50)$$

The X-Y interception trajectory via the CT-based tracking method is also shown in Figure 3.12a. The proposed technique yields an interception-time of $t = 4.56$ seconds, while the pure tracking method yields $t = 6.24$ seconds. In Figure 3.12b, the variation of the interception-times versus the switching time is presented.

Figure 3.13a shows the simulation results for the case where the object is moving along a circular path in CCW direction:
\[ X_t = 1.5 \cos \left( \frac{\pi t}{16} \right) - 0.5, \quad \text{and} \quad Y_t = 1.5 \sin \left( \frac{\pi t}{16} \right). \] (3.51)

The X-Y interception trajectory via the CT-based tracking method is also shown in Figure 3.13a. The proposed technique yields an interception-time of \( t = 4.36 \) seconds, while the pure tracking method yields \( t = 6.00 \) seconds. In Figure 3.13b, the variation of the interception times versus the switching time is presented.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.12a.png}
\caption{Interception via the proposed technique. Target moving along a sinusoidal curve.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.12b.png}
\caption{Real versus estimated interception time.}
\end{figure}
**Figure 3.13a:** Interception via the proposed technique. Target moving on a circle.

**Figure 3.13b:** Real and estimated interception time.
Figure 3.14a shows the simulation results for the case where the object is moving along a sinusoidal path in both X and Y directions. This target-motion has been adopted from [19]. It represents a target with varying acceleration in directions X and Y:

\[ X_t = 1.0 + 0.5 \sin(2\pi f_1 t), \quad \text{and} \quad Y_t = 1.0 + 0.05 \sin(2\pi f_2 t), \]  

(3.52)

where \( f_1 = 0.04 \text{ s}^{-1} \) and \( f_2 = 0.08 \text{ s}^{-1} \). The X-Y interception trajectory via the CT-based tracking method is also shown in Figure 3.14a. The proposed technique yields an interception time \( t = 5.2 \) seconds, while the pure tracking method yields \( t = 7.0 \) seconds. In Figure 3.14b, the variation of the interception times versus the switching time is presented.

Figure 3.14a: Interception via the proposed technique. Target moving on a sinusoidal curve in X and Y directions.

Figure 3.14b: Real versus estimated interception times.
All simulation results shown in this section clearly illustrate the very favorable performance of the proposed interception method over a pure tracking method.

3.5 SUMMARY

A novel robotic interception method is developed. The proposed scheme combines a navigation-based interception system based on an IPNG technique with a conventional trajectory tracking method for intercepting fast-maneuvering moving objects. The Ideal Proportional Navigation Guidance (IPNG) method itself is modified for robotic interception. It is shown that the determination of the switching instant to a PD-type CT-control scheme is critical to the time-optimality of the proposed hybrid interception system.

It is clearly illustrated, through computer simulations, that the proposed hybrid interception technique performs favorably over the pure PD-type CT-control method.
CHAPTER FOUR

ROBOTIC INTERCEPTION VIA AUGMENTED IDEAL PROPORTIONAL NAVIGATION GUIDANCE

PNG-based techniques normally yield time-optimal results only for cruising targets (i.e., targets moving with relatively constant velocity), [e.g., 29,30]. In contrast, Augmented Proportional Navigation Guidance (APNG) schemes have been reported in the literature as optimal interception techniques for maneuvering targets, [e.g., 31,55]. In APNG, it is assumed that (1) the interceptor and target can only accelerate laterally in the direction of their velocities and the target's acceleration amplitude is constant, and (2) autopilot and seeker loop dynamics are fast enough to be neglected when compared to the overall guidance loop behavior. The PNG acceleration command is augmented by adding a term that reflects the target's acceleration.

A novel Augmented Ideal Proportional Navigation Guidance (AIPNG) technique is developed in this chapter to improve on the IPNG method addressed in Chapter 3 for cases where the target's acceleration can be reliably predicted. Figure 4.1 shows a schematic diagram of the proposed hybrid robotic-interception method. The robot initially moves under the AIPNG control. At a "switching point", a conventional tracking method takes over the control of the robot, bringing its end-effector to the interception point matching the target's location and velocity.

Figure 4.1: The hybrid interception scheme utilizing AIPNG technique.
Herein, it will be shown that the proposed hybrid method combining an AIPNG with a PD-type CT-method brings the LOS angular rate to zero regardless of the target's motion type. This, subsequently, indicates the reduction in dimensionality of the interception problem, (i.e., the relative velocity/acceleration between the target and the interceptor lies on the LOS), leading to shortening of the interception time.

4.1 AUGMENTED IPNG INTERCEPTION METHOD

In this section, first, the conventional Augmented Proportional Navigation Guidance (APNG) technique is briefly reviewed. Later on, the proposed Augmented Ideal Proportional Navigation Guidance (AIPNG) and its advantages over an APNG technique are discussed for robotic interception. Thereafter, a methodology for improving the AIPNG technique is addressed in detail and robotic interception via IPNG and AIPNG are compared through numerous simulations.

4.1.1 APNG Interception Technique

Introducing the target's acceleration, when utilizing a Proportional Navigation Guidance (PNG) law, yields time-optimal solution to the interception problem when the target is moving with constant acceleration, [e.g., 55,56]. As PNG-type navigation techniques have been developed with the objective of optimal control for intercepting non-maneuvering targets (i.e., cruising targets), Augmented Proportional Navigation Guidance (APNG) can be seen as a special case of optimal control for intercepting maneuvering targets (i.e., targets moving with non-zero acceleration).

The optimal-interception solution of the APNG has been obtained for cases in which both the interceptor and target can have only velocity-turning maneuvers (i.e., they can only accelerate in a direction normal to their velocities). The time/energy optimal solution\(^1\) to this interception problem yields an acceleration command as follows:

\[ J = \int_0^T \left( 1 + |\dot{\nu}|^2 \right) dt, \]

\(^1\) This solution is derived through optimization of the following objective function:
\[ (a_{\perp})_n = \lambda \dot{\theta}_{\text{LOS}} V + \left( \frac{\lambda}{2} \right) (a_T)_n, \]  

(4.1)

where \( V \) is the interceptor's velocity, \( \lambda \) is the navigation gain, \((a_{\perp})_n\) is the interceptor's acceleration command normal to the \( V \) and \((a_T)_n\) is the target's acceleration normal to its velocity. Equation (4.1) has been derived for the case in which \( \| (a_T)_n \| = \text{constant} \).

It is important to note, however, that, both modeling and measuring a target's acceleration are complex tasks and filtering the noise associated with acceleration measurements, with on-board filters, may be computationally cumbersome.

### 4.1.2 AIPNG Interception Technique

In IPNG, the acceleration command is normal to the relative velocity between the target and the robot, therefore, augmenting it as in the APNG technique would not yield an optimal solution. No closed-form solution has been reported in the literature for this type of navigation guidance for optimal interception. In the proposed augmented IPNG technique, therefore, the target's acceleration is taken into consideration differently from that in the APNG technique, represented by Equation (4.1): the acceleration command computed through the IPNG technique is augmented by the target's acceleration as follows:

\[ a_{\text{AIPNG}} = a_{\text{IPNG}} + a_T = K_d \dot{r} + K_p r + a_T, \]

(4.2)

where \( K_d \) and \( K_p \) are defined in Equation (3.11). The arguments of the coefficients \( K_p \) and \( K_d \) are dropped for simplicity.

The above acceleration command augmentation was developed based on the work of Babu et al., [57]. They showed that the polarity of the LOS angular rate,\( \dot{\theta}_{\text{LOS}} \), plays an important role in the PNG-based laws. This was achieved by invoking the sliding-mode control technique combined with a PNG law with an additive bias term, which depends on the polarity of the \( \dot{\theta}_{\text{LOS}} \). The switching surface was defined in [57] as:

\[ S = \dot{\theta}_{\text{LOS}}. \]

(4.3)

where \( T_f \) is the interception time. It is assumed that target's acceleration is always normal to its velocity, however, its magnitude may vary with time. It is also assumed that the average of the target's acceleration is available to the interceptor's controller.
The following control law, in an acceleration command form, was subsequently proposed:

\[
a_c = \frac{a_{PNG}}{\cos(\sigma_t - \theta_{LOS})} + W \text{Sgn}(\theta_{LOS}),
\]

(4.4)

where \(\sigma_t\) denotes the angle between the interceptor’s velocity and the reference line, \(W\) is a constant, and \(\text{Sgn}\) denotes the sign function.

It will be shown later in this Section that, defining the augmented acceleration command of the IPNG technique as in Equation (4.2) has the ability to drive the robot trajectory onto the switching surface defined in Equation (4.3) and constrain it to slide along this surface for all subsequent times regardless of the target’s motion type.

More specifically, the proposed AIPNG technique has three advantages over the IPNG technique for robotic interception:

(i) AIPNG yields a position-difference error equation similar to that of a PD-type CT-method;

(ii) \(r\) converges to zero, for \(\lambda>1\), regardless of the target’s motion type, (convergence is assured); and,

(iii) \(\dot{\theta}_{LOS}\) approaches zero, for \(\lambda>2\), regardless of the target’s motion type, yielding the

Phase II of our hybrid interception technique (i.e., PD-type CT-method) optimal.

These points are discussed below in more detail.

(i) The AIPNG proposed in Equation (4.2) can be simplified by rewriting it as:

\[
K_p r + K_d \dot{r} + (a_T - a_{AIPNG}) = 0,
\]

(4.5)

and substituting \((a_T - a_{AIPNG})\) with \(\ddot{r}\):

\[
\ddot{r} + K_d \dot{r} + K_p r = 0.
\]

(4.6)

Equation (4.6) represents a second-order differential equation for the position difference between the target and the robot, \(r\). The coefficients of this second-order differential equation are time- and state-dependent scalars, constituting a non-liner system. However, for the case where the target’s velocity relative to the robot’s velocity is in the opposite direction of the LOS, \(r\), from Equation (3.11), one can obtain the following relation between \(K_p\) and \(K_d\),

\[
K_d = \sqrt{\lambda} K_p.
\]

(4.7)
This condition is met after $\dot{\theta}_{LOS}$ approaches zero and the robot closes its distance with the target. By choosing $4$ as the value of the navigation gain, $\lambda$, Equation (4.7) can be re-written as $K_d = 2\sqrt{K_p}$. This set of gains defines a second-order system with critically damped response (i.e., non-oscillating response). This, specifically, shows the close relationship between the proposed augmented IPNG law and a PD-type CT-method controller, whose error equation is similar to that in Equation (4.6) but with time-invariant gains. It can be shown that $\lim_{r \to 0} \dot{r} = -Kr$, where $K$ is a positive constant for $\lambda > 2$, see point (iii) below. Therefore, Equation (4.7) is always achievable.

(ii) Interception (i.e., $r = 0$) is always achievable for $\lambda > 1$, regardless of the target’s motion type, when utilizing the AIPNG technique. This can be shown in a planar interception scenario utilizing polar coordinates, Figure 4.2. One can write $\dot{r} = \dot{r}_e + r\dot{\theta}_{LOS}e_\theta$, where $e_r$ and $e_\theta$ are unit vectors in polar-coordinates. Also,

$$a_T - a_I = \left( \dot{r}_e - r\dot{\theta}_{LOS}^2 \right)e_r + \left( r\dot{\theta}_{LOS} + 2\dot{r}\dot{\theta}_{LOS} \right)e_\theta,$$

(4.8)

where $a_I = a_{AIPNG} \equiv a_{IPNG} + a_T \equiv \lambda \dot{r} \times \dot{\theta}_{LOS} + a_T$. The above equations can be merged to obtain,

$$\begin{cases} \dot{r}_e - r\dot{\theta}_{LOS}^2 = -\lambda \dot{r}\dot{\theta}_{LOS}, \\ r\dot{\theta}_{LOS} + 2\dot{r}\dot{\theta}_{LOS} = \lambda \dot{r}\dot{\theta}_{LOS}. \end{cases}$$

(4.9)

Equation (4.9), which represents the equation of motion of an AIPNG interceptor for any type of target’s maneuver, is exactly the same as the equation of motion of an IPNG interceptor for non-maneuvering targets, [33].

![Figure 4.2: Geometry of the interception in a plane.](image)
By solving the system of differential equations given in Equation (4.9) one can obtain:
\[
\dot{\theta}_{\text{LOS}}(t) = \dot{\theta}_{\text{LOS}}(0) \left( \frac{r(t)}{r_0} \right)^{\lambda - 2},
\]  
(4.10)
where \( r_0 \) and \( \dot{\theta}_{\text{LOS}}(0) \) are the initial values of \( r \) and \( \dot{\theta}_{\text{LOS}} \), respectively. As can be seen from Equation (4.10), during the interception period, \( \dot{\theta}_{\text{LOS}} \) will approach infinity when \( \lambda < 2 \) and will approach zero when \( \lambda > 2 \).

Substituting Equation (4.10) into Equation (4.9) yields:
\[
r^2 = r_0^2 \dot{\theta}_{\text{LOS}}^2 \left[ 1 - \left( \frac{r}{r_0} \right)^{2(\lambda - 1)} \right] + \dot{r}_0^2,
\]  
(4.11)
where \( \dot{r}_0 \) is the initial relative velocity. From Equation (4.11), the capture criterion for the AIPNG is \( \lambda > 1 \). Namely, regardless of the initial condition of \( \dot{r}_0 \) interception can always be achieved successfully when \( \lambda > 1 \).

(iii) When using AIPNG, the final value of \( \dot{\theta}_{\text{LOS}} \) approaches zero when \( r \) approaches zero for targets moving with any type of maneuver, Equation (4.10). Figure 4.3 shows \( \left( \dot{\theta}_{\text{LOS}} / \dot{\theta}_{\text{LOS}}(0) \right) \) versus \( (1 - r / r_0) \) for different values of the navigation gain, \( \lambda \). As can be seen, the final value of \( \dot{\theta}_{\text{LOS}} \) becomes zero for \( \lambda > 2 \) as \( r \) approaches zero. For \( \lambda = 3 \), the relationship is linear. The higher \( \lambda \) is, the faster \( \dot{\theta}_{\text{LOS}} \) approaches zero. This conclusion can be used as a design-guideline for selecting the navigation gain. This conclusion can be generalized to 3D interception, Appendix A.

![Figure 4.3: Angular rate of the LOS versus the closing distance between the interceptor and the target.](image-url)
One can note that the interception time is decreased by increasing \( \lambda \). However, a high navigation gain means a high maneuvering energy expended by the interceptor, Appendix B.

### 4.2 Dimensionality Reduction in AIPNG

\( \dot{\theta}_{LOS} \) is proportional to the cross-product of \( \mathbf{r} \) and \( \mathbf{\dot{r}} \), Equation (3.8). Therefore, at \( \dot{\theta}_{LOS} = 0 \), the two vectors \( \mathbf{r} \) and \( \mathbf{\dot{r}} \) must be parallel. From Equation (4.6) one can conclude that \( \mathbf{\dot{r}} \) has to be parallel to \( \mathbf{r} \). Thus, after reaching a point at which \( \dot{\theta}_{LOS} = 0 \) (i.e., interceptor locking on the target being in right course), \( \dot{\theta}_{LOS} \) is kept at zero for the rest of the interceptor’s motion up to the interception point. Subsequently, the dimensionality of the interception problem, whether in 2D or 3D, is reduced to a 1D tracking problem.

Since the relative acceleration and velocity of the robot and the target lie on a direction parallel to the LOS, the interception problem can be simply re-defined as finding the time at which an interceptor, namely the robotic manipulator, meets a moving object (i.e., \( \mathbf{r} = \mathbf{0} \)) with the assumption that the relative motion between the target and the robot is conveyed in the fixed direction of the LOS. The robotic interception process, however, should yield a smooth grasp of the moving object, defined herein as the match of the position and velocity of the moving object and those of the robot’s end-effector at the intercept:

\[
\mathbf{r}(t_{int}) = \mathbf{r}(t_{int}) = \mathbf{0},
\]

(4.12)

where \( t_{int} \) denotes the interception time.

This reduction in dimensionality specifically minimizes the time during which the robot is under the CT control up to the interception point. The moment at which \( \mathbf{r} \) is parallel to \( \mathbf{\dot{r}} \) (i.e., \( \dot{\theta}_{LOS} = 0 \)), accelerating the interceptor in any direction other than one parallel to \( \mathbf{r} \) will introduce an overshoot in robot’s response in the direction normal to the LOS, prolonging the interception time. This issue will be discussed in more detail in Section 4.5.1.

### 4.3 AIPNG Technique in 3D

The error equation in 3D is the same as that in 2D represented by Equation (4.6). When \( \mathbf{r} \) and \( \mathbf{\dot{r}} \) are parallel in 3D space, from the relation \( \mathbf{\dot{r}} = -K_d \mathbf{\dot{r}} - K_p \mathbf{r} \) derived from
Equation (4.6), one can conclude that \( \ddot{r} \) will be parallel to \( r \) as well as \( \dot{r} \). Namely, at the moment when \( r \) and \( \dot{r} \) become parallel, they remain so up to the interception point.

It can be shown that, an AIPNG interceptor sweeps an inertially-fixed flat-plane, after reaching the condition \( \dot{\theta}_{LOS} = 0 \), when the target is moving with a constant acceleration. This feature can be also stated as follows: The vector represented by \( (V_T \times a_T) \), where \( V_I \) and \( a_I \) denote the velocity and the acceleration of the interceptor, respectively, remains constant after \( \dot{\theta}_{LOS} = 0 \) when, \( a_T = \text{constant} \). From Equation (4.2):

\[
\dot{\theta}_{LOS} = 0 \Rightarrow a_I = a_T \Rightarrow \ddot{r} = 0. \tag{4.13}
\]

Therefore,

\[
\dot{r} = KU_{LOS}, \tag{4.14}
\]

where \( K \) is a constant and \( U_{LOS} \) is a unit vector in the LOS direction, which remains constant when \( \dot{\theta}_{LOS} = 0 \).

One can also write:

\[
(V_I \times a_I) = (V_T - \dot{r}) \times (a_T)
= (V_T - KU_{LOS}) \times (a_T)
= (V_T \times a_T) - \frac{KU_{LOS} \times a_T}{ii}.
\tag{4.15}
\]

The second term of Equation (4.15) is constant when \( a_T = \text{constant} \). In order to prove that the first term of Equation (4.15) is also constant, one can write:

\[
(V_T \times a_T) = S, \tag{4.16}
\]

where the time-rate of change of vector \( S \) is defined as follows:

\[
\frac{dS}{dt} = \frac{d}{dt} (V_T \times a_T) = (a_T \times \dot{a}_T) + V_T \times \frac{da_T}{dt}.
\tag{4.17}
\]

The first term in Equation (4.17) is zero. The second term must also be zero for targets moving with constant acceleration. Thus, since both terms of Equation (4.17) are constant, \( (V_I \times a_I) \) is also constant. Subsequently, the interceptor's acceleration and velocity must both lie on a flat-plane.

Two conditions must be satisfied for \( (V_I \times a_I) \) to be constant: (1) \( V_I \times a_I = 0 \), which indicates that \( V_I \) and \( a_I \) are parallel, and (2) \( a_I \) is normal to the \( V_I \), which is not possible.
Hence, $a_i$ has to be normal to the relative velocity between the target and the robot. Therefore, the interceptor moves on a straight-line after $\dot{\theta}_{LOS} = 0$. This is similar to the performance of the Optimal Guidance Law (OGL) reported in [28].

This suggests that the performance of the AIPNG technique is similar to that of an OGL for targets moving with constant acceleration. One should however note that the advantage of an AIPNG technique over an OGL is apparent since in the OGL it is assumed that the target's motion is a priori known.

4.4 AIPNG FOR ROBOTIC INTERCEPTION

In this section, the necessary modifications to the AIPNG scheme for robotic interception are discussed.

4.4.1 Upgrading the Acceleration Command of AIPNG

The proposed AIPNG must be upgraded for robotic interception. The process is similar to that of the IPNG technique described in Chapter 3. Namely, the acceleration command of the AIPNG is upgraded as follows:

$$a = a_{AIPNG} + \beta(t) u_{LOS} = \lambda \dot{r} \times \dot{\theta}_{LOS} + \alpha a_{T} + \beta(t) u_{LOS},$$

where $u_{LOS}$ is the unit vector in the LOS direction and $\beta(t)$ is a scalar, whose value is computed as:

$$\beta = \max \left\{ \sum_{i=1}^{n} H_i \right\} H_i = \left\{ \beta \parallel T_i \parallel \leq \alpha T_{\max} \right\}, \quad i = 1, 2, ..., n.$$  \hspace{1cm} (4.19)

In Equation (4.19), $T$ denotes the torque needed to produce the acceleration given in Equation (4.18). This torque can be computed by replacing $\dot{\chi}_r$ in Equation (3.15) with $a_c$ given in Equation (4.18). $T_i$ in Equation (4.19) denotes the $i^{th}$ component of the torque vector $T$. The coefficient $\alpha$ represents the user-defined percentage of the maximum available torque to be utilized. This additional acceleration component does not affect the parallelism of lines-of-sight after $\dot{\theta}_{LOS} = 0$. This can be simply proved by substituting $a_{AIPNG}$ in Equation (4.18) by its equivalent given in Equation (4.2). One thus obtains:
\[ a_c = K_d \dot{r} + K_p r + a_T + \beta(t) U_{LOS} \]  

(4.20)

By replacing \((a_T - a_c)\) by \(\ddot{r}\) and \(U_{LOS}\) by \(r/|r|\) and re-arranging the remaining terms in Equation (4.20), one obtains:

\[ \ddot{r} + K_d \dot{r} + \left( K_p + \frac{\beta(t)}{|r|} \right) r = 0. \]

(4.21)

As can be seen from Equation (4.21), when \(r\) and \(\dot{r}\) are parallel, \(\ddot{r}\) will be parallel to the LOS as well. Therefore, the LOS direction remains constant up to the interception point.

Figure 4.4 shows a schematic diagram for upgrading the proposed interception scheme based on Equation (4.20). For the sake of illustration, this figure shows a mapping between the robot's joint torques and permissible accelerations for a 2-DOF planar robot.

\[ \begin{align*}
T_3 \quad \text{Linear mapping} & \quad \beta U_{LOS} \\
\alpha T_{max} & \quad \alpha T_{max} \\
\alpha T_{max} & \quad \alpha T_{max} \\
\alpha T_{max} & \quad \alpha T_{max}
\end{align*} \]

Figure 4.4: Upgrading the acceleration command of the AIPNG.

The additional acceleration component in Equation (4.20) does not affect the speed of convergence of the angular velocity of the LOS angle to zero. This can be shown by rewriting the modified-AIPNG acceleration command as:

\[ a_T = a_{AIPNG} + \beta(t) U_{LOS} = \lambda \dot{r} \times \dot{\theta}_{los} + a_T + \beta(t) U_{LOS}. \]

(4.22)

From Figure 4.2:

\[ a_T - a_c = (\ddot{r} - r \dot{\theta}^2) e_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) e_\theta, \]
\[ \lambda \dot{r} \times \dot{\theta} = -\lambda r \dot{\theta}^2 e_r + \lambda \ddot{\theta} e_\theta, \]
\[ U_{LOS} = e_r. \]

(4.23)

The subscript \(LOS\) is dropped from \(\dot{\theta}\) for simplicity. By substituting Equation (4.23) into Equation (4.22), one can derive the following non-linear system of differential equations:

\[ \ddot{r} - r \dot{\theta}^2 = -\lambda r \dot{\theta}^2 - \beta(t), \]

(4.24a)
From Equation (4.24b):

\[ r\ddot{\theta} + 2r\dot{\theta} = \lambda r\dot{\theta}. \quad (4.24b) \]

From Equation (4.25):

\[ \dot{\theta} = \dot{\theta}_0 \left( \frac{r}{r_0} \right)^{\lambda-2}. \quad (4.25) \]

Equation (4.25) is exactly the same as Equation (4.10). Thus, the behavior of the angular velocity of the LOS angle does not depend on the factor \( \beta(t) \). The contribution of \( \beta \) on the AIPNG technique is to make the relative distance, \( r \), converge to zero as fast as possible.

By utilizing this additional term, interception is guaranteed for \( \lambda > 2 \). The rational behind upgrading the AIPNG is: (1) initially to send the robot toward the current location of the target with maximum permissible acceleration, and (2) to close the distance between the target and the robot with maximum permissible speed when cruising.

### 4.4.2 Limiting the Acceleration Command of the AIPNG

The acceleration command calculated in Equation (4.2) might exceed the maximum torques available at some of the joints. In this case, the acceleration command should be limited. A method of limiting the \( a_{AIPNG} \) similar to that proposed in Chapter 3 is adopted herein. The command acceleration is calculated as:

\[ a_c = K a_{AIPNG}, \quad (4.26) \]

where \( K \) is a scalar computed as follows:

\[ K = \max \left( \bigcap_{i=1}^{n} S_i \right), \quad S_i = \left\{ K \parallel T_i \parallel T_{max} \right\}, \quad i = 1, 2, \ldots, n. \quad (4.27) \]

Once again, \( T_i \) denotes the torque needed to produce the acceleration given in Equation (4.26). Figure 4.5 shows a schematic diagram for limiting the acceleration command of the proposed interception scheme based on Equation (4.26).

However, it should be noted that, limiting \( a_{AIPNG} \), when using Equation (4.26), might violate the parallelism of the LOS direction. In this case, the limiting procedure is suggested to be carried out alternatively as follows:

\[ a_c = a_{AIPNG} + \Gamma U_{LOS}, \quad (4.28) \]
where \( \Gamma \) is a scalar, whose value is computed the same way as in Equation (4.19), where \( T_i \) in Equation (4.19) denotes the torque needed to produce the acceleration given in Equation (4.28). Limiting the acceleration command using this technique will not violate the parallelism of the LOS direction, Figure 4.6.

\[ \Gamma \text{ is computed the same way as in Equation (4.19), where } T_i \text{ in Equation (4.19) denotes the torque needed to produce the acceleration given in Equation (4.28). Limiting the acceleration command using this technique will not violate the parallelism of the LOS direction, Figure 4.6.} \]

\[ \text{when } |T| > |T_{\text{max}}| \]
\[ \begin{align*}
\text{if } \dot{\theta}_{\text{LOS}} &= 0 \\
\text{use limiting technique as in Equation (4.26)}
\end{align*} \]
\[ \text{else} \]
\[ \text{use limiting technique as in Equation (4.28)} \]
\[ \text{end} \]
\[ \text{end} \]
Figure 4.7 shows the proposed overall algorithm for modifying (i.e., upgrading and/or limiting) the acceleration command of the AIPNG technique for robotic interception.

**Figure 4.7: The algorithm for modifying the acceleration command of the AIPNG.**

### 4.5 AIPNG Interception Technique Combined with a CT Method

In order to match the target's position and velocity at the interception point, a PD-type CT-control method is proposed to take over the robot's control at an optimal switching time, Figure 4.8. As can be seen from Figure 4.8, the proposed hybrid interception scheme includes two phases as: Phase I, during which the robot is under the AIPNG control, and Phase II, during which the robot is under the control of a tracking method, namely a PD-type CT control. A methodology for on-line estimation of the Optimal Switching Point (OSP) is discussed in this section.
4.5.1 Optimal Use of the PD-type CT-Control Method (Phase II)

As was mentioned earlier in Chapter 3, the error equation for a PD-type CT-controller can be represented as a second-order system with constant coefficients known as proportional and derivative gains, where the error is defined as the difference between the target and robot's positions, Equation (3.37). $K_p$ and $K_d$ are diagonal proportional and derivative gain matrices, respectively. These gains should be chosen such that the response of the system is critically damped, as represented in Equation (3.39).

For the set of gains defined in Equation (3.39), the time-optimal response is the one with no-overshoot, [58]. Overshoot in a critically-damped system depends on the initial conditions of $r$ and $\dot{r}$. Since $r$ is generally a vector in 3D, overshooting must be avoided in each of $r$'s components. Satisfying this condition on-line is a time-consuming process. However, if $\ddot{r}$ and $\dot{r}$ are both parallel to $r$, the dimensionality of the interception problem is reduced to 1 (i.e., the interception problem would be analogous to one in which the robot tracks an object moving on a straight-line). Thus, overshooting should be considered only in the LOS direction. When $\ddot{r}$ is parallel to $r$ and $\dot{r}$, the matrices $K_p$ and $K_d$ become scalars.

Interception is defined herein as when:

$$|r| \leq (Tol)_p \quad \text{and} \quad |\dot{r}| \leq (Tol)_v,$$

(4.29)
for N consecutive time steps, where N ≥ 2. \((Tol)_p\) and \((Tol)_v\) are tolerances for position and velocity errors at the rendezvous-point, respectively. A trajectory that starts within the overshoot-zone normally renders a larger interception time when \((Tol)_p \to 0\) and \((Tol)_v \to 0\), [32]. However, interception time would also be influenced by the size of aforementioned tolerances. Figure 4.9 shows a schematic diagram of three different trajectories labeled as I, II, and III. There may exist a significant difference between the interception times corresponding to overshooting trajectories II and III. A trajectory that crosses over the \(r\)-axis, renders a larger interception time, Appendix C. The impact of introducing a trajectory which does not cross over the \(r\)-axis on our hybrid interception scheme will be addressed below in Sections 4.6 and 4.7.

\[ r \]

\[ \dot{r} \]

**Figure 4.9:** Phase-portraits and the intercept tolerance square.

The necessary conditions for time optimality of the PD-Type CT-Method can be better understood when studying the phase-plane trajectory generated by the aforementioned controller. The objective, herein, is to show that the parallelism of the position-difference vector, \(r\), and relative velocity between the target and the interceptor, \(\dot{r}\), is a necessary condition for having a time-optimal response. Some properties of the phase-plane trajectory generated by the aforementioned controller are listed below.

(i) In a critically-damped PD-type CT-control system, where \(K_d = 2\sqrt{K_p}\), a phase-plane trajectory will approach the origin while being asymptotically tangent to a line with a slope of \((-K_d/2)\). This can be seen by solving Equation (3.37). The solution of the
ODE given in Equation (3.37) can be computed as: 

\[ \dot{r} = \left[ -\frac{K_d}{2} + \frac{(C_0 + (K_d/2)r)}{r + (C_0 + (K_d/2)r_0)} \right] r \]

(note that \( \frac{dr}{dr} \rightarrow \frac{K_d}{2} \) when \( t \rightarrow \infty \)), see Figure 4.10.

![Phase-plane trajectories](image)

**Figure 4.10**: Phase-plane trajectories are asymptotically tangent to the overshoot-line for critically-damped 2nd order systems.

(ii) The time that takes for the robot to move along a trajectory starting form point \((r_0, \dot{r}_0)\) at time \( t = t_0 \) and ending at point \((r_f, \dot{r}_f)\) at time \( t = t_f \) is inversely proportional to the area confined between the trajectory and the \( r \)-axis in the phase-plane, [32]. This is schematically shown in Figure 4.11. One can write:

\[
\int_{t_0}^{t_f} dt = \begin{cases} 
\frac{\dot{r}_f \, dr}{(\dot{r}_f) - (\dot{r}_0)} \, \text{sign}(r, \dot{r}) < 0 \\
\frac{\dot{r}_f \, dr}{(\dot{r}_f) - (\dot{r}_0)} \, \text{sign}(r, \dot{r}) > 0.
\end{cases}
\]

It can be seen from Equation (4.30) that, if the area confined between the trajectory and the \( r \)-axis is maximum, then, the time \( t_f - t_0 \) is minimum.

(iii) No two trajectories, I and II, generated by a PD-type CT-method can intersect in the phase-plane. This can be proven by "contradiction". Suppose that trajectory I intersects trajectory II at point \((r_m, \dot{r}_m)\) in the phase-plane. One can then write:

\[
\left[ \left( \frac{d\dot{r}}{dr} \right)_m \right]_{\text{I}} \neq \left[ \left( \frac{d\dot{r}}{dr} \right)_m \right]_{\text{II}}.
\]
Equation (4.31) simply states that two trajectories I and II must have different slopes in the phase-plane at the intersection point, namely $m$. One can generally re-write the expression for the slope of any trajectory in the phase-plane as: \( \frac{d\hat{r}}{dr} = \frac{\hat{r}}{r} \). Therefore, Equation (4.31) can be re-written as follows:

\[
\left[ \begin{array}{c} \ddot{r} \\ \dot{r} \end{array} \right] \not= \left[ \begin{array}{c} \ddot{r} \\ \dot{r} \end{array} \right]_{m} \neq \left[ \begin{array}{c} \ddot{r} \\ \dot{r} \end{array} \right]_{m II} \quad (4.32)
\]

However, from Equation (3.37) one can write:

\[
\left[ \ddot{r} \right]_{m} = -K_{d} \left[ \dot{r} \right]_{m} - K_{p} \left[ r \right]_{m} = -K_{d} \left[ \dot{r} \right]_{II} - K_{p} \left[ r \right]_{II} = \left[ \ddot{r} \right]_{II} \quad (4.33)
\]

From Equation (4.33) one can conclude that the Equation (4.32) cannot be correct, (note that \( \left[ \dot{r} \right]_{m} = \left[ \dot{r} \right]_{II} \) and \( \left[ \ddot{r} \right]_{m} = \left[ \ddot{r} \right]_{II} \) at intersection), therefore the two trajectories I and II cannot intersect. However, trajectories in the phase-plane are asymptotically tangent to a line with slope of \(-\frac{\dot{r}}{r} / 2\) as was mentioned earlier in (i).

Figure 4.11: The relationship between the phase-plane trajectory and the interception time.

In order to investigate the impact of the parallelism of \( r_0 \) and \( \dot{r}_0 \) on the interception time, two different cases are studied, Figures 4.12(a) and 4.12(b). Figure 4.12(a) shows a schematic diagram for the interception Scenario #1, where \( r_0 \) and \( \dot{r}_0 \) are parallel, while Figure
4.12(b) shows a schematic diagram for the interception Scenario #2, where \( r_0 \) and \( \dot{r}_0 \) are not parallel. For Figure 4.12(a):
\[
\dot{r}_0 = (V_{t0} - V_{r0}) e_r.
\] (4.34)

For Figure 4.12(b):
\[
\dot{r}_0 = (V_{t0} - V_{r0} \cos(\varphi)) e_r + (-V_{r0} \sin(\varphi)) e_\theta.
\] (4.35)

Now, let us consider the phase-plane trajectory generated by a PD-type CT-method for both interception scenarios. This has to be done for both directions, \( e_r \) and \( e_\theta \):

![Figure 4.12(a): Interception Scenario #1. Figure 4.12(b): Interception Scenario #2.]

(i) The phase-plane trajectory in the direction of the unit vector \( e_r \):

The initial relative velocity between the target and the robot in the direction \( e_r \) for interception Scenario #1 can be computed from Equation (4.34) as; \( \dot{r}_0 = V_{t0} - V_{r0} \). The initial relative velocity between the target and the robot in the direction \( e_r \) for interception Scenario #2 can be also computed from Equation (4.35) as; \( \dot{r}_0 = V_{t0} - V_{r0} \cos(\varphi) \). It can be concluded that the initial relative velocity between the target and the robot in the \( e_r \) direction for interception Scenario #2 is larger than that for interception Scenario #1 for \( 0 < \varphi < \pi / 2 \). Figure 4.13(a) shows a schematic diagram comparing the trajectories generated by a PD-type CT-method for both interception scenarios in the direction \( e_r \). As can be seen from Figure 4.13(a), the phase-plane trajectory corresponding to the interception Scenario #1 yields a larger area with respect to the \( r \)-axis, subsequently yielding a shorter interception time.

(ii) The phase-plane trajectory in the direction of the unit vector \( e_\theta \):

The initial relative velocity between the target and the robot in the direction \( e_\theta \) for interception Scenario #1 is zero. The initial relative velocity between the target and the robot in the direction \( e_\theta \) for interception Scenario #2 can be computed from Equation
Figure 4.13(a): Phase-plane trajectories in the $e_r$ direction.

\[ \dot{r}_0 = -V_{r0} \sin(\varphi) \]

Figure 4.13(b) shows a schematic diagram of the trajectory generated by a PD-type CT-method for the interception Scenario #2 in the direction $e_\theta$. As can be seen from Figure 4.13(b), the phase-plane trajectory corresponding to the interception Scenario #2 overshoots (the trajectory crosses the $r$-axis).

Figure 4.13(b): Phase-plane trajectory in the $e_\theta$ direction.

From (i) and (ii) above, it can be concluded that when $r$ and $\dot{r}$ are not initially parallel, (1) the system generates a larger interception time, and (2) it overshoots in the direction normal to the LOS, namely $e_\theta$. Therefore, parallelism of the LOS is a necessary condition for time-optimality of the Phase II in our hybrid interception technique, namely the PD-type CT-method.
4.6 **AIPNG + CT Interception Scheme**

In the AIPNG-based hybrid interception method presented in this chapter, there exists an Optimal Switching Point (OSP) that renders minimal interception time. The overall interception time, $t_{int}$, is thus a combination of time during which the robot is under the AIPNG control and the time during which the robot is under the CT-method control:

$$t_{int} = t_{AIPNG} + t_{CT}.$$ \hfill (4.36)

$t_{int}$ can be approximated on-line as follows:

$$\tilde{t}_{int} = t_{AIPNG} + \tilde{t}_{CT},$$ \hfill (4.37)

where $\tilde{t}_{CT}$ denotes the estimation of the time during which the robot is under the control of the CT-method.

In Chapter 3, it was shown that, $\tilde{t}_{CT}$ can be approximated on-line and its value is independent of the target motion class. $\tilde{t}_{CT}$ can be found by solving a 2$^{nd}$ order ODE of the position-error given in Equation (3.37) with the initial conditions $r(t=0)=r_0$ and $\dot{r}(t=0)=\dot{r}_0$, and an end condition given by Equation (4.29).

Figure 4.14 shows a schematic diagram of the phase-plane trajectory when utilizing the aforementioned interception technique. Two segments are featured: In segment (A-C) the AIPNG is in control, and in segment (C-O) the CT-method has taken over. Segment (A-C) itself has two parts. In segment (A-B) the angular velocity of the LOS angle has not approached zero yet. In segment (B-C), however, $\dot{\theta}_{LOS}$ approaches zero, namely,

$$a_1 = a_T \Rightarrow \ddot{r} = 0.$$ \hfill (4.38)

---

**Figure 4.14:** Overshooting response of the CT-method.
Equation (4.38) indicates that in segment (B-C), the robot is cruising toward the interception point with zero \( \text{closing-acceleration} \). If the condition \( \dot{\theta}_{los} = 0 \) is satisfied before reaching the optimal switching point, the necessary condition for optimality of the PD-type CT-method is ensured. Otherwise, the AIPNG + CT technique may yield results no better than that for the IPNG + CT technique discussed in Chapter 3.

4.7 AIPNG + MODIFIED CT INTERCEPTION METHOD

A method for modifying the Phase II of the interception trajectory, namely the PD-type CT-method, is discussed in this Section. The objective is to further reduce the overall interception time. In this technique, the AIPNG remains unchanged up to the Optimal Switching Point (OSP).

(i) Relationship Between Interception Time and Phase-plane Trajectory:

For a phase-plane trajectory starting at \( t = t_0 \) and ending at \( t = t_f \), the area confined between the phase-plane trajectory and the \( r \)-axis is inversely proportional to \( (t_f - t_0) \), Equation (4.30). Therefore, the area confined between the phase-plane trajectory and the \( r \)-axis must be maximized in order for \( (t_f - t_0) \) to be minimized.

(ii) The Modified CT Method:

As was discussed in Section 4.6, for a typical phase-plane trajectory of the AIPNG + CT method, Figure 4.14, the CT-method takes over at Point C. The area confined between the Trajectory (C-O) and the \( r \)-axis is inversely proportional to the time during which the robot is under the control of the PD-type CT-method. Point C in Figure 4.14 corresponds to the OSP. The objective here is to increase the aforementioned area by changing the shape of the phase-plane trajectory.

Figure 4.15 shows a typical phase-plane trajectory when utilizing the proposed technique. The phase-plane Trajectory (C-D-E-O) yields an area which is larger than that for a regular CT-method. Thus, the time during which the robot is under the control of this proposed technique is shorter than that for a CT-method, although segment (A-B-C) is the
same for both methods. Three segments are characterized in the proposed modified CT method:

Segment (C-D): The start point of this segment, Point C, represents the OSP. At this point $\dot{\theta}_{LOS}$ must have approached zero (by selecting the navigation gain, $\lambda$, sufficiently high this would be achievable). Segment (C-D) represents the zero-closing-acceleration phase, $\dot{r} = 0$. The robot's control does not switch to a CT-method at Point C, but it keeps moving as instructed by AIPNG.

Segment (D-E): In this segment, the robot moves with constant deceleration. The value of this deceleration and also the location of Point D are found by taking the robot's dynamics into account.

Segment (E-O): At Point E, the conventional PD-type CT-method, exactly the same method used in the AIPNG + CT technique, takes over. Point E is a user-defined point located along the Trajectory C-O, as shown in Figure 4.15. Trajectory C-O is the phase-plane trajectory of the CT-method when it takes over at OSP. The choice of Point E will be discussed below.

![Figure 4.15: Phase-plane trajectory of the AIPNG + modified_CT method.](image)

The concept behind the above-proposed scheme for CT-method modification is that a PD-type CT-method can be considered to be acting as a slowing-down operation for our hybrid interception technique. It continuously tries to match both the position and the velocity of the robot and the target. Clearly, matching the velocities of the interceptor and the target from the beginning (e.g., when the robot is initially far from the target) is not practical.
However, the navigation technique minimizes the distance between the interceptor and the target as fast as possible while bringing the interceptor to the proper heading toward the interception point. In the proposed technique, the use of the PD-type CT-method is postponed. At Point E, the PD-type CT-method takes over matching the terminal position and velocity of the interceptor and the target.

The overall interception time of the AIPNG + modified_CT method is given as:

$$t_{int} = t_{AIPNG} + t_{mod\_CT}. \quad (4.39)$$

Figure 4.16 shows the conceptual algorithm for implementing the AIPNG + modified_CT method.

![Figure 4.16: A conceptual algorithm for implementing the AIPNG + modified_CT method.](image)

(iii) **Selecting Point E Along the Trajectory C-O:**

Point E, as shown in Figure 4.15, is an arbitrary point located along the trajectory presented by C-O. In general, a candidate for Point E would be a point with the following coordinate along the $r$-axis in the phase-plane:

$$r_E = r_o + \gamma (r_c - r_o), \quad (4.40)$$

where $r_c$ and $r_o$ denote the coordinates of Points C and O along the $r$-axis, respectively, (the coordinates of Point O can be computed on-line). The coefficient $\gamma \in [0,1]$ in Equation (4.40) is user defined. The smaller it is, the closer Point E would be to Point O. The coordinate of Point E along the $\dot{r}$-axis can be then calculated analytically.

For a critically-damped 2nd order system, represented by Equations (3.37) and (3.39):
\[ \dot{r} + 2\sqrt{K_p} \dot{r} + K_p r = 0, \quad (4.41) \]

where,

\[ \frac{\partial \dot{r}}{\partial r} = \frac{\dot{r}}{r}. \quad (4.42) \]

From Equations (4.41) and (4.42):

\[ \frac{\partial \dot{r}}{\partial r} = -\frac{2\sqrt{K_p} \dot{r} - K_p r}{\dot{r}} \Rightarrow \frac{\partial \dot{r}}{\partial r} + K_p r + 2\sqrt{K_p} = 0. \quad (4.43) \]

Let \( \dot{r} = \xi r \), where \( \partial \dot{r} = \xi \partial r + r \partial \xi \). Substituting this into Equation (4.43) yields:

\[ \frac{\xi \partial \xi}{(\xi + \sqrt{K_p})^2} = -\frac{\partial r}{r}. \quad (4.44) \]

Integrating both sides of Equation (4.44):

\[ \frac{\sqrt{K_p}}{(\xi + \sqrt{K_p})^2} + \ln(\frac{\xi + \sqrt{K_p}}{r}) = \ln\left(\frac{C}{r}\right), \quad (4.45) \]

where \( C \) is the integration constant. Replacing \( \xi \) in Equation (4.45) by its equivalent \( \dot{r}/r \):

\[ \frac{r\sqrt{K_p}}{(\dot{r} + \sqrt{K_p})} = \ln\left(\frac{C}{\dot{r} + r\sqrt{K_p}}\right). \quad (4.46) \]

The integration constant, \( C \), can now be computed as:

\[ C = \left(\dot{r}_0 + r_0 \sqrt{K_p}\right) e^{\frac{r_0}{\dot{r}_0 + \sqrt{K_p}}}, \quad (4.47) \]

where \( r_0 \) and \( \dot{r}_0 \) are the initial values of the relative position and velocity, respectively.

Replacing the value of \( r_E \), namely the position of Point E projected on the \( r \)-axis, in Equation (4.46) and utilizing Equation (4.47), one can readily calculate the value of \( \dot{r}_E \) analytically.

Control of the robot is switched to a PD-type CT-method, when \( |r - r_E| \leq (Tol)_p \) and \( |\dot{r} - \dot{r}_E| \leq (Tol)_v \). It is proven that, the closer Point E is to Point O, the shorter the overall interception time would be, Appendix D. However, one should not extend the Trajectory (D-E) up to the interception point, namely to Point O, since the robot is assumed to be moving with a constant closing-acceleration along (D-E), and therefore, the condition \( \ddot{r}(t_f) = 0 \), which must be satisfied at Point O, would be violated. (Here \( t_f \) denotes the time of intercept).
Namely, the transition from a non-zero constant closing-acceleration to the almost-zero relative acceleration at the interception point has to be carried out by the PD-type CT-method.

(iv) Implementing the Segment (D-E) On Line:

An important remaining issue is to calculate the starting point of the constant-closing-acceleration-based motion, namely Point D in Figure 4.15. The objective is to move the robot with a constant closing acceleration (or constant deceleration), \( \ddot{r} = \text{constant} \), starting from Point D to Point E. This constant closing acceleration can be readily computed for each arbitrary point on segment C-D as follows:

\[
\ddot{r}_{\text{constant}} = \frac{(\dot{r}_E)^2 - (\dot{r})^2_{\text{AIPMC} + \Delta t}}{2[(\dot{r}_E) - (\dot{r})_{\text{AIPMC} + \Delta t}]} , \quad i = 1, 2, ..., n ,
\]  

(4.48)

where \( \Delta t \) denotes the time-step of the control system. To check whether the acceleration computed in Equation (4.48) is executable, one should compare that with the maximum permissible value. The maximum permissible deceleration, as a reference to closing acceleration, is proposed to be estimated as follows:

\[
\ddot{r}_{\text{permissible}} = \frac{\sum_{i=1}^{n} (\ddot{r}_{\text{max}})_{i} + \dot{r}_E}{i + 1} ,
\]  

(4.49)

where \( \ddot{r}_{\text{max}} \) denotes the maximum permissible closing acceleration computed by taking the robot's dynamics into account. \( \ddot{r}_{\text{permissible}} \) in Equation (4.49) represents the average of the maximum permissible decelerations of the robot along the segment C-D-E. The robot is proposed to start moving with a constant closing deceleration given in Equation (4.48) at the point where the following is satisfied:

\[
\ddot{r}_{\text{constant}} \geq \ddot{r}_{\text{permissible}} .
\]  

(4.50)

This method guarantees that the torque limits of the robot would not be violated when the robot is moving along the D-E trajectory. Thus, moving along Trajectory D-E with the constant closing acceleration given in Equation (4.48) is executable. The algorithmic procedure for implementing the proposed trajectory, C-D-E-O, is given below.
Step 0: Is OSP reached? If yes, solve for the Trajectory (C-O). Assign a value to $r_E$ along trajectory C-O, Figure 4.14. Compute the value of $\dot{r}_E$ using Equation (4.46) and go to Step 1. Otherwise, let the robot move as instructed by AIPNG.

Step 1: Set $i = 1$.

Step 2: Compute the constant deceleration of the robot to bring it from its current state to the state found in Step 0, namely Point E, using Equation (4.48).

Step 3: Compute the permissible deceleration of the robot in the LOS direction using Equation (4.49).

Step 4: Compare the $\ddot{r}_{constant}$, computed in Step 2, with $\ddot{r}_{permissible}$, found in Step 3. If Equation (4.50) is satisfied go to Step 5, otherwise, go to Step 6.

Step 5: Move the robot with $\ddot{r} = 0$ for the next time-step. Set $i = i + 1$. Go to Step 2.

Step 6: Move the robot with $\ddot{r} = \ddot{r}_{constant}$ for the next time-step. Set $i = i + 1$.

Step 7: If $|\dot{r}_i - \dot{r}_E| \leq \{(Tol)_r\}_{CT}$ and $|\dot{r}_i - \dot{r}_E| \leq \{(Tol)_p\}_{CT}$, go to Step 8. Otherwise, go to Step 6.

Step 8: Move the robot with $\ddot{r} = -K_d\dot{r}_i - K_p r_i$. If $r \leq Tol_p$ and $i \leq Tol_i$, stop the interception scheme. Otherwise, go to Step 9.

Step 9: Set $i = i + 1$. Go to Step 8.

In summary, the algorithmic procedure described above generates three trajectory segments; cruising (segment C-D), moving with a constant relative deceleration (segment D-E), and tracking, based on a PD-type CT-method, (segment E-O).

4.8 SIMULATION RESULTS AND DISCUSSION

In this section, computer simulations of the proposed interception scheme are presented. For simplicity a SCARA-type two-link planar robot is utilized, Figure 3.10. The physical parameters of the manipulator are given in Table 3.1. The object to be grasped is assumed to be a point mass moving in the X-Y plane. The X-Y coordinates of the object are assumed to be available to the interception system via a vision system. The dynamic
simulation module, SIMULINK, and a robotic toolbox of MATLAB were used for the simulations.

The grasping tolerances are $Tol_p = 10$ mm (1% of the maximum distance between the robot and the target), and $Tol_v = 10$ mm/s (2% of the maximum target's speed). The coefficient $\alpha$ in Equation (4.19) is chosen as 0.5.

The proposed hybrid interception scheme was applied to a variety of object trajectories. Some of them are given herein to illustrate the most-difficult-case scenarios. In all the simulations a navigation constant of $\lambda = 5.0$, and proportional and derivative gains of $K_p = 1.0$ and $K_d = 2.0$ are employed.

Table 4.1 summarizes the interception times of all the interception methods employed: pure PD-type CT control, IPNG + CT, AIPNG + CT and AIPNG + modified_CT. The results are for two target motion cases:

**CASE # 1:** (target moving with a constant acceleration as a projectile)

\[
X_{T0} = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}, \quad V_{T0} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad a_T = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}
\]

**CASE # 2:** (target moving on a sinusoidal curve)

\[
X_{T0} = \begin{bmatrix} 1.0 \\ 1.2 \end{bmatrix}, \quad V_{T0} = \begin{bmatrix} 0.2 \left( \frac{\pi}{2} \right) \\ -0.2 \end{bmatrix}, \quad a_T = \begin{bmatrix} -0.2 \left( \frac{\pi}{2} \right)^2 \sin \left( \frac{\pi t}{2} \right) \\ 0.0 \end{bmatrix}
\]

where $V_{T0}$ and $X_{T0}$ are the initial velocity and position of the target, respectively. The robot's end-effector is initially located at (0,1) m. As can be seen from Table 4.1, the interception time obtained via the AIPNG + modified_CT technique is better than that of the IPNG + CT method discussed in Chapter 3 by approximately 15% for CASE # 1 and 30% for CASE # 2.
Table 4.1: Comparing the interception times.

<table>
<thead>
<tr>
<th></th>
<th>CASE # 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method</td>
<td>PD-type CT-method</td>
<td>IPNG + CT</td>
<td>AIPNG + CT</td>
</tr>
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<td></td>
<td>Interception time (sec)</td>
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<td>4.15</td>
<td>3.75</td>
</tr>
<tr>
<td></td>
<td>CASE # 2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Method</td>
<td>PD-type CT-method</td>
<td>IPNG + CT</td>
<td>AIPNG + CT</td>
</tr>
<tr>
<td></td>
<td>Interception time (sec)</td>
<td>7.2</td>
<td>5.2</td>
<td>4.45</td>
</tr>
</tbody>
</table>

Figure 4.17a shows the X-Y plots of the robot’s and target’s trajectories for CASE #1 for AIPNG + modified_CT method. Figure 4.17b shows the position and velocity of the target and of the robot in the X and Y directions versus time. The phase-portraits of the AIPNG + modified_CT and the AIPNG + CT methods are shown in Figures 4.18a and 4.18b, respectively. In these figures, only the part of the phase-plane trajectory generated after the condition $\dot{\theta}_{\text{los}} = 0$ has been met is shown. Figures 4.19 and 4.20 show the same results for CASE #2.

Figure 4.17a: X-Y plot of the robot and the target trajectories utilizing AIPNG + modified_CT technique for CASE #1.
Figure 4.17b: X-Y position and velocity of the robot and the target versus time for CASE #1.

Figure 4.18a: Phase-portrait of the AIPNG + modified_CT method for CASE #1.
Figure 4.18b: Phase-portrait of the AIPNG + CT method for CASE #1.

Figure 4.19a: X-Y plot of the robot and the target trajectories utilizing AIPNG + modified_CT technique for CASE #2.
Figure 4.19b: X-Y position and velocity of the robot and the target versus time for CASE #2.

Figure 4.20a: Phase-portrait of the AIPNG + modified_CT method for CASE #2.
Figure 4.20b: Phase-portrait of the AIPNG + CT method for CASE #2.

4.9 SUMMARY

A novel approach to on-line, robot-motion planning for moving-object interception is proposed. This approach utilizes a navigation-based technique that is robust and computationally efficient for the interception of fast-maneuvering objects. The navigation technique utilized is an augmentation of the Ideal Proportional Navigation Guidance (IPNG) technique.

This navigation technique has been modified to reflect maneuvering capabilities of a robotic manipulator over a free-flying interceptor, namely a missile. The overall interception scheme includes two phases as; Phase I during which AIPNG technique is in control, and Phase II during which the robot is under the control of a tracking method, namely a PD-type CT-method. It is shown that since the AIPNG provides the tracker with the optimal interception condition, it is superior to the proposed method in the Chapter 3. However, reliable estimation of the target’s acceleration is necessary for the AIPNG technique.
AIPNG also reduces the dimensionality of the interception problem to one regardless of the target’s motion type. This specifically yields a methodology for further interception-time improvement when combined with a tracker.

The implementation of the proposed technique has been illustrated via simulation examples. It has been clearly shown that the hybrid interception method proposed herein yields results favorable over the pure conventional tracking methods, namely a PD-type CT-method.
CHAPTER FIVE

CONVERGENCE ANALYSIS OF THE PROPOSED IDEAL PROPORTIONAL AND AUGMENTED IDEAL PROPORTIONAL NAVIGATION GUIDANCE TECHNIQUES

In this chapter, the convergence analyses of the IPNG and AIPNG techniques in terms of bringing the robot to the rendezvous with the target are discussed. It will be shown that the AIPNG technique assures interception regardless of the target's motion type. The conditions under which the IPNG yields an interception will also be explained. This analysis will be extended to the convergence of the overall interception scheme combining the navigation-based interception scheme with a CT-based tracking method under ideal conditions.

Next, noise sensitivity analysis of the proposed hybrid interception techniques will be discussed in detail. This will be carried out by adding white noise to the target's position readings. Noise sensitivity analysis is critical since the performance of the proposed hybrid interception scheme depends on the correct estimation of the optimal switching point, which is in turn affected by noisy target-position measurements.

5.1 CONVERGENCE ANALYSIS UNDER IDEAL CONDITIONS

As was mentioned in Chapter 2, the interception of fast-maneuvering targets is guaranteed for the IPNG technique if (1) the target acceleration varies with a known bound, (2) the navigation gain, $\lambda$, is chosen large, and (3) the interceptor has a speed superiority over the target. It was also shown that when the interceptor keeps its heading toward the target during the motion, then, $d(LOS)/dt < 0$ for $t \in [0, T_f]$, (i.e., the distance between the target and the interceptor continuously decreases over time). However, a fast dynamics for the interceptor's control system is assumed in the interception scenario. Namely, the acceleration command found through the navigation technique is applied immediately. The impact of the interceptor dynamics on the convergence of the IPNG method is discussed below.
Let us consider the simplified diagram of the linearized IPNG-loop shown in Figure 5.1, where $t_{go}$ is the time-to-go (i.e., the remaining time up to the intercept, $t_{go} = T_f - t$, where $t$ is the current time and $T_f$ is the interception time), $a_i$ and $a_T$ are the interceptor and target acceleration, respectively. $y$ is the distance between the target and the interceptor projected in a direction normal to the reference line (i.e., reference LOS), $V_{rel}$ is the relative velocity between the target and the robot, $\theta_{LOS}$ is the LOS angle, and $\lambda$ is the navigation gain. The command acceleration of the IPNG is executed by the robotic manipulator with a time-delay, Appendix E. The IPNG law is implemented as: 

$$a_c = \lambda V_{rel} \dot{\theta}_{LOS}.$$ 

It can be shown that, this system is analogous to the one reported in [59,60]. Figure 5.2, where in this figure $V_c$ denotes the closing velocity between the target and the interceptor (i.e., the relative velocity between the target and the interceptor projected along the LOS direction). The analogy between the systems represented in Figures 5.1 and 5.2 is discussed below.

**Figure 5.1:** First order interception loop for the IPNG.

**Figure 5.2:** First order interception loop for the PNG, [59,60].
Yuan et al. [33] showed that for maneuvering targets with an acceleration always normal to the relative velocity between the target and the interceptor, $\dot{\theta}_{LOS}$ approaches $|a_T|/[(\lambda - 2)|V_{rel}|]$. Therefore, for bounded target's acceleration, a large navigation gain, and speed superiority of the interceptor over the target, the steady-state value of the $\dot{\theta}_{LOS}$ will be small. Small $\dot{\theta}_{LOS}$ suggests that the $V_{rel}$ is parallel to the LOS direction, thus, $V_{rel} = V_C$. It should be noted that a target whose acceleration is always normal to the relative velocity, $V_{rel}$, represents a fast-maneuvering target with the utmost resistance to turning the relative velocity toward the LOS, [33]. This, subsequently, suggests that for all other types of the target maneuver, the steady-state value of the $\dot{\theta}_{LOS}$ would be even smaller than the aforementioned terminal angular velocity of the LOS, namely $|a_T|/[(\lambda - 2)|V_{rel}|]$.

By comparing Figures 5.1 and 5.2, one can conclude that the overall transfer function of the systems represented in these figures are analogous, namely the relationship between the miss distance and the target's acceleration must be the same for both systems when $V_{rel} = V_C$. Convergence of the close-loop system represented in Figure 5.1 can be analysed by calculating the miss distance (i.e., terminal distance between the target and the robot's end-effector). Since the system represented in Figure 5.1 includes a time-dependent term (i.e., $t_{go}$), the Laplace transform technique is not adequate to calculate the miss distance. Zarchan [59] suggests using a standard adjoint method for this purpose. The miss distance at intercept time, $T_f$, is, thus, written as:

$$M(T_f) = \int_0^{T_f} f(T_f - t^*) a_r(t^*) dt^*, \quad (5.1)$$

where $t^*$ is the adjoint time and $f$ represents a time-dependent scaling of a unit-step change in target's acceleration. By using the convolution theorem, Equation (5.1) can be written in the frequency domain as:

$$M(s) = F(s)A_r(s) = \frac{F(s)Y(s)}{s^2}, \quad (5.2)$$

where $F(s)$ and $Y(s)$ are the Laplace transforms of the function $f(t)$ and $Y_r(t)$, respectively.

Zarchan [59], has also shown that:
The transfer function \( G(s) \) in Equation (5.3) relates the final miss distance (i.e., steady-state miss distance) to the input target acceleration for the 1st-order guidance system. Hence, steady state refers to the condition when all transients have disappeared and the relation \( \tau/T_f << 1 \) is satisfied.

Targets with step and sinusoidal maneuvers are normally considered in the literature for convergence analysis. By taking the inverse Laplace transform of Equation (5.3), one can obtain the miss distance for a step maneuver in the target's acceleration as follows:

\[
\text{miss} \bigg|_{\lambda=3} = \frac{1}{2} T_f^2 e^{-T_f/T}.
\]

As can be seen from Equation (5.4), for a large interception time, \( T_f \), or a small time-constant of the guidance system, \( \tau \), the miss distance approaches zero.

For targets with sinusoidal maneuvers; \( f(t) = A_T \sin(\omega t) \). In this case, the transfer function \( G(s) \), given in Equation (5.3), may be physically interpreted as follows: If the linear interception scheme given in Figure 5.1 is driven by a sinusoidal input of frequency \( \omega \), then, the final miss distance (after transients have disappeared) is a sinusoidal function of \( \omega T_f \) with an amplitude which is \( H \) times the input amplitude, \( A_T \), and with a phase lag \( \varphi \). Namely, the steady-state miss distance for an input \( A_T \sin(\omega t) \) is given by:

\[ M(T_f) = H A_T \sin(\omega T_f + \varphi), \]

where \( H = |G(j \omega)| = \omega^{\lambda - 2} \tau^\lambda \left(1 + (\omega \tau)^2\right)^{-\frac{1}{2}}, \) and \( \varphi = \angle G(j \omega) = (\lambda - 2) \left(\frac{\pi}{2}\right) - \lambda \tan^{-1}(\omega \tau). \) As can be seen from Equation (5.5), the miss distance remains bounded as long as \( A_T \) remains bounded. One can conclude that the convergence of the IPNG for step- and sinusoidal-maneuver in target's acceleration is guaranteed.

In regard to AIPNG, convergence is assured regardless of the target's motion type. As was mentioned in Chapter 4, since the performance of the AIPNG for fast-
maneuvering targets is similar to the performance of the IPNG for non-maneuvering targets, the convergence conclusion of the IPNG technique for non-maneuvering targets leads to the convergence conclusion of the AIPNG in general.

The convergence of the proposed hybrid interception method also depends on the convergence of the Phase II of our proposed method, namely that of the PD-type CT-method. It is well-known that PD controllers yield a converging solution without any prior restrictions on the gains, [52]. In general, a PD-based controller assures global asymptotic stability for a tracking control problem for robotic manipulators under some simplifying assumptions (e.g., the gravity force exerted on the robot’s links is perfectly compensated, external disturbances are not present and non-linear friction is negligible). This suggests that the convergence of the proposed hybrid technique is guaranteed under the aforementioned conditions.

5.2 CONVERGENCE ANALYSIS UNDER NOISY CONDITIONS

In practice, information on the target’s state provided to the interception scheme through the vision system is corrupted by noise. Thus, the performance analysis of the proposed interception techniques in regard to robustness to noise is addressed in this section.

Based on the target’s position estimates obtained by a vision system, a digital filter is normally used to estimate the target’s state. The target-state estimates can then be substituted into a motion model to predict the target’s future trajectory if needed. In this context, the Kalman filter has been widely used in robotic interception problems, [e.g., 6,16,17,39,61-64], and thus, it will be briefly discussed below.

5.2.1 The Kalman Filter

As was originally proposed in [65], the Kalman Filter (KF) is a computationally efficient recursive filter which generates an optimal least-square estimate from a sequence of noisy observations.

Let a discrete-time linear dynamic system be described by the following system model:
\[
x(k+1) = F x(k) + w(k), \quad w(k) = N(0, Q(k))
\]
where \( x \) is the state vector, \( w \) is a sequence of zero-mean white Gaussian process noise with covariance matrix \( Q \), and \( F \) is the state-transition matrix. Let also the measurement model be given as:
\[
y(k) = M x(k) + v(k), \quad v(k) = N(0, R(k))
\]
where \( y \) is the noisy observed measurement, \( M \) is the measurement matrix, and \( v \) is a sequence of zero-mean white Gaussian noise with covariance matrix \( R \). The problem at hand, is then to define \( \hat{x}(k) \), which is the best linear estimate (filtered value) of \( x(k) \).

Equations (5.8a) through (5.8e) below define the procedure for the recursive formulation of the optimal KF. The recursion is carried out in the following two stages:

(i) Prediction:
\[
\hat{x}(k|k-1) = F \hat{x}(k-1|k-1),
\]
\[
P(k|k-1) = F P(k-1|k-1)F^T + Q(k-1).
\]

(ii) Update or correction:
\[
\hat{x}(k|k) = \hat{x}(k|k-1) + K [y(k) - H \hat{x}(k|k-1)],
\]
\[
P(k|k) = P(k|k-1) - K H P(k|k-1),
\]
\[
K = P(k|k-1)H^T [H P(k|k-1)H^T + R(k)]^{-1},
\]
where \( P, Q, \) and \( R \) are defined as:
\[
E[e(k)e^T(k)] = P(k), \quad E[w(k)w^T(l)] = Q(k)\delta_{kl}, \quad E[v(k)v^T(l)] = R(k)\delta_{kl}, E[w(k)v^T(l)] = 0.
\]
\[
e(k) \text{ in Equation (5.9) denotes the error in state’s estimation, } e(k) = x(k) - \hat{x}(k), \quad \delta \text{ is the Kronecker delta, } K \text{ denotes the optimal gain of the KF, and } E \text{ is the expected value of the random variable.}
\]

Variety of modifications have been applied to the original KF in order to enhance its performance in different applications. Extended Kalman Filters (EKF) have been

---

1 In general, the system model is presented as:
\[
x(k + 1) = F x(k) + G U(k) + w(k),
\]
where \( U(k) \) is the known input signal or control vector. However, in object-tracking algorithms, this term is usually neglected.

2 In general, the measurement model is presented as:
\[
y(k) = H x(k) + L U(k) + v(k),
\]
where \( U(k) \) is the known input signal or control vector. However, in object-tracking algorithms, this term is usually neglected.
suggested to cope with nonlinearities in a system. [e.g., 66]. \(\alpha-\beta-\gamma\) filters, which are an extension of the steady-state Kalman filters, have been developed for time-invariant systems and used in visual-servoing techniques due to their capability of being on-line implementable.

5.2.2 Kalman Filters and Object Motion Estimation

A number of models have been suggested for the object's motion: the constant-velocity model, 1st-order Gauss-Markov object's jerk model, 2nd-order Gauss-Markov object's jerk model, and constant-acceleration model are the most commonly used ones, [67]. The effect of different motion modeling techniques on the Kalman filtering procedure can be better understood through Equation (5.6), where each aforementioned model yields a different state-transition matrix, \(F\), and a different form of \(Q\).

Although these motion models yield different results when being used for long-term prediction of target's motion, they yield similar results for immediate target-motion estimation and/or one-step-ahead prediction, [17]. A constant-acceleration model was utilized in this thesis, due to its simple gain-tuning procedure, Appendix F.

5.2.3 Noise Sensitivity of the IPNG and AIPNG

Two issues have to be addressed in regard to the sensitivity analysis of the proposed interception technique to measurement noise:

(i) The convergence of the proposed technique to a solution in the presence of noise; and,

(ii) The feasibility of achieving a time-optimal solution based on on-line interception-time estimation in presence of noise.

These issues are individually discussed below.

(i) Convergence: As discussed in Chapter 3 and 4, the success of the proposed algorithm depends on our ability to estimate the robot motion time under the control of the CT-based method. In this sub-section, the effect of noise on this estimation is investigated together with the simulated "real" interception time. In order to study the impact of noise
in target’s direct vision-based position readings on the convergence of the proposed interception scheme, the target’s trajectory given in Equation (4.52) is simulated following:

\[ \begin{align*}
    x_t &= 1.0 + 0.2 \sin(2\pi f t) + e_x(t) \\
    y_t &= 1.2 - 0.2 t + e_y(t),
\end{align*} \]

(5.10)

where \( f = 0.25 \text{ s}^{-1} \) and \( e_x(t) \) and \( e_y(t) \) are white Gaussian noise processes with zero means and variances varying from \( \sigma_x = \sigma_y = (0.002)\text{m} \) to \( \sigma_x = \sigma_y = (0.015)\text{m} \).

Figures (5.3a) through (5.3g) show the variation of the simulated real and estimated interception times versus the switching time for the IPNG+CT method for different levels of noise. As can be seen from Figure (5.3a), there exists a good match between the real and estimated interception times for noiseless target trajectories. When noise is added to the system, the estimated interception time starts deviating from the real interception time as the noise variance increases, Figures (5.3b) through (5.3g). Namely, although convergence to a solution (i.e., target interception) is achieved at all noise levels considered, the interception process becomes increasingly unpredictable.

For example, at \( \sigma_x = \sigma_y = 0.015\text{m} \) in Figure 5.3g, the estimated interception time fluctuates quite randomly as a function of switching time, therefore, a reliable estimation of the optimal interception time can no longer be expected.

![Figure 5.3a: IPNG + CT. No noise added.](image-url)
Figure 5.3b: IPNG + CT. $\sigma_x = \sigma_y = 2$ mm.

Figure 5.3c: IPNG + CT. $\sigma_x = \sigma_y = 4$ mm.

Figure 5.3d: IPNG + CT. $\sigma_x = \sigma_y = 6$ mm.

Figure 5.3e: IPNG + CT. $\sigma_x = \sigma_y = 8$ mm.

Figure 5.3f: IPNG + CT. $\sigma_x = \sigma_y = 10$ mm.

Figure 5.3g: IPNG + CT. $\sigma_x = \sigma_y = 15$ mm.
It should be noted that the sensitivity of the system to noise is naturally dependent on the definition of the interception tolerance. The finer this tolerance is chosen, the more sensitive the proposed interception scheme would be to noise. In our examples, the tolerance level was kept constant.

The simulation results given in Figures (5.3a) through (5.3g) were obtained for only one random noise seed value. However, in order to show that the results are statistically significant, three different noise seed values were utilized. The comparative results are given in Table 5.1. Since, the same simulation procedure was carried out for the AIPNG + CT method as well, Figures (5.4a) through (5.4g), the corresponding optimal interception times are also included in Table 5.1.

![Figure 5.4a: AIPNG + CT. No noise added.](image)

**Table 5.1: Comparing the optimal interception times of the pure CT, IPNG + CT, AIPNG + CT and AIPNG + mod_CT for different noise levels in target's position readings.**

<table>
<thead>
<tr>
<th>METHO</th>
<th>PURE CT</th>
<th>IPNG + CT</th>
<th>AIPNG + CT</th>
<th>AIPNG + MOD_CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_x = \sigma_y = 0) mm</td>
<td>5.00</td>
<td>2.80</td>
<td>2.60</td>
<td>2.30</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 2) mm</td>
<td>4.75, 4.75, 4.75</td>
<td>2.70, 2.70, 2.70</td>
<td>2.50, 2.50, 2.50</td>
<td>2.30, 2.30, 2.30</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 4) mm</td>
<td>4.80, 4.80, 4.75</td>
<td>2.75, 2.70, 2.70</td>
<td>2.50, 2.50, 2.50</td>
<td>2.50, 2.30, 2.35</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 6) mm</td>
<td>4.90, 4.90, 4.90</td>
<td>2.70, 2.70, 2.75</td>
<td>2.50, 2.50, 2.50</td>
<td>2.50, 2.50, 4.30</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 8) mm</td>
<td>5.20, 5.25, 5.25</td>
<td>2.65, 2.70, 2.70</td>
<td>2.45, 2.45, 2.50</td>
<td>4.90, 4.75, 4.90</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 10) mm</td>
<td>5.25, 5.25, 5.25</td>
<td>2.65, 2.65, 2.70</td>
<td>2.45, 2.50, 2.50</td>
<td>2.50, 5.00, 5.20</td>
</tr>
<tr>
<td>(\sigma_x = \sigma_y = 15) mm</td>
<td>5.30, 5.20, 5.20</td>
<td>2.50, 6.50, 6.50</td>
<td>2.45, 6.50, 6.30</td>
<td>5.00, 5.00, 5.30</td>
</tr>
</tbody>
</table>
As expected and shown in Table 5.1, the AIPNG + mod_CT shows more sensitivity to noise in comparison to IPNG techniques. This is due to the fact that, in this method, an accurate estimate of Point E, see Figure 4.15, is needed in order to avoid overshooting. Noise in the system leads to an erroneous estimation of the coordinates of Point E in the phase-plane.

![Figure 5.4b: AIPNG + CT. $\sigma_x = \sigma_y = 2$ mm.](image1)

![Figure 5.4c: AIPNG + CT. $\sigma_x = \sigma_y = 4$ mm.](image2)

![Figure 5.4d: AIPNG + CT. $\sigma_x = \sigma_y = 6$ mm.](image3)

![Figure 5.4e: AIPNG + CT. $\sigma_x = \sigma_y = 8$ mm.](image4)
(ii) Time optimality: As shown in Chapters 3 and 4 and discussed above, determining accurately the Optimal Switching Point (OSP) is critical to the proposed hybrid interception scheme. The OSP represents the point at which the robot's trajectory is at the threshold of overshooting. Thus, unnecessarily prolonging the robot's motion under the control of the IPNG, in the IPNG + CT method, or the AIPNG, in the AIPNG + CT method, beyond OSP, jeopardises the time-optimality of the interception. However, as discussed above, when noise is present in the system, the predictability of the interception time becomes increasingly difficult.

Figure 5.5 shows a conceptual diagram representing the estimated and real interception times versus the switching time in presence of noise. As can be seen from this figure, the estimated interception time may have local minima in Region #1 and Region #2 of the interception-time curve. This suggests that switching from the IPNG techniques to the CT-phase may be caused prematurely very early in the motion, namely in Region #1. In order to account for this, one should carry out an uncertainty analysis associated with the interception-time estimation at each time step of the robot motion. This uncertainty analysis would determine if a local minimum in Region #1 is due to noise in the system or it really reflects the occurrence of the OSP. This uncertainty estimation is discussed below.
Figure 5.5: Conceptual diagram of the real and estimated interception times in the presence of noise.

One can recall that for the IPNG + CT method, the interception time is estimated as $\tilde{t}_{int} = t_{IPNG} + \tilde{t}_{CT}$, and for the AIPNG + CT method as $\tilde{t}_{int} = t_{AIPNG} + \tilde{t}_{CT}$. One can, thus, conclude that, at any time during the robot motion, the uncertainty related to the interception-time estimation, $\tilde{t}_{int}$, can be determined by computing the uncertainty related to the calculation of $\tilde{t}_{CT}$ (since $t_{IPNG}$ and $t_{AIPNG}$ are already known past-time variables). Therefore,

$$\delta \tilde{t}_{int} = \delta \tilde{t}_{CT},$$

(5.11)

where the notation $\delta$ denotes the uncertainty in the random variable.

$\tilde{t}_{CT}$ can be related to the uncertainty on the target’s position readings (i.e., noisy measurements) by solving the position-error equation of a PD-type CT-technique given as:

$$\ddot{r} + K_d r + K_p r = 0.$$

(5.12)

For a critically-damped system (i.e., $K_d = 2\sqrt{K_p}$) the solution of Equation (5.12) for $r$ and $\dot{r}$ are as follows:
Equations (5.13a) and (5.13b) can be numerically solved to estimate the remaining time to intercept, i.e., $\tilde{t}_{\text{CR}}$. Herein, intercept is defined as the point at which:

$|r| \leq (\text{Tol})_p$ and $|\dot{r}| \leq (\text{Tol})_v$. Now, let us assume that the interception point has been computed as the point at which $|r| = A$, and $|\dot{r}| = B$, where $A \leq (\text{Tol})_p$ and $B \leq (\text{Tol})_v$. By substituting $r$ and $\dot{r}$ in Equations (5.13a) and (5.13b) with $A$ and $B$, respectively, and solving for time, $t$, one can obtain:

$$
\tilde{t}_{\text{CR}} = \frac{2}{K_d} \ln \left( \frac{\tilde{r}_0 + (K_d/2)r_0}{B + (K_d/2)A} \right).
$$

Equation (5.14)

As can be seen from Equation (5.14), the uncertainty associated with $\tilde{t}_{\text{CR}}$ is related to the uncertainties associated with $r_0$ and $\dot{r}_0$. In general, the basic equation of uncertainty analysis of a variable $R$ dependent on $N$ individual variables, is given as:

$$
\delta R = \left\{ \sum_{i=1}^{N} \left( \frac{\partial R}{\partial X_i} \delta X_i \right)^2 \right\}^{1/2},
$$

where $\delta X_i$ denotes the uncertainty in an individual variable. Using Equation (5.15), one can write:

$$
\delta \tilde{t}_{\text{int}} = \delta \tilde{t}_{\text{CR}} = \sqrt{\left( \frac{\partial \tilde{t}_{\text{CR}}}{\partial r_0} \delta r_0 \right)^2 + \left( \frac{\partial \tilde{t}_{\text{CR}}}{\partial \dot{r}_0} \delta \dot{r}_0 \right)^2}.
$$

Equation (5.16) can be rewritten considering Equation (5.14) as follows:

$$
\delta \tilde{t}_{\text{int}} = \delta \tilde{t}_{\text{CR}} = \frac{4}{2\tilde{r}_0 + K_d r_0} \sqrt{\left( \delta r_0 \right)^2 + \left( \frac{2}{K_d} \delta \dot{r}_0 \right)^2}.
$$

Equation (5.17)
As can be seen from Equation (5.17), the uncertainty in the estimated interception time depends on the uncertainties in \( r_0 \) and \( \dot{r}_0 \). These can be related to the uncertainties in the target’s state-estimation as follows:

\[
\delta r_0 = \delta (X_T - X_R) = \delta X_T, \quad (5.18a)
\]

\[
\delta \dot{r}_0 = \delta (\dot{X}_T - \dot{X}_R) = \delta \dot{X}_T, \quad (5.18b)
\]

where \( X_T \) and \( X_R \) denote the target’s and robot’s positions, respectively. Equations (5.18a) and (5.18b) are derived based on the assumption that the uncertainty levels in the robot’s position and velocity are negligible when compared with those of the target.

Since the information on the measurement noise (i.e., its variance) is not available to the interception scheme a priori, the best way to assign an uncertainty to the target’s position readings is to use the innovation sequences in the KF, which are established in the matrix \( P \), given in Equation (5.8d), [68]. \( \delta X_T \) and \( \delta \dot{X}_T \) can be computed at each time-step using Equation (5.8d) of the KF as:

\[
\delta (X_T)_i = \sqrt{P(n(i-1)+1, n(i-1)+1)}, \quad i = 1, 2, ..., n \quad (5.19a)
\]

\[
\delta (\dot{X}_T)_i = \sqrt{P(n(i-1)+2, n(i-1)+2)}, \quad i = 1, 2, ..., n \quad (5.19b)
\]

where \( n \) denotes the dimension of the space through which the target moves (i.e., in the most general form \( n \) is 3).

Figures 5.6a through 5.6d show the individual estimated interception times and their associated uncertainties at each time step for the IPNG + CT method for a moving object whose motion is given in Equation (5.10). Figures 5.7a through 5.7d show the same results for the AIPNG + CT method. These figures clearly show that the sharp change in the interception-time estimation at the OSP is beyond the uncertainty level associated to this estimate. This suggests that the sharp change in the interception-time estimation has not been generated by the target’s position measurement-noise, but it is solely due to the overshoot in the robot’s trajectory. In contrast, at the beginning of the motion, i.e., Region #1 in Figure 5.5, the interception time might show an increase in interception, which actually is within the uncertainty level of estimation of the interception time if one were to consider the next time step as the OSP. One might, thus, consider to devise a decision-making algorithm to cope with such a situation.
Figure 5.6a: Estimated interception time for IPNG + CT. $\sigma_x = \sigma_y = 2$mm.

Figure 5.6b: Estimated interception time for IPNG + CT. $\sigma_x = \sigma_y = 4$mm.

Figure 5.6c: Estimated interception time for IPNG + CT. $\sigma_x = \sigma_y = 8$mm.

Figure 5.6d: Estimated interception time for IPNG + CT. $\sigma_x = \sigma_y = 10$mm.

Figure 5.7a: Estimated interception time for AIPNG + CT. $\sigma_x = \sigma_y = 2$mm.

Figure 5.7b: Estimated interception time for AIPNG + CT. $\sigma_x = \sigma_y = 4$mm.
Figure 5.7c: Estimated interception time for AIPNG + CT. $\sigma_x = \sigma_y = 8$mm.

Figure 5.7d: Estimated interception time for AIPNG + CT. $\sigma_x = \sigma_y = 10$mm.

5.3 SUMMARY

In this chapter, the convergence of the IPNG and AIPNG techniques in terms of bringing the robot to a rendezvous point with the target were discussed. The behavior of aforementioned techniques when facing either a step- or a sinusoidal-maneuver in target’s acceleration under ideal conditions was explained. In this context, the robot’s dynamics was modelled as a simple-lag system. It was shown that the AIPNG technique assures interception regardless of the target’s motion type.

Noise sensitivity analysis of the proposed hybrid interception technique was also discussed. This was carried out by adding Gaussian noise to the target’s position readings. A conventional Kalman filter was used to estimate target’s state (i.e., target’s position, velocity and acceleration).
CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY AND CONCLUSIONS

Two methods have been presented for on-line-robotic-interception of fast-maneuvering objects which do not require a priori information on the moving-object’s motion. These techniques combine a navigation-based method with a conventional object-tracking technique. Thus, they can be classified as hybrid interception schemes with two phases: Phase I, during which the robot is under the control of a navigation-based interception technique, and Phase II, during which the robot’s control is switched to a conventional tracking method. Both proposed techniques perform favorably over the pure tracking method when the transition from Phase I to the Phase II occurs at an optimal time rendering a non-overshooting robot’s trajectory.

An Ideal Proportional Navigation Guidance technique has been used in Phase I. This technique seeks to nullify the angular rate of the LOS, bringing the interceptor toward the interception point as fast as possible. As an alternative, the augmented form of this technique has been suggested when a reliable estimation of the target’s acceleration can be provided to the interceptor. Since these techniques have been originally designed for missile guidance, they do not attempt to match the target’s velocity at the interception point. For smooth interception, a tracking method has been proposed to be switched on to at an optimal time to bring the robot to the interception point matching both the target’s position and velocity.

On-line selection of the aforementioned time-optimal switching point has been discussed. A technique has been developed for estimating the interception time on-line. This scheme allows us to determine the switching point regardless of the target’s motion type. The time-optimal solution yields a non-overshooting robot’s trajectory toward the interception point. The effect of the noise in target’s position readings through a vision system, and also the impact of the uncertainties associated with these readings on the on-line estimation of the interception time, and, subsequently, on the overall performance of the proposed technique have also been studied.
(i) Benefit of IPNG Interception Schemes

The IPNG is a navigation technique that has its roots in the PNG law. The control input is in an acceleration command form, which is proportional to the angular rate of the LOS and the relative velocity between the target and the interceptor. It tries to send the interceptor toward the interception point, while keeping its course with the target constant, as fast as possible. PNG-based techniques provide a time-optimal solution to the interception problem for cruising targets (i.e., targets moving with constant speed).

The AIPNG is an augmentation of the IPNG technique taking the target's acceleration into consideration. This technique has been suggested for fast-manuvering targets whose acceleration can be reliably estimated on-line. This technique needs the on-line use of filters/estimators for estimating the target's acceleration. Although this may seem as a drawback of the AIPNG, some of its significant advantages are: (1) reduction in dimensionality of the interception problem, and (2) guaranteed convergence of the motion regardless of the target's motion type. These make the AIPNG very attractive for robotic interception.

Robotic manipulators are different from airborne missiles with regard to their maneuvering capability. In order to reflect these capabilities, a methodology for further upgrading the acceleration command of the IPNG and AIPNG has been developed in this thesis. It has been addressed that fast interception is achievable by upgrading the $a_{IPNG}$ and $a_{AIPNG}$ in the LOS direction.

(ii) Need for a Hybrid Interception Scheme

The navigation-based techniques bring the interceptor into a collision course with the target rather than a smooth grasp. A hybrid interception method combining a navigation technique with a conventional tracking method has been proposed to overcome this potential drawback. A PD-type CT-method has been utilized for the tracking part. A methodology for on-line estimation of the overall interception time has been developed. The time-optimal performance of this hybrid scheme depends on the on-line selection of the switching point. The estimated interception time versus the switching time (i.e., the current clock time) generally yields a single-minimum curve. The minimum of this curve represents the
minimum interception time achievable by this hybrid interception technique. Proper estimation of this point is very crucial to the proposed method, hence going beyond this point yields an overshooting in robot’s trajectory and subsequently a longer interception time.

(iii) Modified CT for AIPNG

As was mentioned earlier, the AIPNG reduces the dimensionality of the interception problem to one (i.e., the relative velocity and acceleration between the target and the robot lie along the LOS). This facilitates the use of an effective methodology for further interception-time improvement. In this technique, instead of switching at the OSP, one can let the robot move as instructed by the AIPNG, then, at a certain point, the robot starts moving with its maximum permissible deceleration to avoid overshoot. Namely, the robot keeps moving under the AIPNG control up to a point at which any further attempt by the AIPNG to close the distance between the target and the interceptor would induce an overshoot in the robot's trajectory. At this point, a PD-type CT-method takes over.

It is important to note that, there exists no need for on-line estimation of the interception time for this technique, thus, making it more suitable for on-line implementation. However, as was explained in the thesis, this technique shows sensitivity to the accuracy in on-line estimation of the switching point (i.e., the point at which a PD-type CT-method takes over).

(iv) Convergence Analysis

Convergence of the IPNG technique for slow- and fast-maneuvering objects has been discussed. It has been shown that under ideal conditions, interception is guaranteed for slow-maneuvering objects when a navigation gain greater than 1 is selected for IPNG. It has also been shown that, interception for fast-maneuvering targets is also guaranteed when (1) the target's acceleration is bounded, (2) a large navigation gain is chosen, and (3) the robot is initially heading toward the target. The convergence of the IPNG for the interceptor dynamics, modeled as a simple-lag, has been discussed in regard to the step- and sinusoidal-maneuvers in target's acceleration.
It has been shown that AIPNG guarantees interception regardless of the target's motion type when (1) the interceptor has speed superiority over the target, and (2) the target's acceleration is bounded. In the AIPNG, the LOS and its angular rate approach zero for $\lambda > 1$ and $\lambda > 2$, respectively.

Sensitivity of these techniques to noise in the target's position readings has also been addressed. It has been shown that the general trend of the simulated interception time versus the switching time does not change until a significantly high noise level. The effect of measurement noise on on-line estimation of the interception time, and subsequently, on the selection of the time-optimal switching point has also been discussed.

6.2 RECOMMENDATIONS

Further potential research related to the robotic object-interception problem addressed in this thesis is briefly discussed below. The first issue is in regard to attempting to improve upon the techniques proposed in this thesis. The second issue, on the other hand, is potential extension of the research area.

(1) Points (i) through (iii) below, explain the potential tasks for increasing the robustness of the proposed method.

(i) In the AIPNG + modified CT method, accurate estimation of Point E and bringing the robot to the vicinity of this point is a key factor in achieving shorter interception times, Figure 4.15. Inaccurate estimation of this point might cause an overshoot in robot's trajectory. In this case, the robot's trajectory in the phase-plane does not precisely follow the Segment (E-O). A Variable Structure Control (VSC) technique [69] can be utilized to maintain the robot's trajectory along the Segment (E-O). Thus, the robot's trajectory slides along the desired phase-plane trajectory within a tolerance, avoiding an overshoot.

(ii) The effect of the unmodeled dynamics and disturbances on the performance of the proposed method may be investigated. How to integrate a robust object-tracking method with a navigation-based technique in order to account for the aforementioned uncertainties in the system can be studied.

(iii) The potential of improving the Kalman filter to estimate the target's state in presence of measurement noise can also be studied. One drawback of Kalman filters is their
slow transient response when the target performs sudden maneuvers (e.g., a step-change in target's velocity). The effect of these maneuvers on the performance of the Kalman filters, and subsequently, on the proposed interception technique can be investigated.

(2) The proposed navigation techniques in this thesis can be extended by integrating them with an APPE-like approach for achieving more efficient interception time for slow-maneuvering objects. It is recommended to integrate these two techniques by possibly combining their acceleration commands as follows:

\[ a_{total} = \alpha a_{IPNGIAIPNG} + \beta a_{APPE}, \quad (6.1) \]

\( \alpha \) and \( \beta \) in Equation (6.1) are weighting factors, where \( \alpha, \beta \in [0,1] \), and \( \alpha + \beta = 1 \).

In APPE techniques, a global-time-optimal solution to the interception problem is achievable for slow-maneuvering targets. The acceleration profile of the robot's end-effector along the time-optimal trajectory can be used by a decision-making mechanism, which chooses on-line approximation of the weighting factor values. For example, a large value for \( \alpha \) may be selected if the target is fast-maneuvering, and a large value of \( \beta \) may be selected for slow-maneuvering objects.
References


Let us consider the spherical coordinates \((r, \theta, \phi)\) with the origin of the frame being located on the interceptor, where \(r\) is the relative distance between the target and the interceptor and \(\theta\) and \(\phi\) are the azimuth angles. Let also \((e_r, e_\theta, e_\phi)\) be unit vectors along the coordinate axes, Figure A1. One can write:

\[
\dot{r} - r\dot{\phi}^2 - r\dot{\theta}^2 \cos(\phi) = a_r - a_r', \tag{A1a}
\]

\[
r\dot{\theta} \cos \phi + 2r\dot{\phi} \cos \phi - 2r\dot{\phi} \sin \phi = a_\theta - a_\theta', \tag{A1b}
\]

\[
r\ddot{\phi} + 2r\dot{\phi} + r\dot{\theta}^2 \cos \phi \sin \phi = a_\phi - a_\phi', \tag{A1c}
\]

where \(a_r', a_\theta', a_\phi'\) are the acceleration components for the target and the interceptor, respectively.

Yang et al. [32] showed that the angular momentum of the unit mass can be used to simplify the above equations. This angular momentum for the interceptor-target relative motion is defined as:

\[
h = r \times \dot{r}, \quad \tag{A2}
\]

where \(r\) is the relative-position vector along the LOS and \(\dot{r}\) is the relative velocity. Yang et al. [32] also showed that the set of unit vectors \((e_r, e_\perp, e_h)\) constitutes a new moving coordinate system that is more convenient for describing 3D guidance laws than the conventional spherical coordinates \((e_r, e_\theta, e_\phi)\). The kinematic equations given in Equation (A1) can then be rewritten as follows:

\[
\dot{h} = r \times \dot{e}_h, \quad \tag{A3a}
\]

\[
\dot{e}_h = \left(\frac{r}{h}\right) \dot{e}_h e_\perp, \quad \text{and} \quad \tag{A3b}
\]

\[
\ddot{r} - \left(\frac{h^2}{r^3}\right) = a_r'. \tag{A3c}
\]

The command acceleration of the interceptor based on the AIPNG technique is:

\[
a_i = \lambda \dot{\theta} \times \dot{\theta}_{\text{LOS}} + a_T. \tag{A4}
\]

Therefore, the relative acceleration between the target and the interceptor can be computed as:
\[ \ddot{r} = a_r - a_r = -\lambda \dot{r} \times \dot{\theta}_{\text{LOS}}, \quad (A5) \]

It is known that \( \dot{\theta}_{\text{LOS}} = (r \times \dot{r})/r^2 = h/r^2 \). By placing this expression in Equation (A5) and rearranging the remaining terms, one obtains:

\[ \ddot{r} = \lambda \left( \frac{h \dot{r}}{r^2} \right) e_r - \lambda \left( \frac{h^2}{r^3} \right) e_r. \quad (A6) \]

By substituting Equation (A6) into Equation (A3), one can rewrite the kinematic equations of the interception as follows:

\[ \ddot{r} + (\lambda - 1) \frac{h^2}{r^3} = 0, \quad (A7a) \]

\[ \dot{h} = \lambda h \left( \frac{\dot{r}}{r} \right), \quad (A7b) \]

\[ \dot{e}_h = 0. \quad (A7c) \]

As expected, \( \dot{e}_h \) is zero.

Equation (A7b) can be solved, leading to the following result:

\[ h = h_0 \left( \frac{r}{r_0} \right)^\lambda. \quad (A8) \]

The angular rate of the LOS is, then, calculated as:

\[ |\dot{\theta}_{\text{LOS}}| = \frac{h}{r^2} = \left( \frac{h_0}{r_0} \right)^\lambda \left( \frac{r}{r_0} \right)^{\lambda-2}. \quad (A9) \]

From Equation (A9), \( \dot{\theta}_{\text{LOS}} \) approaches zero when \( r \) approaches zero for a navigation gain selected as \( \lambda > 2 \).

\[ \text{Figure A.1: Three-dimensional interception geometry.} \]
APPENDIX B. A DESIGN GUIDELINE FOR SELECTING THE NAVIGATION GAIN, $\lambda$, FOR AIPNG TECHNIQUE

The interception time for the AIPNG technique can be computed as:

$$ t_{\text{int}} = \int_{r_0}^{r_f} \frac{dr}{\dot{r}} \quad \text{(B1)} $$

where $r_0$ and $r_f$ are the initial and final relative distances between the target and the robot's end-effector, respectively. By solving Equation (A7) for $\dot{r}$ and substituting the result into Equation (B1), one obtains:

$$ T = \frac{t_{\text{int}}}{r_0/\dot{r}_0} = \int_0^1 \frac{dx}{\sqrt{1 - \sin^2 \phi_0 \cdot x^{2-2}}} \quad \text{(B2)} $$

where $T$ is the dimensionless interception time, and $\phi_0$ is the initial angle between the LOS and the relative-velocity vector, $\dot{r}_0$. As expected, Equation (B2) is similar to the one obtained for the IPNG technique for slow-maneuvering objects, [33]. Yuan et al. [33] showed that Equation (B2) can be rewritten as follows:

$$ T = \frac{(\sin \phi_0)^{(\lambda-2)/(\lambda-1)} \cdot \int_0^{(2-\lambda)/(\lambda-1)} (\sin \phi) \cdot (\lambda-1) \cdot \dot{\phi}_{\text{LOS}}(0)}{\lambda - 1} \quad \text{(B3)} $$

It has been shown that $T$ decreases when $\lambda$ increases and it asymptotically approaches zero for large $\lambda$s, namely $\lim_{\lambda \to \infty} T \to 0$. However, large navigation gain yields large expended energy, hence it necessitates a large acceleration effort to bring the interceptor into the right course with the target, [e.g., 29,30,32]. A navigation gain chosen in the range of $3<\lambda<5$ is known to be an effective navigation gain value, [e.g., 70]. However, one should note that the selection of a navigation gain for robotic interception highly depends on the application at hand. A navigation gain $\lambda=5$ was chosen in the proposed AIPNG technique.
The objective of this appendix is to show that in a critically-damped 2nd order system, the overshooting phase-plane trajectory (i.e., a trajectory that crosses the r-axis in the phase-plane) yields longer interception time than one with a non-overshooting trajectory. This suggests that there always exists a sharp increase in the interception time in transition from a non-overshooting to an overshooting trajectory. Figure C1 shows a schematic diagram representing two phase-plane trajectories. In Trajectory I, the interception point is shown as the point with coordinates \((A_I, B_I)\), where \(A_I = -(Tol)_p\) and \(B_I = -(Tol)_v\), and \((Tol)_p\) and \((Tol)_v\) are interception tolerances along \(r\) and \(\dot{r}\) axis, respectively. This trajectory does not intersect the \(r\) axis. For simplicity, it is assumed that the overshooting line lies along the diagonal of the interception tolerance rectangle, Figure C1. In Trajectory II, the interception point is shown as the point with coordinates \((A_2, B_2)\), where \(A_2 = A_1 = -(Tol)_p\) and \(|B_2| < |B_1|\). This trajectory intersects the \(r\) axis. As can be seen from Figure C1, the start point of the two trajectories are assumed to have the same coordinate along the \(\dot{r}\) axis, but with different \(r_0\) values, namely \((r_0)_I > (r_0)_II\).

As described in Chapter 5, the interception time for the CT-based method can be computed as:

\[
\bar{\tau}_{CT} = \frac{2}{K_d} \ln \left( \frac{|\hat{r}_0 + (K_d/2)r_0|}{|B + (K_d/2)A|} \right),
\]

where \(A\) and \(B\) are the coordinates of the interception point in the phase-plane. Therefore, for Trajectory I,

\[
(\bar{\tau}_{CT})_I = \frac{2}{K_d} \ln \left( \frac{|\hat{r}_0 + (K_d/2)r_0|}{|(Tol)_v + (K_d/2)(Tol)_p|} \right)
\]

and for Trajectory II,

\[
(\bar{\tau}_{CT})_II = \frac{2}{K_d} \ln \left( \frac{|\hat{r}_0 + (K_d/2)r_0|}{|-B_2 + (K_d/2)(Tol)_p|} \right).
\]

Now let us compare \((\bar{\tau}_{CT})_I\) and \((\bar{\tau}_{CT})_II\). First, let us examine the denominator of the argument of the logarithm. Since \(B_2 < (Tol)_v\), (note that Trajectory II has to be asymptotically tangent to the overshoot-line), thus, \(|-B_2 + (K_d/2)(Tol)_p| < |(Tol)_v + (K_d/2)(Tol)_p|\). Second,
by comparing the numerators of the logarithmic term, one can readily conclude that
$|\dot{r}_0 + (K_d / 2)(r_0)| < |\dot{r}_0 + (K_d / 2)(r_0)|$, hence $(r_0) < (r_0)$.

It is shown that for any infinitesimally small $\varepsilon$, where $|(r_0) - (r_0)| = \varepsilon$, the condition
$|B_2 + (K_d / 2)(\text{Tol})_p| < |(\text{Tol})_p + (K_d / 2)(\text{Tol})_p|_{\varepsilon}$ is always satisfied. This will subsequently imply that in a transition from a non-overshooting trajectory, namely Trajectory I in Figure C1, to an overshooting trajectory, namely Trajectory II in Figure C1, there always exists an increase in the interception time, namely $(\tau_{\text{CT}}) > (\tau_{\text{CT}})$.

The above statement can be deduced by understanding the facts that (1) any phase-plane trajectory for a critically-damped 2nd order system must be asymptotically tangent to the overshoot-line at origin, and (2) phase-plane trajectories must be orthogonal to the $r$-axis. The latter can be obtained using the following equation:

$$\lim_{r \to 0} \frac{d\dot{r}}{dr} \rightarrow \lim_{\dot{r} \to 0} \dot{r} \rightarrow \infty.$$  \hfill (C2)

This suggests that $|B_2| < (\text{Tol})_p$, thus, $|B_2 + (K_d / 2)(\text{Tol})_p| < (\text{Tol})_p + (K_d / 2)(\text{Tol})_p$ for any value of $\varepsilon$.

![Diagram](image)

**Figure C1:** An overshooting trajectory intersecting the $r$-axis and a non-overshooting trajectory.
APPENDIX D. SELECTION OF TRANSITION POINT E

The objective of this appendix is to show that the closer the point E in Figure 4.15 is to the origin of the phase-plane, the shorter the interception time would be. In order to prove the above statement, one should first derive the characteristics of the Segment (D-E). For simplicity, the square-shaped interception tolerance zone is chosen infinitesimally small. In general, one can write:

\[
\frac{d\dot{r}}{dr} = \frac{\dot{r}}{\dot{r}}. \tag{D1}
\]

By letting \( \dot{r} = C \), where \( C \) is a constant, one can conclude:

\[
\dot{r} \frac{d\dot{r}}{dr} = C \Rightarrow \frac{\dot{r}^2}{2} + K = C \Rightarrow \dot{r} = \sqrt{\frac{2K}{C}} + \sqrt{\frac{2K}{C}}, \tag{D2}
\]

where \( K \) denotes the constant of integration. Equation (D2) suggests that Segment (D-E) can be represented as a parabola.

Figure D1 shows a schematic diagram of the parabolic trajectory given in Equation (D2) represented as Segment (D-E). The question at hand is “Is another parabolic segment, namely Segment (D-E1), also executable by the robot?”. One should note that the significance of choosing Point E1 over Point E2 is that Segment (D-E1) yields a shorter interception time than does Segment (D-E2). The necessary condition for parabolic segments (D-E1) and (D-E2) to intersect at Point D is given as:

\[
|\ddot{r}_{D-E1}| < |\ddot{r}_{D-E2}|. \tag{D3}
\]

In Equation (D3), \( \ddot{r}_{D-E1} \) and \( \ddot{r}_{D-E2} \) denote the constant closing-accelerations of the robot along the Segments (D-E1) and (D-E2), respectively. It is also known that:

\[
\ddot{r}_{D-E1} = \frac{\dot{r}_{E1}^2 - \dot{r}_{D}^2}{2(r_{E1} - r_{D})}, \tag{D4a}
\]

\[
\ddot{r}_{D-E2} = \frac{\dot{r}_{E2}^2 - \dot{r}_{D}^2}{2(r_{E2} - r_{D})}. \tag{D4b}
\]

From Equations (D3) and (D4), one can conclude:

\[
\frac{\dot{r}_{D}^2 - \dot{r}_{E1}^2}{2(r_{D} - r_{E1})} < \frac{\dot{r}_{D}^2 - \dot{r}_{E2}^2}{2(r_{D} - r_{E2})}. \tag{D5}
\]
For simplicity, let us assume that $K_p = 1$. Therefore, the overshoot-line is a line with slope 1 for a critically-damped PD-type CT-method: $\dot{r}_{E_i} = r_{E_i}$ and $\dot{r}_{E_2} = r_{E_2}$. Equation (D5) can be rewritten as follows:

$$\left(\dot{r}_{E_i}^2 - r_{E_i}^2\right)<\left(\dot{r}_{E_2}^2 - r_{E_2}^2\right).$$  \hspace{1cm} (D6)

By re-arranging the terms in Equation (D6), it can be written as:

$$\frac{\dot{r}_{E_i}^2 (r_{E_2} - r_{E_i}) + r_{E_i} \left(\dot{r}_{E_i}^2 - r_{E_i}^2\right) + r_{E_2} \left(\dot{r}_{E_2}^2 - r_{E_2}^2\right)}{\dot{r}_{E_i}} > 0.$$  \hspace{1cm} (D7)

The second term in Equation (D7) is greater than zero, hence, $0 < r_{E_i} < r_{E_2}$. Let us consider the first term in Equation (D7). It can be rewritten as:

$$\frac{(r_{E_i} - r_{E_2}) \left\{ \dot{r}_{E_i}^2 - r_{E_i}^2 \right\} - r_{E_2}^2}{\dot{r}_{E_i}}.$$  \hspace{1cm} (D8)

The first term in Equation (D8) is always less than zero, thus, the second term must be less than zero in order for the inequality of Equation (D7) to be valid.

One should note that for $K_p = 1$, $|\dot{r}_{E_i}| > r_{E_i}$. Therefore, one can conclude that when $(r_{E_i} + r_{E_2}) \leq r_{E_2}$, the following condition is satisfied:

$$\left\{ \dot{r}_{E_i} \left(\dot{r}_{E_i} + r_{E_2}\right) - r_{E_2}^2 \right\} < 0.$$  \hspace{1cm} (D9)

This suggests that under the above condition, Equation (D5) is satisfied, thus, the Segment (D-E1) is (1) executable by the robot, and (2) yields shorter interception time than one by the Segment (D-E2).

In summary, for each Point $E_2$, one can select another point, namely Point $E_1$, satisfying the condition $0 \leq r_{E_i} \leq (r_{E_2} - r_{E_2})$ that (1) is reachable, and (2) yields a shorter interception time.
Figure D1: A segment with smaller value of $r_e$ yields shorter interception time.
APPENDIX E. CLOSED-LOOP IPNG FOR ROBOTICS

The objective of this appendix is to show that the acceleration command computed through the IPNG technique is executed by the robotic manipulator with a time delay due to robot’s actuators dynamics.

Let us consider the linearized dynamics of the robotic manipulator at an operating point P. Seraji [71], showed that the robot’s dynamic equations represented in Equation (2.2), can be linearized as:

\[ \ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} = \mathbf{T} \]  

(E1)

\( \ddot{\mathbf{q}}, \mathbf{B}, \) and \( \mathbf{C} \) in the above linear differential equation are constant \((n \times n)\) matrices which depend on P. One should note that, \( \mathbf{q} \) and \( \mathbf{T} \) in Equation (E1) represent slight perturbations in the robot’s joint angle and torque vectors, respectively.

Equation (E1) can be rewritten in the task-space as follows:

\[ \mathbf{A}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F} , \]  

(E2)

where \( \mathbf{A} = (\mathbf{J}_p^T)^{-1}\ddot{\mathbf{A}}\mathbf{J}_p^{-1} \), \( \mathbf{B} = (\mathbf{J}_p^T)^{-1}\dot{\mathbf{B}}\mathbf{J}_p^{-1} \), \( \mathbf{C} = (\mathbf{J}_p^T)^{-1}\ddot{\mathbf{C}}\mathbf{J}_p^{-1} \), \( \mathbf{F} = (\mathbf{J}_p^T)^{-1}\mathbf{T} \). \( \mathbf{J}_p \) denotes the Jacobian of the manipulator calculated at the operating point P. One can then rewrite Equation (E2) in the frequency domain as:

\[ \mathbf{x}(s) = \left( \mathbf{A}s^2 + \mathbf{B}s + \mathbf{C} \right)^{-1}\mathbf{F}(s) , \]  

(E3)

or as,

\[ \mathbf{a}(s) = \left( \frac{\mathbf{A}s^2 + \mathbf{B}s + \mathbf{C}}{s^2} \right)^{-1}\mathbf{F}(s) , \]  

(E4)

where \( \mathbf{a}(s) \) denotes the Laplace transform of the robot’s tip acceleration. Now, by assuming that a field-controlled DC motor is used at each joint actuator, one can write the dynamics of each actuator as follows:

\[ \frac{E_f(s)}{I_f(s)} = R_f + L_f s , \]  

(E5a)

\[ T = K_f I_f , \]  

(E5b)

where \( E_f \) denotes the armature voltage, \( I_f \) denotes the armature current, \( K_f \) denotes the motor constant, and \( R_f \) and \( L_f \) denote the resistance and inductance of the DC motor field loop, respectively.
Figure E1 shows a block diagram relating the incremental change in the acceleration command to that in robot's tip acceleration. By choosing the calibration factor between the constant armature voltage and the motor torque command, $K_e$, as $R_f/K_e$, one can conclude:

$$\frac{a_f}{a_c} = \frac{1}{1 + \tau s},$$

(E6)

where $\tau = L_f/R_f$.

Equation (E6) suggests that an incremental change in the acceleration command will be executed by the robotic manipulator with a delay represented through a simple-lag system.

**Figure E1:** The acceleration command is executed with a time delay.
APPENDIX F. CONSTANT-ACCELERATION MOTION MODEL

A commonly used target-motion model assumes a constant-acceleration target $\ddot{x}(t)=0$. In practice, however, acceleration does undergo slight changes. Bar-shalom et al. [72], suggests to model this by a continuous-time Gaussian noise $\dot{w}(t)$ as:

$$\ddot{x}(t) = \dot{w}(t)$$  \hspace{1cm} (F1)

where $E[\dot{w}(t)]=0$, and $E[\dot{w}(t)\dot{w}(\tau)]=(\sigma_w)^2 \delta(t-\tau)$.

For this model, the continuous-time state and output equations are:

$$\dot{x}(t) = A x(t) + \dot{w}(t), \quad \text{and} \quad y(t) = C x(t) + \dot{v}(t).$$  \hspace{1cm} (F2)

where

$$x = \begin{bmatrix} P_x \\ V_x \\ a_x \\ P_y \\ V_y \\ a_y \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & b_3 \\ 0 & b_3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0_x & 0 & 0 \end{bmatrix}$$

and

$$\dot{w}(t) = \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}, \quad \dot{v}(t) = \begin{bmatrix} \nu_x \\ \nu_y \end{bmatrix}$$

are the system and observation noises, respectively.

The discrete-time state equation with sampling interval $T$ is:

$$x(k+1) = F x(k) + \Gamma w(k),$$  \hspace{1cm} (F3)

where

$$F = e^{AT} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0_{3x3} \end{bmatrix}$$

and

$$\Gamma = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix}$$
Figure 1: Noisy measurement and estimated position of the target using constant-velocity motion model.

Given the initial expected values of the state and its covariance at $x(0)$ and $P(0)$, the recursive formulation of the Kalman filter represented in Equation (5.8) were used to estimate the target motion represented in Equation (5.10) with the observation noises. A practical range is $0.5 \leq \sigma_{a} \leq 1$.

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$x^T \sigma_{a} x \leq \sigma_{a} x^T \sigma_{a} x$

where $x_{0}$ and $c_{0}$ are the variances of the Gaussian noises in moving objects' jerk model in time.

The covariance of the process noise is then calculated as:

$$[I \gamma^{T}]_{k} w(k) \gamma^{T} = \Theta$$

where $\Theta$ is computed based on the jerk that is the constant acceleration during the $k$-th sampling period (of length $T$), the increment in the velocity during this period is $w(k)$. Above $f$ is constant.