Cartography and the geometry of space

Re-imagining the legacy of Claudius Ptolemy

by

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Abstract

It is generally recognized that during the span of time from the mid-15th to mid-17th centuries, European cosmology underwent a fundamental change. The new idea of the universe was theorized as substanceless extension, or space, infinite and homogeneous but divisible into any number of finite, absolute spaces by the imposition of an abstract grid. This action has been characterized as the 'geometrization of space,' and it resulted in a deep philosophical shift. This thesis develops a general chronology and context for the hypothesis that cartographic representation provided a perceptual pre-requisite for the new cosmological ideas. The key idea behind this proposition is that the mapping model explicated in the *Geographia* by Claudius Ptolemy took Euclidean geometry as its basic principle, and that this led to the adoption of Euclidean geometry as the basis for what was eventually known as 'Newtonian' cosmology. The thesis therefore develops three parallel thematic histories – cosmological ideas, geometry, and cartography – and explores and surmises connections between them.

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For my father, Thomas Desmond Earl (1921-1993)

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General Chronology

1160		Ptolemaic planetary system available in Latin transl. of the *Almagest*
1202	Fibonacci	*Liber abaci*
1210-15		church in Paris forbids teachings of Aristotle's works
1217		translation of *Liber astronomiae* contrasts Ptolemaic and Aristotelian planetary systems
1220	Robert Grosseteste	optics - lux (English)
	Fibonacci	*Practica Geometriae* (Ital). Euclidean geometry and trigonometry
		stereographic projection described (French)
1220-30	Sacrobosco	*De sphaera* (John of Holywood)
1230	Robert Grosseteste	on scientific methodology
1231	Pope Gregory IX	rescinds ban on study of physics
1231-35	Robert Grosseteste	optics
1250	Albertus Magnus	botanical classification (German)
1255	Campanus	translation of Euclid's *Elements* from Arabic to Latin (first printed in 1482)
1267-68	Roger Bacon	*Opus Maius*: mathematics, optics, experimental science, geographical co-ords
1269	Peregrinus	magnetism (celestial poles) (French)
		translation of many of the works of Archimedes and Aristotle from Greek into Latin
1270	Witelo	optics, mirrors, refraction (Silesian)
	St. Thomas Aquinas	'Christianization' of Aristotle (substitution of God for Prime Mover)
	William of Moerbeke	translation of Archimedes
1272		Alfonsine Tables: predictions of celestial motion based on Ptolemaic system, used for 3 c.'s
1277	Etienne Tempier	Bishop of Paris, condemns 219 propositions of Aristotle
1277-79	John Pecham	*Perspectiva communis*, standard work on optics for 3 centuries (English)
1281		major work on planetary system, cosmography, geodesy, geography, mechanics (Persian)
1290	William of St. Cloud	latitude of Paris (English-French)
1296-97	Marco Polo	descriptions of travel in Far east
1303	Giotto	begins work on frescoes at Padua, new way to represent relationship between visual space and 2D surface
1306		popular treatise on farm methods
1328	T. Bradwardine	mathematics of motion (English)
1330	William of Ockham	motion, logic (Ockham's razor) (English)
1344		mathematical concept of infinity (Italian)
1350	Jean Buridan	impetus theory: no need for continual intervention of God in universe, only initial push (Fr)
1360	Nicole Oresme	2D graphs: 'latitude of forms' (French)
1391	Chaucer	treatise on the astrolabe (English)
1420	Henry the Navigator	establishes school for navigation, astronomy, geography, storage of maps (Portuguese)
1416-34		voyages add to geographical knowledge
1425	Brunelleschi	demonstrates linear perspective
1435	Alberti	publishes book on linear perspective, *Della pittura*
1440	Nicholas of Cusa	*On Learned Ignorance* (*De docta ignorantia*): universe indeterminate, no centre
	Gutenburg	moveable type
1447	Ghiberti	*Third Commentary*: survey of mathematical tradition in optics and perspective theory
1450	Nicolas of Cusa	*Idiota*
	Papal Jubilee celebrations in Rome, Nicolas of Cusa hosts visitors	
1453		fall of Byzantium
1455	Gutenburg	*Bible* published, first printed book in Western world
1464	Regiomontanus	trigonometry, Law of Sines
		death of Nicholas of Cusa
1472	Peurbach	*New Theory of the Planets*
1473	Michelangelo	Sistine Chapel
1473		atomic theories of Democritus, Epicurus, Lucretius introduced into Europe
1476	Caxton	sets up printing press (England), by end 15th c.: 9 million books in Europe
1482	Euclid	*Elements* printed (Campanus' translation of 1255)
1482-1515		translations and printing of classical texts
1489		algebraic symbolism, + and - signs
1491		long division
1492	Columbus	sails the ocean blue, discovery of America
1494	Luca Pacioli	*Summa de arithmetica*
1500	Leonardo da Vinci	speculations
1502	Amerigo Vespucci	concludes South America not part of Asia but separate continent
1505	Viator	three-point perspective

1509	Luca Pacioli	*De divina proportione*
1514	Copernicus	writes first version of treatise on planetary system, not published until 1543
1518	Francesco Feliciano	*Scala Grimaldella: Libro d'aritmetica*
1522	Waldseemuller	theodolite
	Magellan/Elcano	roundness of the earth verified, round-the-world voyage completed
1531	Juan Luis Vives	condemns dogmatic acceptance of Aristotelianism, advocates experimental evidence (Spanish)
1533	Gemma Frisius	triangulation
1543	Copernicus	publication of *De revolutionibus*, centre of universe moved from earth to near the sun (Polish)
	Tartaglia	Italian translation of Euclid
		publication of Moerbeke's Archimedes
1544	S. Munster	*Cosmographia*, 6-vol compendium of world geography (German)
1545	Cardano	initiates study of probability theory (Italian)
1546	Georgius Agricola	mineralogical classification system
1551	Reinhold	tables of planetary motions based on Copernican system
1553	Servetus	theory on circulation of blood, theology in book leads to Inquisition and death
1570	Euclid	*Elements* translated into English
1572	Tycho Brahe	new star
1576		Georgian calendar replaces Julian (adopted in 1582)
	Thomas Digges	stellar distances variable, universe infinite (English)
1577-78	Brahe/Mastlin	observation of comets, acceptance of Copernican theory
1584	Giordano Bruno	defends Copernican theory, postulates infinite universe and non-existence of absolute truth
1591	Vieta	algebra, generalities, symbolism (French)
1600	Bruno	burned at the stake for heretical theories
	Gilbert	magnetically-based concept of solar system
1602	Tycho Brahe	posthumous publication of observations
1604	Kepler	mathematics (postulate re parallel lines) and optics
1605	Francis Bacon	*The Advancement of Learning* encourages uses of experiment (English)
1608	Jungius	importance of mathematical foundations for theory asserted (German)
1609	Kepler	gravity, also *Astronomia nova*: first 2 laws of planetary motion (elliptical orbits)
	Galileo	telescope
1610	Galileo	observes sky with telescope, publishes *Sidereus nuncius*, support for sun-centred universe (Ital)
1610-13	Galileo	discoveries of nebulae, sun-spots, etc.
1613	Galileo	inertia
1614	John Napier	logarithms (Scottish)
1616	Catholic church forbids teaching Copernican theory, bans books, reprimands Galileo	
1619	Kepler	3rd law
1620	Francis Bacon	*Novum organum*: inductive scientific rather than medieval *a priori* method
1621	Kepler	systematic treatment of astronomy, first since *Almagest*, theory of gravitational force, vacuum
	William Oughtred	sliderule (English)
1622	Gunter	magnetic declination
1623	Galileo	methods - measurable properties, mathematics
	Bauhin	binomial classification for plants (Swiss)
1624	Gassendi	rejection of Aristotelian dogma in science and philosophy (French)
1627	Kepler	*Rudolphine Tables*: motions of planets, positions of stars
1629	Fermat	analytical geometry, not published so does not influence mainstream, also number theory
(French)		
1632	Galileo	dialogue on planetary system published
1633	Galileo	arrested by Roman Inquisition
1635-48		France: scientific conferences
1636-39	Desargues & Pascal	synthetic projective geometry (French)
1637	René Descartes	*Discours de la méthode*: bases method on Euclidean geom., deductive, rational, mathematical I La Géometrie (appendix): analytical geometry, properties of curved lines, classif of curves by degrees, theory of equations, revision of exponential notation, powers greater than 3 II La Dioptrique: optical theory III Les Météores: meteorology (French)
1638	Fermat	differential calculus
	Galileo	major work on mechanics: motion, acceleration, etc., discourse on finite and infinite classes
1639-40	Pascal	conics (Pascal's theorem)
1640	Wilkins	defense of Copernican theory, reconciliation with Bible (economy of nature) (English)
	Fermat	number theory
	Gassendi	biography of Epicurus, revival of atomic theory of Greeks
1642	Torricelli	refutes impossibility of vacuum (mercury barometer) (Italian)
		death of Galileo, birth of Newton
1644	Descartes	*Principia philosophiae*: solar system a vortex, sun at centre, orbits circular (rejects poss. of void) conservation of momentum (initial push by God), inertia

1645	Guericke	vacuum pump (German)
1650	Varenius	*Geographia generalis*, standard geography text for over a century
1650-54	James Ussher	Irish Archbishop, dates the creation at 4004 B.C.
1651	Riccioli	geocentric system of universe (Italian)
1654	Pascal, Fermat	probability theory
1655	Huygens	Saturn's rings and satellites (Dutch)
1656	Huygens	pendulum clock
1659	Pascal	calculus
	Huygens	centripetal force
1660		Royal Society of London chartered by Charles II
1664	Robert Hooke	comet's path deflected by gravitational force of sun (English)
1664	Descartes	posthumous publication of theory that animals are mechanical
1664-66	Isaac Newton	develops ideas on gravitation, calculus, angular momentum, inertia
1665	Robert Hooke	theories of light (wave theory)
1666	Borelli	orbital theory of Kepler, forces maintaining planets in orbit (Italian)
	Robert Boyle	atomic theory of matter (Irish-English)
	Leibniz	begins study of symbolic logic (German)
		French Academy of Sciences
1668	Cassini	tables of Jupiter's satellites for finding longitude (Italian-French)
	Isaac Newton	builds reflecting telescope
	Nicolaus Mercator	calculates area under a curve, infinite series $\ln(1+x)$
1669	Steno	fossils the remains of once-living organisms, forms of crystals, mountain formation (Danish)
1670	James Gregory	binomial series (Scottish)
1671	Isaac Newton	polar co-ords, calculus, differential equations (unpublished)
1672		computation by triangulation of distance to Mars gives scale of solar system
	Isaac Newton	physics of white light
1673	Huygens	law of centripetal force
1673-76	Leibniz	differential and integral calculus
1675		Greenwich Observatory
1675	Isaac Newton	wave theory of light
	Romer	1st measurement of speed of light, substantiates heliocentric solar system (Danish)
1676		protozoa found in rain water
	Isaac Newton	Binomial Theorem
1678	Huygens	wave theory of light, ether
1679	Hooke, Wren, & Halley	all deduce from Kepler's 3rd law & Huygens' Law of Centripetal Force that gravitational force is inversely proportional to the square of distance from source
	Leibniz	*Characteristica geometrica*: mathematical analysis
	Fermat	posthumous publ. of analytical geometry
1681	Thomas Burnet	sacred history of the earth, Noah's flood (English)
1683	Isaac Newton	mathematical theory of tides
1684	Leibniz	differential and integral calculus, notation
1685	Newton	derives elliptical orbits from centripetal force and inverse square law (for Halley)
1686	Halley	1st meteorological chart: world map showing winds
1687	Newton	*Principia mathematica*: theory of universal gravitation, definition of mass, laws of motion, earth an oblate sphere due to rotation
	Hevelius	star map (Polish)
1690	John Locke	at birth the mind is a blank sheet (*tabula rasa*), knowledge from experience
1691	Bernoulli	polar co-ordinates
1693	Newton	calculus

Cartographic Chronology

1154	al-Idrisi	world map with curved parallels, derived from Ptolemy
1235	Ebstorf map	
1250	Matthew Paris	*Flores Historarum*, 4 maps of Britain with roads & towns (1st maps of N. Eur. with north at top)
1290	Hereford map	
13th c.	Ibn Said	Arabic world map
1300	first Spanish and Portuguese portolan charts	
1313	Pietro Vesconte	earliest dated portolan chart
1320	Fra Paolino	
	Pietro Vesconte	map of Palestine w. rectangular grid
	Ranulf Higden	Polychronicon
1325-30	Gough	map of England
1330	potolan chart form combined with concept of world map	
1375	Catalan Atlas	
1377	Nicole Oresme	manuscript shows the earth as a sphere
1406-10	Jacopo de Scarperia	Ptolemy's *Geographia* transl. into Latin from Greek
1411-15	de Virga	*mappamundi*
1414	Pirrus de Noha	rectangular *mappamundi* with Ptolemaic influence
1417	Pomponius Mela	*De cosmographia*, contains world map by Pirrus de Noha
1426	Battista Beccario	portolan chart
	C'nrd of Dyffenbach	maps of Germany use co-ordinate system
1433-57	Paolo Toscanelli	maps of the paths of comets
1436	Andrea Bianco	world map according to Ptolemy's co-ords with graticule (Ital)
1440	use of meridional line to solve a territorial dispute	
	Toscanelli	describes geometrical grid system
	Vinland map	
1442	Giovanni Leardo	*mappamundi*
1443	Fra Mauro	*mappamundi*
1444	Nicholas of Cusa	celestial globe (earliest extant European one)
1448	Andreas Walsperger	map with graphic scale
1450	Alberti	*Ludi Mathematici*: ways of constructing maps to scale
	Paolo Toscanelli	estimate of distance across Atlantic on a map for feasibility of sailing from Europe to Asia
1450-51	Nicholas of Cusa	compiles scale map(s) of Germany (not publ. until 1490s)
1450-60	Catalan world map	
1452	Alberti	*De re aedificatoria*
1457	Genoese map	*mappamundi* with bar scales, shows influence of portolani and Ptolemy's *Geography*
1459-60	Fra Mauro	last of the medieval *mappaemundi*
1472	Isidore of Seville	T-O map of the world
1475-77	Vicenza/Bologna	1st published transl of *Geographia*
1481	Caxton	T-O world map
1483	Macrobius	circular world map, zonal type
1485-90	anon.	circular world map, Venice (?)
1486	Apian	shows diagrammatically how to use Ptolemy's co-ordinates in index to edition of *Geographia*
1490	H.Martellus Ger.	earliest world map revised from Ptolemy, with latitude and longitude
	"	ms. of Cusanus map, rectangular projection (modified Marinus)
1491	Eichstätt engraving of Cusanus map for an edition of Ptolemy uses trapezoidal (Donis) projection, unfinished, later purchased by Peutinger	
1492	Roselli	unsigned world map on the homotheric projection (conical)
	Behaim	1st terrestrial globe
1494	Treaty of Tordesillas	
1500	Jacopo de Barberi	bird's-eye view of Venice, woodcut
	Juan de la Cosa	world map, first to show the New World
	Etzlaub	first printed itinerary map
1502	Nicolo de Caveri	world chart
1503	Gregor Reisch (Freiburg)	
1504	Reisch	Ptolemaic map, woodcut (Strasburg)
1506	Contarini	world map, engraved by Roselli, first to show America
1507	Waldseemuller	Ptolemaic world map (Strasburg), shows N. and S. America as separate from Asia (woodcut)
	"	globe gores (earliest known)
1508	Johannes Ruysch	world map based on Contarini's (Rome)

	Francesco Roselli	world map in oval form, meridians near ellipses
1511	B. Sylvanus	Ptolemy (Venice)
1512	Jan ze Strobnicy	after Waldseemuller
1513	Piri Re'is	Turkish world map
1514	Leonardo da Vinci	spherical octant world map
1515	Dürer	woodcut astronomical maps (Nuremburg)
1519	Oronce Fine	
1524	Vespucci	world map, polar projection
	Peter Apian	*Cosmographia*, creation of map projection for large areas
1527	Robert Thorne	squared plane chart
1528	Benedetto Bordone	oval projection, woodcut (Venice)
1529	F. Monachus	hemispheric world map
	Diego Ribeiro	world chart (longitude) (Portuguese)
1531	Oronce Finé	hemispheric world maps
1533	Gemma Frisius	invention of technique of triangulation
1535-42	Battista Agnese	Ptolemaic world map, oval, Portolan Atlas
1536	Orontius Fonaeus	cordiform world map
	Gerardus Mercator	double cordiform, engraved (Leuven)
1536-37	Gemma Frisius	earliest known pair of globes, celestial and terrestrial, for geographical teaching
1540	Munster	Ptolemy (Basle)
1542	Jean Rotz	world chart
1541	Mercator	globe
1544	Gemma Frisius/Peter Appian (Antwerp)	
	Oronce Finé	stereographic projection described in treatise
1548	Andrea Mattioli	Italian translation of Ptolemy, maps by Gastaldi
1550	Ramusio	*Navigatione et viaggi*, 3 vols: 1550, 1553, 1559, maps by Gastaldi
	Desceliers	world map
1556	Le Testu	manuscript sea atlas, six different projections used
1561	Girolamo Ruscelli	Ptolemy in Italian (and 1564, 1573)
	Giacomo Gastaldi	*Cosmographia universalis*, incl. 9-sheet world map
1561-65	Gastaldi	*Universale descrittiones del mondo*
1562	Paolo Forlani	
1564	Ortelius	and up to at least 1590
1566	Le Testu	mappemonde, double cordiform type
1568	Mercator	projection for world maps described and published
1570	Ortelius	*Theatrum Orbis Terrarum*, 1st atlas, oval projection, 53 maps, used for generations
1588	1st printed star atlas with co-ordinate system, Venice	
1594	Mercator	posthumous publication of Atlas, begun in 1585, finished by his son
1599	Edward Wright	*Celestial Errors in Navigation*, mathematics of Mercator projection, construction of navigational charts using it (English)
	Jodocus Hondius	world map using Mercator projection
1600	Richard Hakluyt	world map using Mercator projection, compiled by Edward Wright

Schematic Timeline

Aristotelian concepts of space and cosmology
arithmetic, Euclidean geometry, tactile-muscular spatial intuitions
finite universe, non-existence of void space

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shift to the visual
Euclid re-discovered
clocks, musical notation, portolan charts, accounting

|

Ptolemy's <u>Geographia</u>, linear perspective
the grid, continuous space

|

new philosophy
humanism, Neoplatonism, hermeticism, Copernicanism
universe unbounded

|

mathematization of reality
Galileo: mechanics, experiment; Descartes: geometry and algebra

|

concept of absolute space
Newton, <u>Principia</u> 1687
mechanical world, calculus

|

non-Euclidean geometries
Lambert, Gauss, Lobachevski, Riemann

|

relativity
Einstein, field theory
(Leibniz vindicated ?)

|

cyberspace
digital world, the icon map, the cartographic boundary file

Introduction

Claudius Ptolemy's *Geographia,*[1] a work comprising eight books, arrived in Florence ca. 1400 and was translated into Latin by 1406-07.[2] Although the actual geography was incorrect and out-of-date, the system of projection described, the location of places by means of co-ordinates, and the grid of parallels and meridians[3] were quite new to Europeans and can be shown to have stimulated, in an extraordinary way, concepts in geography and art and, possibly, in astronomy and physics.

Although historians of both art and science have demonstrated awareness of the influence of Ptolemaic cartography, it has not been sufficiently well examined in the context of cosmology, where its potential influence has largely gone unnoticed. Cartography is usually assigned a reactive, secondary role and thus it has generally been assumed that astronomy influenced cartography, but not that cartography may have influenced astronomy and physics.[4]

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[1] In the Renaissance, the *Geography* of Ptolemy was more commonly called the *Cosmography*. See Samuel Y. Edgerton Jr., "From Mental Matrix to *Mappamundi* to Christian Empire: The Heritage of Ptolemaic Cartography in the Renaissance" in *Art and Cartography*, ed. David Woodward (1987), p. 12. I will refer to it throughout this paper in the Latin form as the *Geographia*.

[2] Woodward, "Medieval *Mappaemundi*," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 316. Paul Lawrence Rose, *The Italian Renaissance of Mathematics* (1976), states (p. 27) that the first translation was started by the Greek humanist scholar, Chrysoloras, and finished by his pupil Jacobus Angelus (Jacopo Angeli de Scarperia) between 1406 and 1410. In 1410, it was dedicated to Alexander V and thereafter widely circulated.

[3] *Latitude* and *longitude* are co-ordinate values. A line of constant latitude is a *parallel* while a line of constant longitude is a *meridian*. Collectively, the system of parallels and meridians is known as the *graticule*.

[4] It should be noted that 'astronomy' was heretofore regarded as combining the study of aspects of both terrestrial and celestial phenomena. In the eighteenth century, the confusion between 'mathematical geography' and 'mathematical astronomy' which had come about because of the inclusion of study of the earth in what was called astronomy, was resolved by use of the term cosmography, a branch of which was practical geometry which included the construction of globes,

The goal of this paper is to develop a general chronology and context which gives cartographic representation the role of providing a perceptual pre-requisite to the development of the conception of the universe which reached its full flower with the thought of Isaac Newton. The key conception is that of absolute space, a concept which was required for Newton's laws of motion, and a concept which was, I contend, depicted on maps well before it was explicitly theorized. This proposition is concerned primarily with world- or small-scale mapping[5] and does not deal with the history of topographical mapping which is a later canon and which developed fully only after the acceptance of the Newtonian model, though with prototypical antecedents.[6]

The idea for this paper grew out of an early topic on the instrumentality of cartographic depictions of space on the emergence of the nation state. One aspect of this topic was the necessity of a concept of space as a container and as abstract extension and this notion of

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maps, and marine charts. Eric G. Forbes, in "Mathematical Cosmography," *The ferment of knowledge: Studies in the Historiography of Eighteenth-century Science*, eds. G.S. Rousseau and Roy Porter (1980), attempts to unravel the confusion of terms (p. 418): "Once it became evident that the physical world could be interpreted in terms of the geometrical axioms of Newtonian mechanics, the distinction between physical and mathematical cosmography and their sub-divisions became blurred, being replaced by the more obvious distinction between the science of the heavens and that of the earth – astronomy and geography as we now understand them."

[5] Small-scale mapping is sometimes termed chorography. See Edney, "Cartography without Progress", *Cartographica* 30, 2&3 (1993), p. 60: "Chorography is the mode of small-scale mapping both of regions and the world (1:1,000,000 or less). While such mapmaking occurred sporadically in medieval Europe, it began to expand in the fifteenth century." In contrast to Edney's typology, Cosgrove, in "Mapping New Worlds:Culture and Cartography in Sixteenth Century Venice," *Imago Mundi*, 44 (1992), p. 66, uses Ptolemy's distinction between *geography* and *chorography* to characterize chorography as a "pictorial 'impression' of a local area without regard to quantitative accuracy." He notes that *geography* represented the world at smaller scale using astronomical observations and geometrical principles, and the term *cosmography* implied the setting of the earth within the celestial spheres.

[6] P.D.A. Harvey in "Local and Regional Cartography in Medieval Europe," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), draws a distinction between the small-scale geographic

space is intrinsic in the cartographic model which developed during the Renaissance, derived from the Ptolemaic tradition. As well, the apparent contradiction between the infinite space implied by the geometry of the map and the finite surface of the earth was an enigma. Solving this puzzle inspired the central idea for my thesis which then became the following proposition: inherent in the cartographic paradigm employed by the Greek astronomer, Claudius Ptolemy, in the *Geographia*, a work written in ca. 150 A.D. and lost to knowledge in Europe until the fifteenth century, was a radically different concept of space for this age. It was an assumption that space was homogeneous, continuous, isotropic, and infinitely extended. These are qualities of Euclidean space and they became the paradigmatic qualities of space in the physics of the European Renaissance. A map image was responsible for perception of the world as a flat inscription and, as two-dimensional space is governed by the laws of Euclidean geometry, I shall argue that the Ptolemaic mapping model was at least partly responsible for perception of the space of the universe as infinite and absolute.

In elaborating this proposition, my paper will present a brief study of the history of the conceptualization of space and an overview of the complex interactions between mathematics, cosmology, and cartography. The intricacies of these interactions are reflected in the overlapping nature of the central questions about the influence of cartography on changing notions of space and cosmology. These central questions are:

> i) did the new cartography of the 15^{th} and 16^{th} centuries, stimulated by Ptolemy's *Geographia*, provide a perceptual pre-requisite for the new ideas about space which were evolving in post-medieval Europe and which finally matured into the concept of absolute, or Newtonian, space?

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mapping of the fifteenth century and local or regional mapping, strongly suggesting that they arose from different imperatives. See p. 473.

ii)how and why did the developing modes of depiction of space on maps interact with or influence the changing philosophy of the cosmos, that is, the change from the medieval spiritual paradigm to the modern conception of the universe?

iii) has the study of mathematics and the study of cartography been so closely intertwined through the interests of individuals who have contributed to the development of modern science and philosophy that the study of cartography could have had an influence on their ideas?

There were certainly many factors other than the influence of cartographic images which contributed to a change in how the world was perceived, a change which has been described as the 'mathematization of nature' and led to the birth of modern science. They include internal factors such as the new experimental methodology in science, the rise of Platonism, and the re-introduction of atomic theory, as well as external factors such as the voyages of discovery undermining beliefs, Protestantism, the increasing use of the vernacular, and new technologies like print and gunnery. The many factors which may have had an influence on such a revolutionary change demonstrate how complex a process it is to understand its history. I do not wish to imply that the change was, as it has been traditionally viewed, an emergence from darkness and confusion into light and rationality. This, as Latour has pointed out, is "too complicated a hypothesis."[7] The fact remains that the revolution in consciousness which began in the fifteenth century in Europe has never been completely satisfactorily explained and an examination of the role of cartography is a line of enquiry not previously pursued.

As no prior analysis of just this aspect of the history of ideas exists to my knowledge, I have had to chart my own course through the literature. I will draw upon the work of some of the

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[7] Bruno Latour, "Visualization and Cognition: Thinking with Eyes and Hands," *Knowledge and Society: Studies in the Sociology of Culture Past and Present. A Research Annual*, eds. Kuklick and Long (1986), p. 2.

many scholars who have written about the history of science, cosmology, art, cartography, and mathematics; and who have touched upon concepts of space and the cognition of space. I will adduce the authority of these scholars and from the network of ideas and meanings assembled, and the historical picture constructed, present general conclusions. The scope of this paper is broad and the potential literature so large that in the brief time allotted for research, only a small selection has been studied. The selected literature will demonstrate the plausibility of the argument. However, the ideas presented here touch on a variety of fields and further research needs to be done to investigate such areas as the history of surveying and the development of theories of vision and optics, and how they bear upon the thesis of this paper.

The increasing importance of visual imagery in the late Middle Ages and early Renaissance is a crucial element of the argument for the influence of cartographic representation. An awareness of this development should provide the reader with an important context in which to view and assess the ideas presented in this thesis. Maps belong to a special class of imagery – they appear as factual representations while serving as vehicles for ideas about the world in much the same way that art does. A discussion of the increasing dominance of the visual and how maps actively structure beliefs about the world is introduced at the outset, forming the material of Chapter 1.

The theorizing of the concept of absolute space can be seen, in the simplest terms, as requiring a fundamental change in the idea of the cosmos, that is, a shift from understanding it as finite and composed solely of matter to understanding it as infinite, and consisting of a substanceless extension which contains matter. Chapter 2 will present an historical overview of the basic ideas about space and the cosmos in the late medieval and early modern period, and how they evolved during the fifteenth and sixteenth centuries until Newton presented an authoritative formulation.

The Newtonian laws of motion were dependent on an absolute frame of reference so that a body's position could be determined unambiguously. This frame of reference was the space of the universe and thus space had to be independent of the objects in it. This space of the universe, the 'substanceless extension' which is the principal idea of the Newtonian cosmos, is abstract space, and it cannot be thought about except in terms of geometry. The new cosmology was thus dependent on mathematical concepts of space. To understand how cartography is involved, it is necessary to understand that Euclidean geometry came to be seen intuitively as the geometry of the space of the cosmos as it is my contention that this resulted from the use of Euclidean geometry to depict the physical world on maps. In Chapter 3, therefore, I will survey mathematical concepts of space from antiquity to the discovery of non-Euclidean geometries in the nineteenth century, and discuss the paradoxical use of plane geometry to portray the spherical earth.

The use of Euclidean geometry as the basis for cartographic representation is not as obvious as it may seem. Chapter 4 describes the mapping models used in medieval Europe, and the adoption of the totalizing framework of the Ptolemaic model after the re-discovery of the *Geographia* in ca. 1400 by a group of Florentine scholars. The change from the dominance of the medieval representational paradigms for cosmological ideas and knowledge, to the dominance of the Ptolemaic representational paradigm was not instantaneous – the ancient, entrenched forms persisted long, surviving in iconographic decoration even after the general acceptance of the new cosmology. However, the innovative elements of Ptolemy's cartography were so compelling that they formed the basis for the standard cartography of the following six centuries. The major implication of the adoption of Ptolemaic tradition was that the mathematical space constituting the cartographic model was ultimately believed to be the underlying physical space of the cosmos. The network of parallels and meridians also created a sense that the world was rationally ordered which satisfied a need for certainty and gave it additional appeal. The grid was of fundamental importance and Chapter 4 will conclude with a discussion of its significance.

Interest in mathematics and astronomy has, since ancient times, been manifested in cartographic activities, especially the development of map projections, and, often, the same individuals have been attracted to all of these fields. To understand this is to realize that a cartographic image may have had an unrecognized, and perhaps important, effect on these individuals through its representational quality. To put the question into perspective, Chapter 5 will survey the history of linkages between the fields of mathematics and cartography as manifested by the activities of individuals engaged by both. While it is not possible to prove that cartographic activity stimulated mathematical or cosmological ideas, the conjunction of cartographic and mathematical work in the lives of many of the key individuals in the history of ideas provides contextual support for the central idea of this thesis.

Ptolemy's map did not represent the entire globe; all three of his projections relate to the inhabited world, the *oikumene*, which was about 180° east to west, by 110° north to south. Nevertheless, the entire globe was implied: his directions for construction could be applied to the sphere as a whole (his third projection actually draws the earth as it would look from a point in space) and eventually they were.[8] The use of mathematics in the construction of maps may have given the process, in the cabalistic and Neoplatonist quattrocento, an aura of higher truth.[9] Really, the depiction of the entire globe in two dimensions, is a logical error – a convenience, but an absurdity. It could be said that such a depiction is nothing but a symbolic representation and to understand it as scientifically correct because it employs the principle of map projection is wrong, illusory. The representation of the entire globe came into increasingly frequent use following the voyages of Columbus with the desire to display the new geographical discoveries. This prompted intensive work on map projections with revisions aimed at making them suitable for showing the whole of the known world.[10] The implication of belief in the factual truth of the representation of the earth, or a portion of it, in the plane is the acceptance of plane geometry as axiomatic for the physics of the universe. That facts ascertained from the senses were true and real was a system of knowledge coming to hold increasing sway in European philosophy, the legacy of Thomas Aquinas and William of Ockham, among other 13th century thinkers. Sensory experience followed by intellectual cognition and abstraction came to be preferred over either revelation, or deduction from principles. Mapping both tenders sensory experience and *is* symbolic abstraction.

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[8] The earliest map to show the entire globe is the 1508 world map with oval projection by Francesco Roselli. The date is given as 1508 by David Woodward in "The Meanings of Cartographic Co-ordinates," an unpublished paper. John P. Snyder in *Flattening the Earth* (1993) attributes this map more vaguely to the first decade of the 16th century (p. 38).
[9] The fifteenth century in Italy, the quattrocento, was a period of intellectual and religious ferment. The Byzantine Empire was crumbling and both Greek and Jewish scholars sought refuge in Italy's city states. See Jammer, *Concepts of Space (1954)*, 2nd edition, 1969, p. 33. Jewish esoteric teachings, in particular the cabala, generated considerable interest. Pico della Mirandola (d. 1494), active in the Neoplatonist movement, is also credited by Jammer with the introduction of the cabala into Christianity. It later spread north and, in the seventeenth century, influenced Henry More.
[10] See Snyder (1993), p. 29: "Perhaps the first to adapt Ptolemy's projections to post-Columbian mapping was Giovanni Contarini (d. 1507). On his only known map, a world map of 1506, Contarini modified Ptolemy's first coniclike projection ... in three ways: he doubled the span of meridians from 180° to the full 360°; he extended latitudes to the North Pole (shown as a circular arc); and he continued the meridians unbroken to about 35° S without a bend at the equator."

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Image

The shift to the visual

The discovery of Ptolemy's *Geographia* generated tremendous interest in 15th century Italy. The excitement over the new cartography also ushered in a dramatic shift from the primacy of word to the dominance of image. It is a commonplace of contemporary mentality that thought is imagination and imagination – the etymology of the word is a demonstration in itself – is the act of making pictures in the mind. Thus we concentrate, in ideation, on making mental pictures, interpreting these images, classifying and symbolizing them. In the act of classification, thoughts evolve from concrete images to more abstract ideas and, frequently, the abstract ideas discard the root images from which they originated, or which prompted their development. Conversely, vaguely-imagined things usually take on aspects of horror until they find concrete expression in visual or symbolic form. Vision has long enjoyed what Martin Jay calls "a special status as the noblest of the senses."[1]

While the keenness of our sense of sight has made it our favoured mode of perception, it has not always been as dominant as it is now, at the end of the twentieth century. It is generally believed that the oral tradition was the stronger in the Middle Ages, and that word prevailed

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[1] Martin Jay, "Ideology and Ocularcentrism: Is There Anything Behind the Mirror's Tain," in *Force Fields. Between Intellectual History and Cultural Critique*, p. 134.

over image for descriptive and intellectual power.[2] Europeans distrusted visual perception as a means to understanding, preferring a spiritual path to truth. Janet Soskice comments on an ambivalent attitude towards vision in the Middle Ages. She says:

> *The medievals received from Platonism, and later Aristotelianism, philosophical systems that privileged vision above the other senses. But this primacy of vision was qualified, particularly in matters epistemological, by at least two important reservations. The first was the accepted inadequacy of all the senses, including vision, to obtain the most important kind of knowledge, viz, knowledge of God; and the second is the importance accorded to "the Word."*[3]

The late medieval period, however, was marked by an increasing visual tendency which came to what may be termed a 'flash point' in the fifteenth century. Johann Huizinga comments:

> *Literature and art of the fifteenth century possess both parts of that general characteristic that we have already spoken of as being essential for the medieval mind: the full elaboration of all details, the tendency not to leave any thought unexpressed ... so that eventually everything could be turned into images as distinctly visible and conceptualized as possible.*[4]

As early as the thirteenth century, Roger Bacon had called for the use of maps and pictures in understanding geography.[5] It was an emphasis that was unusual and his ideas were

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[2] For example, see Johann Huizinga, *The Autumn of the Middle Ages (1921)* (1996), Alfred W. Crosby, *The Measure of Reality: Quantification and Western Society 1250-1600* (1997), David Woodward, "Medieval *Mappaemundi*" (1987), and Ernst Cassirer, *The Individual and the Cosmos in Renaissance Philosophy (1927)* (1963). Cassirer used the word *form* rather than image which is less precisely visual, but his analysis makes it clear that image was the predominant form he had in mind.
[3] Janet Soskice, "Sight and Vision in Medieval Christian Thought," in *Vision in Context: Historical and Contemporary Perspectives on Sight*, eds. Teresa Brennan and Martin Jay, p. 31.
[4] Johann Huizinga (1996), p. 333.
[5] David Woodward and Herbert M. Howe, "Roger Bacon on Geography and Cartography," in *Roger Bacon and the Sciences*, ed. Jeremiah Hackett (1997), p. 219.

disregarded by his contemporaries.[6] Early in the fourteenth century, however, Fra Paolino Veneto, a Minorite friar, echoed Bacon's plea:

> *Without a map of the world, I say it would be not so much difficult as impossible to imagine or conceive in the mind the dispersal of the sons and grandsons of Noah and the four kingdoms ... [There] is needed a map with both pictures and words for one without the other will not suffice.*[7]

There was certainly a complex of changing social, intellectual, and technological conditions which promoted what Alfred W. Crosby has termed the 'shift to the visual' in the fifteenth century in Europe. One such change was the transformation of the concept of time into measureable pieces – hours – which became visually perceivable and thus comprehended in terms of an image following the invention of the mechanical clock, and the installation of clocks in public places.[8] As well, the advances in mathematical symbolic notation and double-entry bookkeeping were influential.[9] However, two of the most important discoveries of the quattrocento to contribute to the growing dominance of the visual were the development of geometrically-based perspective in art and, concomitantly, the invention of moveable type and the printing press. The latter led to the prevalence of a written tradition of literacy which is visual in character. Samuel Y. Edgerton Jr. has argued in a well-known

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[6] Woodward and Howe (1997), p. 217.

[7] Amended translation of "Sine mapa mundi ea, que dicitur de filiis ac filiis filiorum Noe et que de IIII[or] monarchiis ceterisque regnis atque provinciis tam in divinis quam humanis scripturis, non tam difficile quam impossibile dixerim ymaginari aut mente posse concipere. Requiritur autem mapa duplex, picture ac scripture. Nec unum sine altero putes sufficere ..." in Juergen Schulz, "Jacopo de' Barbari's View of Venice: Map Making, City Views, and Moralized Geography Before the Year 1500," *Art Bulletin* 60, 1978, p. 452. A translation also appears in Woodward and Howe (1997), p. 219.

[8] Paul Lawrence Rose in *The Italian Renaissance of Mathematics* (1976) says, p. 7: "One of the most noticeable ways in which mathematics penetrated Renaissance culture was in the spread of the composite clock which both divided up the day mathematically and depicted some of the motions of

and persuasive book, *The Renaissance Re-discovery of Linear Perspective*, that the technique of linear perspective was inspired, or stimulated, by the appearance of the freshly discovered and eagerly studied *Geographia* and Ptolemy's map projections, particularly his third projection. Alfred Crosby also cites Ptolemy as the match which kindled the shift in perception. Jacob Bronowski has said that human intellectual activity is 'eye-conditioned' and that "the world of science" is "wholly dominated by the sense of sight."[10] The shift to the visual was a key process in the development of early modern science and in the visualization of the universe as two-dimensional extension subject to the assumptions and axioms of Euclidean geometry.

It has been claimed by Panofsky, Ivins, Edgerton, Cassirer[11] and other historians of art that the modern idea of space was first anticipated by artists. William M. Ivins Jr. remarked:

> *As much of the subsequent history of physics and philosophy, as well as of picture making and geometry, is centred about the idea or problem of space, it is interesting and important to trace the development of perspective from its discovery or invention as a quasi-mechanical procedure to a logical scheme or grammar of thought.*[12]

In an examination of the development of linear perspective, Ivins asserted that "the most important thing that happened during the Renaissance was the emergence of the ideas that

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the heavens. From the fourteenth century onwards public clocks were erected in the main towns in Italy – at Milan, Modena, Padua, Bologna, Ferrara, Parma, Pavia, Mantua, Venice and Florence."
[9] See, for example, Crosby (1997), chapter 10, p. 199-223.
[10] Jacob Bronowski, *The Origins of Knowledge and Imagination* (1978), p. 11.
[11] Irwin Panofsky, "Perspective as Symbolic Form," (in *Vorträge des Bibliothek Warburg, 1924-25*), translated by Christopher Wood (1991), William M. Ivins Jr., *On the Rationalization of Sight (1938)* (1973), Samuel Y. Edgerton Jr., *The Renaissance Re-discovery of Linear Perspective* (1975), Cassirer (*1927*) (1963).
[12] William M. Ivins Jr., *Art and Geometry: A Study in Space Intuitions* (1946), p. 68.

led to the rationalization of sight."[13] By 'rationalization of sight', he meant the transformation of a visual awareness represented in art from a kind dominated by how things feel when touched to how things look from a particular viewpoint. The former is understood by imagining running one's fingers along parallel ridges on an architectural molding – they will never meet but remain exactly parallel indefinitely. The same ridges, when viewed from a particular location at a distance from the structure, will seem to tend towards one another so that they would meet at a point if extended far enough.

This new visual awareness was rationally, i.e., mathematically, defined and thus correspondences between reality and pictorial representation could be made precise and systematic. The 'tactile space awareness' inherited from Greek (Aristotelian) space intuitions was replaced by a spatial consciousness which recognized that the characteristic shape of an object remained constant through change of spatial location even though its visual appearance changed.[14] A logical system of perspective replaced the medieval picture-making paradigm based on representing "what it felt like to walk about, experiencing structures, almost tactilely, from many different sides, rather than from a single overall vantage."[15] The new technique enabled a perception of a reality which was virtual mathematics. Norman Davidson designates the psychological shift in human consciousness as the greatest change in centuries: "[A] fundamental division between sense experience and thought," he writes,

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[13] Ivins (1973), p. 7.
[14] Ivins (1973), p. 9. Ivins explains: "Either the exterior relations of objects, such as their forms for visual awareness, change with their shifts in location, or else their interior relations do. If the latter were the case there could be neither homogeneity of space nor uniformity of nature, and science and technology as now conceived would necessarily cease to exist."
[15] Samuel Y. Edgerton Jr., *The Renaissance Re-discovery of Linear Perspective* (1975), p. 9.

"lies unnoticed at the root of our modern condition. It is the divorce of thinking from living experience, which results in the domination of an abstract thinking."[16]

Though William Ivins credits Leon Battista Alberti with the discovery of linear perspective in 1435-36,[17] Edgerton and Damisch[18] convincingly attribute the discovery and demonstration of it to Filippo di ser Brunelleschi in 1425, in Florence. Brunelleschi's mentor in geometry was the doctor, mathematician, and geographer, Paolo dal Pozzo Toscanelli. The first descriptive treatise on perspective drawing was that by Alberti, *Della pittura*, first published in 1435. The newly-realized system was distinguished from both Greek and medieval artistic conventions by its use of a central point for projection and the assumption of the homogeneity of space. Though congruence and parallelism had been basic assumptions since antiquity, homogeneity of space had not. The Aristotelian doctrine of place and space was concerned with objects as described by their boundaries and not with location *in* space, which did not exist as a container in awareness. Instead general space was discontinuous, an aggregate of differentiated particular spaces, as we will see in chapter 2.

The attribution of causality, or perhaps, more tentatively, influence, to Ptolemy's cartography is contradictory in some respects. Perspective is not the same representational paradigm as mapping, in artistic terms. The former represents space as volumetric and views objects as from a single point, while the latter represents space as two-dimensional extension and views objects from a continuously-changing point – as the eye roves over the map, the projection is everywhere orthogonal. Despite these essential distinctions, however, there are

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[16] Norman Davidson, *Astronomy and the Imagination: A New Approach to Man's Experience of the Stars* (1985), p. 4.
[17] Ivins (1973), p. 9-10.

some clear resonances in the genesis of both the system of linear perspective and the modern mapping paradigm. Both regard the geometry of space as Euclidean and spatial geometry is both positional and metrical (used for measurement), and there is a significant conjunction in the use of the geographical grid for locating objects in geographical space with the use of the 'pavimento,' checkerboard or mesh net described by Alberti as a practical device for locating objects in the space of the painting. The use of this abstraction almost compels the assumption of space as an empty container.

Hubert Damisch, in *L'Origine de la perspective*, denies the influence of Euclidean geometry on the emergence of the method of linear perspective, suggesting that artists of the Renaissance continued to perceive space as discontinuous while using the method as a device to define the outlines of objects. He asserts that:

> *[I]t is somewhat abusive, from a historical point of view, to invoke, by way of Nicolas of Cusa and Giordano Bruno, the cartesian idea of extension as accounting for the invention, at the beginning of the fifteenth century, of* costruzione legittima *[the technique of perspective construction]: as if the concept of homogeneous, unlimited extension had been, at such an early date, within the domain of the* representable.[19]

Having thus identified the concept of space with Descartes, he further criticizes the idea as anachronistic. Damisch clearly has not considered that the product of Ptolemy's map projection theory was indeed a *representation* of just this concept of space, a concept of which Descartes was not the author but for which he was a developer of a systematic description at a later date. The identification of mathematical space with physical space gradually become more and more acceptable until its full realization following Newton. The Ptolemaic mapping paradigm had made it visible from the early fifteenth century on.

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[18] Hubert Damisch, *The Origin of Perspective (1987)*, translated by John Goodman (1995).

Image and cognition

The importance to the conceptualization of continuous space of having it visible, in graphic form or image, cannot be over-emphasized, in my opinion. Ivins argues that without natural phenomena being "brought within the range of visual recognition ... and thus become subject to that logical symbolization ... rational thought and analysis are impossible."[20] Cassirer expresses the same thought in simpler terms: "the limit of vision is also, of necessity, the limit of conception," he writes.[21] In examining the relationship between image, or inscription, and cognition, Bruno Latour reinforces this observation. In particular, he discovers a significant advantage to the representation of phenomena as two-dimensional images in that the complexity of reality is reduced to a comprehensible level:

> *Scientists start seeing something once they stop looking at nature and look exclusively and obsessively at prints and flat inscriptions. In the debates around perception, what is always forgotten is this simple drift from watching confusing three-dimensional objects, to inspecting two-dimensional images which have been made less confusing.*[22]

Latour observes that latter-day scientists are obsessed with papers, diagrams, archives, photographs, and statistical curves, and notes that these flat inscriptions inspire them with confidence and generate credibility. There are a number of advantages to two-dimensional representation proposed by Latour.[23]

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[19] Damisch (1995), p. 18.
[20] Ivins (1973), p. 13.
[21] Cassirer (1963), p. 157.
[22] Latour (1986), p. 16.
[23] Latour's nine advantages are: 1. mobility: they are easily portable; 2. immutability: they may be transported unchanged; 3. flatness: this enables mastery, or dominance, of the subject; 4. congruence: scale may be modified without changing ratios; 5. reproducibility; 6. ease of recombination; 7.

The merging with geometry, is the greatest advantage, Latour states. Space on paper "can be made continuous with three-dimensional space"[24] so that while working on paper in two-dimensions, we are actually manipulating three-dimensional reality. As an example, Latour cites Galileo's work on the physics of motion, one of the key issues in the abandonment of Aristotelian physics. Galileo was able to visualise the solution to a problem of dynamics in terms of two-dimensional geometry in a way his predecessors were not. Latour stresses that Galileo believed the *diagram* for inertial motion against other evidence, that of the senses. or scriptural dicta. Alexandre Koyré, he says, "has shown that Galileo believed in the inertia principle on mathematical grounds even against the contrary evidences offered to him not only by the Scriptures, but also by the senses."[25] Whatever induced Galileo to ignore these contrary indications, he, in the end, believed the inscription, written in the language of figurative geometry.

The importance of image may have been cardinal to human nature for millennia, but, during the Middle Ages in Europe, and until the Renaissance and the invention of printing, descriptive graphical images were rarely used and so had little power to affect ideas. To use Latour's typology, they were neither mobile, nor were they immutable. It would have been extremely difficult to precisely copy images by hand. Prior to the advent of printing, inscriptions that were copied were inevitably corrupted and the few copies that would have been made would, in any case, have decayed and dropped out of circulation before they could have had far-reaching impact. In any case, evidence of the senses was not regarded as

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possibility of superimposition: allows the emergence of pattern; 8. ease of embedding within written text; 9. merging with geometry. (Latour (1986), p. 20-22).

[24] Latour (1986), p. 22.

having value to a theology based on contemplation of the eternal. Following the radical work of the Scholastic philosophers Thomas Aquinas and William of Ockham, natural philosophy became divorced from moral philosophy and sensing of the material nature of the world became an important basis for knowledge. Use of image to represent and abstract this material nature was a natural step. Leonardo da Vinci, artist and scientist of genius, habitually used sketches to aid thought. Cassirer, responding to Olschki's 1919 criticism of Leonardo's reliance on visual imagery, counters: "It is ... true that representational, pictorial material preponderates even in his notes on mechanics, optics, and geometry. For Leonardo, 'abstraction' and 'vision' collaborate intimately."[26]

Cassirer stresses that Leonardo insisted on visibility for "graspability of the form" and continues, "it was artistic 'vision' that first championed the rights of scientific abstraction and paved the way for it."[27] A century earlier than Galileo, Leonardo may well have inspired the greatly influential astronomer and physicist.

Mapping as representational practice

If knowledge begins with sensory perception followed by intellectual cognition, it matures into abstraction, or the construction of universals.[28] One of the most natural and effective techniques of abstraction is the reduction of complex information to conventional signs which take on qualities of universalism. These signs become axiomatic substitutions for

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[25] Latour (1986), p. 26.
[26] Cassirer (1963), p. 158.
[27] Cassirer (1963), p. 158.
[28] The shift from the medieval valuing of the ideal, to a system of thought based on sensory perception of the material world is readably described by Wade Rowland in *Ockham's Razor* (1999). See especially, p. 151-153.

individual experience and mediate our perceptions by categorizing them according to convention. These agreed-upon meanings are primal in our interaction with our environment where they intervene with *a priori* interpretations of experience. Shapiro has stated this effect as that "the real is never wholly present to us -- how it is real for us is always mediated through some representational practice."[29]

Though maps may appear as simple graphical devices, the popular conception of a map as a factual representation of physical data and "a mirror of culture and civilization,"[30] is too uncritical. It is now generally acknowledged that maps do more than passively reflect the changing thought of a culture.[31] They do not "imitate reality;"[32] instead, in the same way that language controls how we think, maps control and actively constitute their subject: the world - or, at least, the taken-for-granted image of the world. A map is an instance of a representational practice; as a conventional sign, culturally based, it mediates our perceptions of reality and, this paper proposes, of space. "[A]ny idea of geography," John Gillies writes "... will always seek a cartographic (as distinct from material) form, for the reason that geographical ideas require cartographic assumptions. An imagined geography is

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[29] Michael J. Shapiro, *The Politics of Representation: Writing Practices in Biography, Photography, and Policy Analysis* (1988), p. xii. Shapiro has written on the way in which representational practices give meaning and value to things: "[T]o the extent that a representation is regarded as realistic," he writes (p. xi), "it is because it is so familiar it operates transparently."
[30] Norman J.W. Thrower, *Maps and Civilization. Cartography in Culture and Society* (1996), p.1.
[31] For example, J.B. Harley, "Silences and Secrecy: The Hidden Agenda of Cartography in Early Modern Europe," *Imago Mundi* 40 (1988), p. 57-76; J.B. Harley, "Deconstructing the Map," *Cartographia* 26, 2 (1989), p. 1-20; Denis Wood, "How Maps Work," *Cartographia* 29, 3&4 (1992), p. 66-74; Matthew Edney, "Cartography without 'Progress': Reinterpreting the Nature and Historical Development of Mapmaking," *Cartographia* 30, 2&3 (1993), p. 54-68; Barbara Belyea, "Images of Power: Derrida/Foucault/Harley," *Cartographica* 29, 2 (1992); and others.
[32] Shapiro (1988), p. xi.

itself a kind of map in the very real sense that it must presuppose a cartographic idiom of one kind or another."[33]

Harley and Woodward, focussing on the history of this idiom, have described it as "a history, at least in part, of the means by which [a] developing picture of reality – what was actually perceived – was modified with the help of maps."[34] This history, they aver, "is a reciprocal process of cognition in which both perception and representation become increasingly structured by different map models."[35] That perception and representation are mutually interdependent in cognition of space is fundamental to my proposition. They are at least to some extent reciprocal, as Harley and Woodward have claimed, and to support this conception it is helpful to resurrect the structuralist theory of *symbolic forms* of the German philosopher, Ernst Cassirer.

Maps have served to generalize experience, whether religious or secular, since antiquity and have thus functioned as abstract, conventional signs, or representations. They function very much like the symbolic forms of Ernst Cassirer. Cassirer first used this term in 1923 with the publication of the first volume of his *Philosophie der Symbolischen Formen* on language.[36] Samuel Edgerton points out that the symbolic forms of Ernst Cassirer were not meant to suggest 'shorthand devices,' but that Cassirer was describing the human ability to structure or to 'modulate' reality by creating intellectual schema which "have an autonomy all their

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[33] John Gillies, *Shakespeare and the geography of difference* (1994), p. 51.
[34] J.B. Harley and David Woodward, "Concluding Remarks," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 504.
[35] Harley and Woodward (1987), "Concluding Remarks," p. 504.
[36] In *The Individual and the Cosmos* (1963), which first appeared in 1927, Cassirer re-iterated his conviction that the question of *form* was central to Renaissance culture. See, for example, p. 51.

own."[37] These autonomous schema may be independent of or unrelated to naturally occurring order and represent, instead, human social or cultural interests.

> *Thus language, myth, and art each constitutes an independent and characteristic structure, which does not achieve its value from an outward, transcendent existence that is somehow "mirrored" in it. What gives each of these forms its meaning is that it builds up a peculiar and independent, self-contained world of meaning according to an inherent formative law of its own.*[38]

The content of each symbolic form thus determines our perception of the thing it attempts to describe or represent. If, as Edgerton puts it, words are the symbolic form for objects in verbal communication, pictures for visual communication, and numbers for scientific concepts, then maps may be thought of as the symbolic form for ideas about space. They are the "primary medium for transmitting ideas and knowledge about space."[39]

First published in 1924-25, Erwin Panofsky's "Perspective as Symbolic Form" became a classic. In it, Panofsky extended Cassirer's theory of symbolic forms to the history of art, elevating the technique of linear perspective to equal status with language and myth. He claimed that every historical age reifies its concepts of physical and metaphysical space in its visual arts. Panofsky mistakenly construed perspective as the 'sign' for a geometrized space, however, failing to take his idea out of the realm of the history of fine art to discover the probable original symbolic form in a cartographic paradigm. Hubert Damisch says of Panofsky:

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[37] Edgerton (1975) p. 156.
[38] Ernst Cassirer, *The Philosophy of Symbolic Forms III: The phenomenology of knowledge (1929)*, translated by Ralph Manheim (1957), p. 383.
[39] Harley and Woodward (1987), "Preface," p. xv.

> *[Panofsky] only introduces [Ernst Cassirer's term 'symbolic form'] after extended developments, supposedly based on psychophysiology, that directly contradict Cassirer's arguments because they take the retinal image, which has nothing to do with the symbolic order, to be the touchstone of perspective construction. And as for the definition he uses, which holds that perspective is one of those "symbolic forms" by means of which "intellectual meaning becomes so closely linked to a concrete sign as to be indistinguishable from it," it is sufficiently vague and generalized to justify any interpretation one would like. Perspective, a system of signs?*[40]

The attribution of the concept of symbolic form to cartography may be a bold, and perhaps even facile, suggestion, particularly as no attempt will be made to develop this idea as a contribution to the philosophy of symbolic forms. It is no more naïve, however, and I believe it is more reasonable, than Panofsky's nomination of linear perspective as a symbolic form. There can be no question that cartographic representation is a system, a system of signs which has a more profound meaning than the obvious connotative meaning of the content of the map. The map is not a reflection of reality but constitutive of reality. Barbara Belyea expresses this succinctly, referring to the writings of Michel Foucault:

> *Foucault insists that far from passively inscribing a pre-existent object, the énoncé* forms *that object. Hence maps determine our perception of the capes and bays rather than the reverse. We see the earth's surface in terms of the cartographic convention we are familiar with: "[le discours] a constitué son objet."*[41]

Maps determine our perceptions not only of 'capes and bays' but of spatial relations. The system of representation creates a concept of space because it is responsible for a perception of the world in terms of this concept. Cassirer expresses this logic as that "the conception of an aesthetic form in the sensible world is possible only because we ourselves create the

[40] Damisch (1995), p. 11.
[41] Belyea (1992), p. 6.

fundamental elements of form."[42] It is true, of course, that maps are not the only vehicles for communication of spatial relationships. Denis Wood contends,

> *[W]hat distinguishes maps from writing* is not *their embodiment of spatial relationships. Words, after all, embody spatial relationships too. It is a mark of "cartocentricity" to imagine that spatial relationships have to be embodied in map form.* [P.D.A.] *Harvey takes pains to demonstrate the extent to which medieval Europeans produced* written descriptions *where today we might be inclined to draw a map.*[43]

However maps, and pictures, make spatial relationships *visible* and the graphic form of the embodiment makes the underlying idea of space an unconscious perception.

An essential aspect of Cassirer's theory of symbolic forms is the conjunction between perceiving the natural world and conceiving it. As conceptions of space have developed, and as these have been reified on maps, our perceptions have been modulated accordingly. As concepts of space changed from the Aristotelian, qualitative, experiential model to the Newtonian/Euclidean rational, homogeneous, infinite and cold extension – so perceptions moved from the experience of immanence to perspective view from a distance. The position of common viewpoint arched from eye level above the ground – as it would have been for the medieval European – to the lofty but distanced position of the divine ruler. This 'axial shift' also prepared the way for a modern detachment from and objectification of the world, the better to reduce it to measured quantities and mathematical terms. As we shall see, the

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[42] Ernst Cassirer, *The Philosophy of Symbolic Forms I: Language*, translated by Ralph Manheim (1953), p.88.

adoption of a mechanistic world picture was characteristic of the 'new philosophy' of the Renaissance which became modern science.

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[43] Denis Wood, "P.D.A. Harvey and Medieval Mapmaking: An Essay Review," *Cartographica* 31. 3 (1994), p. 53.

2

Cosmology

It is generally recognized that during the span of time from the mid-fifteenth to the mid-seventeenth centuries, European cosmology underwent an upheaval that radically altered the framework of existence for every member of society, and resulted in a shift in patterns of thought which has been characterized as the "secularization of consciousness."[1] The influence of this shift has determined contemporary political and economic systems as surely as it profoundly affected the history of science and philosophy. The 'new philosophy' of the seventeenth century destroyed the medieval cosmic certainty of a changeless universe and immutable structure of being within it, replacing it with an infinitely extended, centreless universe[2] in which God was ultimately an unnecessary hypothesis.

The change in world-view was produced by a double-barrelled assault on the medieval spiritual paradigm:[3] (1) the geo-centric model of a changeless hierarchy of perfect spheres in which all being had its permanent rank and where God determined and preserved this harmonious order was discredited by the discoveries and postulations of Copernicus, Kepler and Galileo, and (2) the new idea of the universe was theorized as substanceless extension, or space, infinite and homogeneous but divisible into any number of finite, absolute spaces

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[1] Alexandre Koyré, *From the Closed World to the Infinite Universe* (1957), p. 1.
[2] Koyré (1957), p. vii.
[3] Koyré (1957), p. viii.

by the imposition of an abstract grid of dimensions and boundaries. The latter action is characterized by Alexandre Koyré as the "geometrization of space."[4]

The notion of absolute space is associated with Isaac Newton who explicitly theorized it in conjunction with his laws of motion; Newton's conceptualization was not a spontaneously original idea, however, but the synthesis by a physicist of a long philosophical debate which had involved thinkers such as Giordano Bruno, Descartes, Henry More, Spinoza and Leibniz. In this chapter, we will consider the genesis of the new cosmology and follow the trajectory of changing concepts of space from the earliest idea of an unbounded cosmos as far as Newton's formulation. The details of the complicated theological debate, finally won by Isaac Newton, will not be recapitulated here. An historical overview will furnish a broad picture to compare with a parallel history of cartography and mathematics developed in later chapters.

The genesis of the new cosmology was considerably earlier than Bruno or Descartes; it lay in the explanatory power of the Aristotelian system of philosophy and science. During the twelfth and thirteenth centuries, the works of Aristotle were translated into Latin and medieval Europe suddenly inherited a classical cosmology fully developed and founded on theories of physics and astronomy which were far more advanced than any that were, at that time, known in Europe. Medieval natural philosophers not only had Aristotle's major treatises, but commentaries on them by Islamic and Greek scholars such as Avicenna, Averroes, Proclus and Plato. The Aristotelian world picture was so persuasive that it

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[4] Koyré (1957), p. viii.

relatively quickly became the universally accepted cosmology of the late Middle Ages;[5] it provided the most complete and satisfying account of the perceived world, but the implications for theology and for philosophy were profound. The Aristotelian system was to impel an unconscious shift away from theological explanation towards factual explanation, and thence to the destruction of the Aristotelian edifice itself, and the construction of a radical new philosophy of science.

Despite the alluring credibility of Aristotelian philosophy, medieval philosophers by no means accepted Peripatetic paganism and all concomitant ideas, such as Aristotle's belief that the world had no beginning and no end.[6] The Middle Ages were steeped in Christian theology and Aristotelian concepts had to be made compatible with the doctrines of Christianity. Medieval scholars made it their project to try to bring Aristotelian physics and cosmology into harmony with scripture and with Christian precepts. The Catholic Church had reason to be extremely concerned about the popularity of Aristotle's teachings as, early on, it recognized the danger they posed to the belief that truth was revealed through contemplation of the divine, not through sensory perception.[7] In 1210, the Parisian council of bishops forbad the teaching of his works, a decree which was renewed in 1215 and upheld by Pope Gregory IX in 1231.[8] Nevertheless, scholars and ecclesiastics outside Paris continued to study Aristotle, to write commentaries on questions in physics and astronomy, and to harmonize his ideas with Christian theology and by the 1240's the Parisian bans were

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[5] For an account of the adoption of Aristotelian thought, see David C. Lindberg, *The Beginnings of Western Science* (1992), chapter 10, p. 215-244.
[6] Edward Grant, "Cosmology," *Science in the Middle Ages*, ed. David C. Lindberg (1978), p. 268-270.
[7] See note 3, Chapter 1.
[8] Lindberg (1992), p. 217.

losing their effectiveness.[9] Thomas Aquinas (1224/5-1274) devoted his life and energies to resolving the tensions between theology and philosophy. Despite his deep spiritualism, Thomas' concession of the independent reality of the material world opened the door to secular scientific enquiry. In 1277, in face of the perceived peril to the doctrine of faith, Etienne Tempier, Bishop of Paris, condemned 219 propositions of Aristotle on the grounds that they contradicted the omnipotence of God by limiting His possibilities.

The condemnations of 1277 have been seen by Pierre Duhem in particular, and others, as a pivotal point in the dismantling of the Aristotelian framework and the birth of modern science.[10] It is probably not as decisive, however, as these writers have claimed. If a case can be made for a radical discontinuity between medieval and modern metaphysics and methodology, it must be dated considerably later, to post-Copernican science. Despite the Bishop of Paris' decree, Aristotelian philosophy became well entrenched in late medieval Europe, was taught at the universities and served as the basis for all intellectual exertion.[11] Although Duhem in his magisterial *Système du monde* made the confident assertion, indeed the chauvinistic assertion, that seventeenth-century science was based on fourteenth-century French ideas which, as a result of Tempier's decree, triumphed over the ideas of Aristotle and Averroes, it is generally considered that he overstated the case for the French medieval Scholastics as forerunners. David C. Lindberg has interpreted the condemnations in a more

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[9] Lindberg (1992) p. 217.
[10] See Pierre Duhem, *Medieval Cosmology*. Selections from *Le Système du monde*, a ten-volume opus written between 1909 and 1916. Edited and translated by Roger Ariew (1985). S.L. Jaki adhered to Duhem's view that thirteenth century thinkers anticipated the ideas and discoveries of Galileo, Kepler, and Newton and Alastair C. Crombie also subscribed to the "continuity thesis" (Lindberg (1992), p. 361). H. Floris Cohen gives a detailed review of the theories of a number of the most eminent of the historians of science in *The Scientific Revolution. A Historiographical Inquiry* (1994). See also Cohen's acerbic assessment of the historiography of S.L. Jaki, p. 528, note 13.
[11] Lindberg (1992), p. 241.

subtle light than Duhem and his adherents.[12] While there is no doubt that the condemnations of 1277 prompted scholars to explore alternative explanations for natural phenomena, they did not suddenly destroy the Aristotelian edifice. Lindberg calls the condemnations "a conservative backlash against liberal and radical efforts to extend the reach and secure the autonomy of philosophy, especially Aristotle's philosophy,"[13] and argues that they revealed the anxiety of the Church about the growing influence of thinkers outside the Church's own authority, thinkers who were prepared to speculate on the basis of reason rather than faith. Indeed, other historians of science[14] have regarded the contributions of the Parisian school – Oresme and Buridan especially - as significant but not decisive. The medieval Scholastics made changes and additions to the Peripatetic system of thought but without replacing its overall structure as they were unable to free themselves from the ingrained tenets of Aristotelianism. Hooykaas states the position as follows:

> *In the Paris decree, a purely theological issue was at stake, but an issue that could indeed have great, positive consequences for the freedom of scientific theorizing and for the choice between a rationalistic and an empiricalist approach to nature. For the time being, however, it had no influence on scientific speculation, let alone on practical research. Although the Aristotelian world picture was deprived of its absolute authority, it was not seriously criticized or replaced by an alternative system.*[15]

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[12] Lindberg (1992), p. 234-244.
[13] Lindberg (1992), p. 238.
[14] For example, E.J. Dijksterhuis, *The Mechanization of the World Picture (1950)*; R. Hooykaas, "The Rise of Modern Science: When and Why?" *British Journal for the History of Science*, 20 (1987); Marshall Clagett, *The Science of Mechanics in the Middle Ages*, 1959; Alexandre Koyré, *Etudes Galiléennes (1939)*; William A. Wallace, "The Philosophical Setting of Medieval Science," *Science in the Middle Ages*, ed. David Lindberg (1978); and A. Maier, *Die Vorläufer Galileis im 14. Jahrhundert (1949)*.
[15] R. Hooykaas (1987), p. 457.

Aristotelian natural philosophy

Aristotle was born in 384 B.C. in Northern Greece, studied with Plato in Athens, and in a long and fruitful career as scholar and teacher wrote more than 150 treatises of which about thirty have come down to us.[16] He created a comprehensive philosophy of nature based upon the reality of sensible objects in the world, and a theory of knowledge that began with experience. He devised methods for investigating natural phenomena and causes, and expounded elaborate and influential theories about the universe.[17] The achievement of Aristotle in devising a systematic treatment of the major physical problems of the time, an analysis of natural phenomena, and a persuasive theory for the structure of the cosmos is monumental. His influence was to resonate in the late Middle Ages in Europe.

The Aristotelian world-view postulated the universe as a series of concentric spheres with the earth at the centre, the shell where the moon was situated constituting the division between the upper and lower regions. The Pythagorean infatuation with the perfection of the circle as mathematical and as physical form had seduced all men into thinking that the world must be organized in circular or spherical forms. The pre-eminence of the sphere influenced beliefs about both terrestrial and celestial order and it was unanimously agreed that the universe consisted of a series of concentric spheres moving with uniform rotational motion while the earth remained motionless at the centre.

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[16] Lindberg (1992), p. 47.
[17] For a summary of Aristotle's philosophy of nature, see Lindberg (1992), p. 46-68. For background on Aristotle's cosmological theories, Roger Ariew provides a concise summary in the preface to his translation of selections from Pierre Duhem, *Medieval Cosmology* (1985), p. xxv-xxx.

Aristotle elevated the concept of the hierarchy of concentric spheres governed by an external God - the *Unmoved Mover* - into a 'dogma of astronomy'[18] and established detailed theories of physical properties and processes. Central to Aristotelian physics was a theory of motion which was to be one of the key concepts to be contradicted by experiment and lead to the destruction of the Aristotelian edifice, and of the comforting certainty of the immutable, organic medieval world. Additionally, the Aristotelian system set out theories of space and place, infinity, the void, and whether or not there could be other worlds existing. It is beyond the scope of this study to present a detailed treatment of the Aristotelian system. Its central tenets, and the implications for medieval cosmology, have been thoroughly discussed by modern scholars.[19] The issues of significance to this thesis are concepts of space, infinity, and the void.

The Greeks did not have a sense of unified space.[20] A systematic idea of space was to develop in the fifteenth century with the application of geometry to physical space, a step not taken by Greek mathematicians. Aristotelian spaces were worlds within worlds, a set of loci differentiated on the basis of particular differences, where emplacement was relative - that is, spatial relations were relations between specific places. Place was defined by the boundary of an object that was contained by that boundary, while the spaces between containing boundaries were qualities of the objects themselves. This notion of space is extremely removed from present-day conceptions and difficult to adequately explain. There was no

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[18] Arthur Koestler, *The Sleepwalkers. A History of Man's Changing Vision of the Universe* (1959), p. 58.

[19] For example, Grant, Lindberg, Cohen. See also Duhem.

[20] William M. Ivins Jr. (1946) states, p. 53: "We have seen how the Greek method in conics prevented them from having any idea of geometrical continuity, and how their lack of a geometry of central projection and section prevented them from having any logical idea of a unified space." See also Hubert Damisch, *The Origin of Perspective* (1995).

sense of empty space, or of space as container of objects, as is the commonplace perception of the twentieth century. Space was discontinuous, discrete; the space of the universe was a composite of individual spaces. The result was that, to quote Wade Rowland, "[t]o the medieval mind, the very space contained by [the basilica of] St. Semin would have been animated: empty space was a meaningless concept prior to the scientific revolution of the mid-sixteenth century, and the vastness of the structure would have been inhabited by that ineffable wisdom, or *Logos*, that accounted for the continuing functioning of the universe."[21]

Aristotelian, and medieval, space was alive with mind, but was an unsystematic assemblage. Representational practices reflected this concept. "One of the most important tasks of Renaissance philosophy and mathematics," writes Ernst Cassirer, "was the creation, step by step, of the conditions for a new concept of space. The task was to replace *aggregate space* by *system space*, i.e., to replace space as a *substratum* by space as a *function*."[22]

Aristotle's doctrine on infinity was somewhat complex, reflecting an ambiguous position. While he admitted the existence of *potential* infinites, he opposed the idea that there could be *actual* infinites in magnitude.[23] It is not difficult to see the reason for this position. The world being an aggregation of particular places, it would be illogical to suggest that it could be infinitely large by addition. Medieval Scholastics took differing views on the existence of infinites which formed one of the main subjects of debate. The question of infinites in general is not one which is germane to this discussion until it is associated with the possibility of void space.

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[21] Wade Rowland, *Ockham's Razor* (1999), p. 141.
[22] Cassirer (1963), p. 182.
[23] Duhem (1985), p. 3-72, or for a concise summary, see Ariew's Preface, p. xxv-xxvi.

The Greek Stoicist tradition had believed in an infinite three-dimensional void space beyond the world.[24] The Stoic belief was set forth in a sixth-century commentary by Simplicius, translated into Latin in 1271 by William of Moerbeke and available to European scholars of the fourteenth century. A few of them – Thomas Bradwardine, Nicole Oresme and Johannes de Ripa – interested in this theory, modified it to accord with their religious beliefs by identifying the 'immensity' beyond the world with God, Himself. They did not extend this concept so far as to seriously impugn the Aristotelian framework, which firmly denied the possibility of void, or the corresponding perceptions of space. As Edward Grant explains,

> *With the emphasis on God's absolute power remaining strong throughout the fourteenth century as a significant legacy of the condemnation of 1277, it must have seemed incongruous to some that the presence of an infinitely powerful God should extend no further than the finite world that He chose to create. But if His presence extended beyond the world, it seemed plausible to assume that He was infinitely extended, since no good reasons could be offered for supposing that His presence ceased at some finite distance beyond.*[25]

Some recondite speculations followed these thoughts but did not seriously influence cosmological beliefs. The world itself was still held to be finite for at least another century and the identification of void space only with the infinitude of God persisted into the seventeenth century.

Thus, as far as modern scholarship and speculation can tell, in the medieval cosmos space was finite, knowable, immutable and full. A motionless, fixed earth centred a timeless, rigidly-organized, sacred universe in which "virtues were defined by God in natural law and

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[24] Grant (1978), p. 270-271.
[25] Grant (1978), p. 271.

not by human opinion."[26] It was a very secure, comforting world and a view which provided an antidote to the dangerous instability of life in the period. No one who acted virtuously acted from personal motivation but only as a "correspondent agent in the divine cosmic order."[27] In this world, there could be no question of the domination and control of nature by humanity - this was the prerogative of God. For humanity, pure contemplation was the highest order of relationship with nature and the divine; a biblical poetic dominated discourse,[28] philosophy conformed to theological beliefs, and Christian spirituality alone offered redemption. The destruction of this conception of the world would entail discarding considerations of quality such as harmony and goodness and this would, in the end, result in what Koyré has described as "the utter devalorization of being, the divorce of the world of value and the world of facts."[29]

The waning of the Aristotelian philosophical heyday

The repudiation of Aristotelian cosmology and degradation of medieval Scholastic ideas was to allow the development of the concept of absolute space. The first thinker to suggest that the Aristotelian concept of the universe could be usefully modified, or whose suggestions had an impact on philosophical debate, was the ecclesiastic, Nicholas of Cusa. Koyré describes him as the "last great philosopher of the dying Middle Ages, who first rejected the medieval cosmos-conception and to whom, as often as not, is ascribed the merit, or the crime, of having asserted the infinity of the universe."[30]

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[26] Stephen L. Collins, *From Divine Cosmos to Sovereign State: An Intellectual History of Consciousness and the Idea of Order in Renaissance England* (1989), p. 23.
[27] Collins (1989), p. 2.
[28] See Gillies (1994).
[29] Koyré (1957), p. 2.
[30] Koyré (1957), p. 6.

Nicholas of Cusa is an important figure in this study, and we will have a closer look at him in Chapter 5, as I feel he forms an important link between natural philosophy and cartography. Modern scholars have differed substantially in their assessments of his contributions. He has been disregarded and ridiculed on one hand, and given an important role on the other.[31] The more brilliant aspects of his work were offset by plainly foolish notions such as descriptions of the inhabitants of the moon, and the quadrature of the circle. His cosmological ideas, however, and his epistemological assumptions were bold and creative and may be said to be transitional between the solid beliefs of medieval Aristotelianism, and the Copernican revolution. His most important treatise, *De docta ignorantia*, was issued in 1440. Of this work, Cassirer says: "In speculative philosophy, *De docta ignorantia* represents the first real breach. It attacked the vital centre of the Aristotelian doctrine."[32]

Nicholas of Cusa retained the concentric spheres in his cosmology but they were not bounded by an outer shell which would limit the size of the world. It was indeterminate in size and, not only was it unbounded by an outer shell, but its constituents were also undetermined in any precise way. This suggestion was quite original but had little influence on his contemporaries. It was only after Copernicus had removed the earth from the centre of the cosmos, placing it among the planets, that Giordano Bruno linked this with Nicholas of Cusa's theories and hypothesized the decentralized, infinite universe:

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[31] Koyré (1957) and Cassirer (1927) consider the thought of Nicholas of Cusa important. Pierre Duhem (1985) was scathing in his assessment of the contribution of Nicholas of Cusa, describing his ideas as "neither extremely daring nor extremely useful" (p. 505). H. Floris Cohen does not mention Nicholas of Cusa.

[32] Cassirer (1963), p. 176.

> *The whole universe then is one, the heaven, the immensity of embosoming space, the universal envelope, the ethereal region through which the whole hath course and motion. Innumerable celestial bodies, stars, globes, suns and earths may be sensibly perceived therein by us and an infinite number of them may be inferred by our own reason. The universe, immense and infinite, is the complex of this vast space and of all the bodies contained therein.*[33]

And,

> *For there is a single general space, a single vast immensity which we may freely call void; in it are innumerable (innumerabili et infiniti) globes like this one on which we live and grow. This space we declare to be infinite, since neither reason, convenience, possibility, sense-perception or nature assign to it a limit.*[34]

Bruno, a reputed mystic fired with wild and heretical ideas, was burned at the stake in 1600 for theories which Nicholas of Cusa had been allowed to freely express one hundred and fifty years earlier.[35] Bruno attempted a critique of the church based upon a Neoplatonic elevation of spirit over matter. The attitude of the Catholic Church towards radical speculation had hardened greatly under the increasing threat to established authority posed by the rise of humanism and individualism, Copernican astronomy, and Brahe's celestial observation, as well as a swelling enthusiasm for new mercantilism. But Bruno's execution was too late in the day to be a deterrent. As Crosby puts it: "The cat, already out of the bag, was having kittens. If space were homogeneous and measurable, and therefore susceptible to mathematical analysis, then human intellect could reach around the world and out into the interstellar void."[36]

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[33] Giordano Bruno, *On the Infinite Universe and Worlds. Third Dialogue* (1584). In *Giordano Bruno, His Life and Thought* by Dorothea Waley Singer (1968), p. 302.
[34] See Singer (1968), p. 363.
[35] Cassirer (1963), p. 37; Stephen Toulmin, *Cosmopolis* (1990), p. 77.
[36] Crosby, (1997), p. 105.

Before such a revolution in ideas came to fruition, however, making the persecution of Bruno, and of Galileo, ultimately futile, came the foundational work of the astronomer, Nicolas Copernicus. The cosmological system of Copernicus, expounded in his major work, *De revolutionibus*, published in 1543, was an attempt to correct the unsatisfactorily complicated and dissonant Ptolemaic planetary system. Copernicus, called a 'typical humanist' by R. Hooykaas,[37] studied the ancient Greeks and adduced the sun-centred system of Aristarchos of Samos, a Pythagorean astronomer (b. 310 B.C.), to lend credibility to his planetary system. It is generally agreed that Copernicus' main contribution was his introduction of motion to the earth and removal of its centrality. This brought into question the terrestrial-celestial hierarchy which was the foundation of the traditional world-view. He did not, however, launch the Scientific Revolution. His physics was not mechanistic, not an explanation of fact or observation, or a search for value-free laws. He sought harmony and meaning in a Platonic sense, and retained belief in the sphericity of the world. Moreover, he does not assert that the world is infinite, only that it is too large to be measurable.[38]

Both Johannes Kepler and Galileo Galilei were empiricists rather than philosophers and they are credited with initiating an astronomy based on factual observation and the use of a 'mechanistic' model. The switch from the use of an organic metaphor for universal systems (as in Aristotelian science) – a physics based on qualities – to a mechanistic metaphor, one based on quantities, is a momentous change. Mechanicism is one of the fundamental characteristics of modern science and is attendant upon the 'mathematization of nature.'[39]

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[37] Hooykaas (1987), p. 464.
[38] Koyré (1957), p. 32.
[39] Both Koyré and Dijksterhuis use this characterization. For mechanicist philosophy, see Hooykaas (1987) and John A. Schuster, "The Scientific Revolution," in *Companion to the History of Modern Science*, eds. R.C. Olby, G.N. Cantor, J.R.R. Christie, and M.J.S. Hodge (1990).

Kepler and Galileo were conventional in their beliefs despite the radical implications of their discoveries. Kepler did not believe in an infinite universe and adhered to Aristotelian cosmology notwithstanding his brilliant rationalization of the planetary system. Nevertheless, Kepler's celestial mechanics was seen as a new field of science and, as Schuster asserts,

> *Kepler contributed to the ripening of a cosmological crisis in the minds of early-seventeenth-century thinkers by vigorously asserting, in the light of his overriding philosophy of nature, that empirically determinable simple mathematical harmonies expressed and governed the motions and structure of the heavens and that their existence established the truth of his brand of Copernicanism.*[40]

Galileo's well-known championing of the Copernican planetary system paved the way for its universal acceptance, and his original work on the physics of motion was extremely important as well. His conflict with the Church sprang from his insistence on a mechanical explanation of experience and a cosmology based on the axioms of geometry.[41] Galileo did not engage in the debate about the infinity or finiteness of the world but he clearly considered the question privately as he writes, in *Letter to Ingoli*: " it is as yet undecided ... whether the universe is finite or, on the contrary infinite."[42] Both Galileo and Kepler evidently were able to conceive of infinity in theory, even if they denied its possibility in fact. What was the nature of this conception? What mental image of infinite space may they have had? It has already been noted, in chapter 1, that Galileo was able to merge three-dimensional space with two-dimensional geometry more successfully than his predecessors and his facility with, and confidence in, mathematics may be the clue. To quote Cassirer

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[40] Schuster (1990), p. 232-33.
[41] See Rowland (1999), p. 157.
[42] Galileo, quoted in Koyré (1957), p. 97 (Koyré cites: *Opere*, VI, Firenze: Edizione Nazionale, 1896, p. 518, 529).

again: "For Galileo," he says, "the homogeneity of the world follows from the necessary homogeneity of geometrical space"[43] and Galileo, himself, proclaimed: 'The book of nature is written in the language of mathematics.'

From Descartes to Newton

It was neither Kepler nor Galileo, however, who formulated a new theory of the structure of the universe based on the new findings in astronomy, but René Descartes (1596-1650), a French mathematician and philosopher. With Descartes, philosophy definitively shifts from a preoccupation with the particular to consideration of universals.[44] The social, political, and theological confusion engendered by the Thirty Years' War (1618-48) induced a desire to search for abstract truths, unassailable certainties, and ideas about the cosmos partook in this movement. For the coldly rational Descartes, the new science negated the medieval world-view absolutely and reduced its merry confusion to abstract mathematics.

> *The world of Descartes ... is by no means the colorful, multiform and qualitatively determined world of the Aristotelian, the world of our daily life and experience – that world is only a subjective world of unstable and inconsistent opinion based upon the untruthful testimony of confused and erroneous sense-perception – but a strictly uniform mathematical world, a world of geometry made real.*[45]

Descartes' world consisted only of space and motion defined mathematically but, according to Descartes' theory, the space that bodies occupy is not different from the bodies themselves. He rejected the notion of empty space: the void could not exist because space is something that has extension and exists; it is not nothing and therefore it must have

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[43] Cassirer (1963), p. 183.
[44] Toulmin (1990), p. 31-36.
[45] Koyré (1957), p. 100.

substance. The identification of extension with matter leads to the conclusion that the universe cannot be supposed to have a limit and Descartes thus declared the world to be unlimited, or *indefinite*. This assertion is, moreover, applied to the real world, and linked to the limitlessness of Euclidean space.

> *We likewise recognize that this world, or the totality of corporeal substance, is extended without limit, because wherever we imagine a limit we are not only still able to imagine beyond that limit spaces indefinitely extended, but we perceive these to be in reality such as we imagine them, that is to say that they contain in them corporeal substance indefinitely extended. For as has been already shown very fully, the idea of extension that we perceive in any space whatever is quite evidently the same as the idea of corporeal substance.*
>
> *It is thus not difficult to infer from all this, that the earth and heavens are formed of the same matter, and that even were there an infinitude of worlds, they would all be formed of this matter; ... we clearly perceive that the matter whose nature consists in its being an extended substance only, now occupies all the imaginable spaces where these other worlds could alone be, and we cannot find in ourselves the idea of any other matter.*[46]

As Descartes distinguishes between the infinite, which he applies only to God – an infinite being – and the indefinite, Koyré suggests that he echoes the traditional differentiation between actual and potential infinity. This, explains Koyré, is most likely in deference to theology, but the infinite proved to be such a serviceable concept that it became detached from the personification of God and took on a more general metaphysical quality. Cassirer affirms that it became an important analytical tool. "The metaphysical transcendence of infinity," he says, "transforms itself ... The concept of space strips itself of the last vestiges of

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[46] René Descartes, *Principles of Philosophy, Part II, Principles XXI and XXII*, in *The Philosophical Works of Descartes*, translated by Elizabeth S. Haldane and G.R.T. Ross (1931), p. 264-265.

materiality ... This transformation is most clearly evident in the *concept of co-ordinates* introduced by Fermat and Descartes."[47]

Descartes overreached the reasonable in his well-known identification of matter with extension, or space. Henry More, though one of Descartes' early admirers, disagreed with his conception of space and matter and conducted a correspondence with the French philosopher during which he advanced his objections to Descartes' theory. More believed that spirit (soul) is also extended though it lacks substance and he therefore rejected the identification of matter and extension. More's void is void of matter, but not empty. Space is divine, eternal, uncreated and infinite. It is separated from matter, the material world, which is finite or, at most, indefinite. According to More, says Koyré:

> *Matter is mobile* in *space and by its impenetrability* occupies *space; space is not mobile and is unaffected by the presence, or absence, of matter in it. Thus matter without space is unthinkable, whereas space without matter, Descartes notwithstanding, is not only an easy, but even a necessary idea of our mind.*[48]

Henry More's conception of space was an imaginative, yet satisfying concept which must have had considerable appeal in the seventeenth century. It validated the idea of the existence of an infinite extension of space by identifying it with God who thus retained His pre-eminence in the cosmos. Though this idea is similar to the postulations of the Stoics and the medieval scholars, Bradwardine and Oresme, it is significantly different, as More's infinite space is co-extensive with the space of the real world – God is everywhere – and thus, inevitably, the space of the world must be infinite. As Koyré explains, More could not avoid

[47] Cassirer (1963), p. 186.
[48] Koyré (1957), p. 127.

this conclusion.[49] Infinite space is, necessarily, an absolute and so is God. If matter is also infinite, there is no need for divine creation or direction and so no need for God; but if matter and space are separated, then space may be made an attribute of God through which He governs and maintains the material world, a world indeterminately large yet ultimately smaller than the cosmos itself. An axiom of More's theory is then that matter is finite.[50]

Henry More is not given a prominent place in the historiography of science. Alexandre Koyré describes him as hermeticist and occultist with a 'partially anachronistic standpoint'[51] and Jammer adds that More was a foremost rabbinical student.[52] Nevertheless, in our survey of the development of the modern idea of the infinity of space, the concept developed and articulated by More in his long correspondence with Descartes is an important foundational principle. Although his purpose was to identify extension with divine spirit rather than with a mundane physical matter, as in the Cartesian concept, More's achievement went beyond what he perhaps intended. It effectively took Descartes' 'indefinite extension' and advanced it to the concept of 'infinite space.' Through its identification with divine spirit, his space was eternal and absolute and it was characterized as immobile, homogeneous, and indivisible. Sir Isaac Newton, with whom the concept of absolute space is most closely associated, must have been at least indirectly influenced by it as there is clearly considerable similarity between their explanations of space. In fact Jammer surmises that More's demonstration of the reality of space in his *Divine Dialogues* probably had a direct influence on Newton as

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[49] See Koyré (1957), p. 152.
[50] Koyré (1957), p. 152-154.
[51] Koyré (1957), p. 125.
[52] Max Jammer, *Concepts of Space. The History of Theories of Space in Physics (1954)*, 2nd edition (1969), p. 41.

More's example of a rotating cylinder is very similar to Newton's famous demonstration with the rotating pail.[53]

Newton, however, was not the passionate metaphysician that More was but, rather, a cautious scientist (though physics was not yet 'science', but natural philosophy) dedicated to carefully reasoned explanations for phenomena, who looked at the evidence for the structure of the universe and took it to an apparently logical conclusion in a positivist sense. R. Hooykaas describes Newton's achievement:

> *Newton ... fitted the Copernican model of the universe definitively into a mechanistic system of nature. In his 'Principia' (1687), the synthesis of astronomy and physics, the mathematization that united terrestrial and celestial mechanics, was finally accomplished.*[54]

Newton's approach was a practical, mechanical one. E.A. Burtt, in his 1924 critique of Newtonian science[55] deplored the lack of metaphysical dimension to Newton's thought. H. Floris Cohen summarizes Burtt's analysis:

> *Newton, so Burtt insisted, had been a far greater scientist than a philosopher. Large chunks of the new world-view that had been created by the previous generations of Kepler, Galileo, Descartes, Boyle, and others were adopted fairly uncritically by Newton, who added little but his own positivist leanings to what he had found readily available. The resulting, essentially metaphysical conceptions of space, time, causality, ... were then uncritically carried along with his scientific achievement and adopted by the European intellect.*[56]

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[53] Jammer (1969), p. 45-46. Jammer includes a direct quote of a dialogue in More's *Divine Dialogues*, London, 1668, I:XXIV.
[54] Hooykaas (1987), p. 463.
[55] E.A. Burtt, *The Metaphysical Foundations of Modern Physical Science: A Historical and Critical Essay (1924)*, 2nd edition (1932).

Newton's method exemplified the maturation of the new principles of modern science: use of the mechanical model, and the Baconian ideal of the prime role of experience over speculative theorizing. There appeared to him to be a dichotomy between intellectual notions of time and space, and sense perceptions. Although the shift from sensual to intellectual cognition can be dated back to Nicholas of Cusa and Giordano Bruno, Newton articulated the distinction between the intelligible and the sensible. Accordingly, he defines time, space, place, and motion as being distinguishable into "absolute and relative, true and apparent, mathematical and common."[57]

For Newton, as for More, space is an attribute of God and a receptacle for the creation of a finite matter and the infinity and continuity of absolute space are mutually implied. Thus, the Cartesian concept of the identification of space and matter is rejected. As there is both absolute and relative motion, there is both absolute and relative space: the former is immoveable, indivisible, imperceivable by the senses; the latter is the part of space taken up by a body in it and is the locus of that body relative to other bodies.

> *Space void of body, is the property of an incorporeal substance. Space is not bounded by bodies, but exists equally within and without bodies. Space is not inclosed between bodies; but bodies, existing in unbounded space, are, themselves only, terminated by their own dimensions.*
>
> *Void space, is not an attribute without a subject, because, by void space, we never mean space void of every thing, but void of body only. In all void space, God is certainly present, and possibly many other substances which are not matter; being neither tangible, nor objects of any of our senses.*[58]

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[56] H. Floris Cohen, *The Scientific Revolution: A Historiographical Inquiry* (1994), p. 90.
[57] See "Scholium to Definition VIII," *The Leibniz-Clarke Correspondence, Appendix A: Extracts from Newton's Principia and Opticks*, (ed.) H.G. Alexander (1956), p. 152. By common is meant common sense notions, the latter being designated as "vulgar." (Koyré (1957), p. 160.)
[58] See "Clarke's 4th Reply," *The Leibniz-Clarke Correspondence*, ed. H.G. Alexander (1956), p. 47.

The Newtonian God is the ever-vigilant Biblical ruler and master. God is an 'intelligence' which is everywhere and always, infinite and eternal. His perspective is a single over-arching viewpoint in a homogeneous, infinite and eternal extension or space, a space which can then be ordered in a rational, objective and abstract way. Though space is indivisible in real terms – meaning parts may not be separated – it may be divided in a logical or abstract way by the imposition of an ordering principle. And the material world, existing in infinite space but necessarily finite so not self-sufficient and therefore requiring creation,[59] is subject to God's power of disposition. The problem for the Newtonians, however, became the objection that God would not limit His creative action to only a portion of infinite space, that portion occupied by matter. Thus, the material world, even though it exists *in* an infinitely large void space, must itself become infinite and, paradoxically it seems, no longer needing the intervention of divine power. Thus God, as Koestler puts it, becomes "reduced to the part of a constitutional monarch, who is kept in existence for reasons of decorum, but without real necessity and without influence on the course of affairs."[60]

The Newtonian natural philosophy ultimately overcame resistance from the Cartesians, and from Leibniz and his adherents, and became the accepted cosmological model of modernity. The abstract infinite extension of More and Newton merged with matter, and any distinction between infinite space and the real world disappeared. The obliteration of this distinction is implied by the application of Euclidean geometry to the material universe, as is precisely the mode of 'scientific' cartographic representation. With the publication of Newton's *Principia mathematica* in 1687, the geometrization of the universe became inevitable and the more

[59] Koyré (1957), p. 257.

'fluid' concepts of space and time that the medieval being understood experientially were displaced by mathematical, abstract conceptions. The old view of the world had gradually lost its explanatory and mythic power to be replaced by a rational, mathematical, quantifiable, invariant universe. Today it is generally realized that the epistemology of science is contingent, and the theories of Einstein, Heisenberg, and Gödel have made faith in the rationalistic, quantifiable, mechanical conception of the universe untenable. Jeans stated, in 1930, that "the universe begins to look more like a great thought than like a great machine,"[61] recapitulating the medieval idea of 'a mind at work in the universe,'[62] but this is to leap ahead of ourselves. For the time being, the new world-view of modernity as an infinite, substanceless extension subject to quantification and classification permeated all intellectual, political, and economic thought.

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[60] Koestler (1959), p. 509.
[61] Sir James Jeans, *The Mysterious Universe* (1930), p. 148.
[62] Rowland (1999), p. 141.

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Geometry

As we have seen in the previous chapter, one of the important roots of the concept of absolute space was the abandonment of medieval Scholastic ideas. The new cosmology which developed from the repudiation of Aristotelian concepts and reached its fruition with Isaac Newton's physics, resulted in the 'geometrization of space,' to use Koyré's evocative phrase. It was dependent on mathematical conceptions. It will be important, therefore, in establishing the particular mathematical conceptions it relied upon, and how cartography was implicated, to review mathematical ideas of space. In this chapter, I will review these ideas chronologically, from the concepts of space of the early Greeks to the formulation of the theory of non-Euclidean geometry by Riemann. The chapter will conclude with a discussion of the geometry of cartographic space.

Concepts of space in Greek mathematics

Mathematics was of paramount importance to philosophical thought in Greek civilization. Plato regarded the study of mathematics as the path to the highest order of truth, and characterized geometry as "knowledge of the eternally existent."[1] The Greeks were doing geometry from the earliest times and employing deductive logic; they devised the earliest map projections and demonstrated an interest in what we would now call scientific rigour and validity. The Pythagorean school was founded on the belief that all things could be

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[1] *The Republic of Plato*, translated by Francis Macdonald Cornford (1962), p. 244.

reduced to numbers and proportions and that "the symbols of mythology and the symbols of mathematical science were different aspects of the same, indivisible reality."[2] As Koestler puts it: "Nobody before the Pythagoreans had thought that mathematical relations held the secret of the universe. Twenty-five centuries later, Europe is still blessed and cursed with their heritage."[3]

The earliest conceptualization of absolute space may be attributed to the Pythagoreans. They accorded a kind of spatiality to numbers and this required the existence of void space in between numbers, to assure their discreteness.[4] The Pythagorean approach to mathematics might appear naive or spiritualistic to contemporary culture and their concept of abstract space was rudimentary compared to later formulations, but the studies of the Pythagorean school may have had an important facet: they were concerned not just with theory but with proofs, and for proofs of theorems relating to ideas about space, the Greeks systematized geometry. Geometry evidently had intuitive appeal in the search for a deductive system of mathematics. Jeremy Gray describes this appeal eloquently:

> *Certainly one appeal of geometry is that it treats existing things clearly, mathematically existing that is, but that is if anything better. Geometry then becomes an analysis of (true) reality, and the deductive method an enquiry into the world. It is a paradigm of the philosopher's quest for truth, easy enough to be a forcing ground in logic and comprehensive enough to be saying things. I cannot quite believe that the Greeks were interested in their triangles and circles just for their own sake, fascinating though they can be. If the theorems are, as I suggest, part of a programme which makes numerical work intelligible and which in its methods holds the promise of explaining things, then it is truly exciting. It raises the possibility that we might obtain deductive knowledge of the world, knowledge which starts from undeniable premises and by an irrefutable process yields a description of reality. We would then*

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[2] Koestler (1959), p. 37.
[3] Koestler (1959), p. 40.
[4] Jammer (1969), p. 9.

> *truly know the nature of things. It is an awe-inspiring thought, truly a godlike thought, and one we have never wholly abandoned.*[5]

Indeed, the Pythagoreans applied their ideas to the geometry of the universe and postulated a non-geocentric cosmology and circular celestial motion. The theory of circular motion dominated astronomy for approximately two millenia and Copernicus was to point to Greek scholarship as corroboration for his planetary model.

The atomists, Democritus, Epicurus, Leucippus, and Lucretius considered theories of space which accorded with their view of matter as being composed of an infinite number of atoms. That space is infinite is inherent in early Greek atomism, but space is not defined as unbounded extension. Rather it is "complementary to matter and is bounded by matter; matter and space are mutually exclusive."[6] Thus space exists in the interstices between objects.

In contrast to the earlier Greek atomists, Lucretius explicitly argues in favour of space as a receptacle for matter and as infinite and unbounded, using the familiar example of the hand or spear extended through beyond the outermost boundary of the universe.[7] However infinite and homogeneous space is, however, Lucretius and Epicurus do not characterize it as isotropic. They conceived of space as having a preferred direction in which atoms move along parallel lines.

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[5] Jeremy Gray, *Ideas of Space: Euclidean, Non-Euclidean, and Relativistic (1979)*, 2nd edition (1989), p. 14-15.
[6] Jammer (1969), p. 11.
[7] Jammer (1969), p. 12.

Plato's theory of space is expounded mainly in *Timaeus*.[8] Plato identifies matter with its geometry, conceiving of bodies as being those parts of space occupied by geometric surfaces. Although Plato pointed out that solid geometry is the branch of mathematics most relevant to astronomy,[9] his study of geometry did not include solids but remained confined to the plane. He, himself, stated that the study of solids was in "a pitiable state."[10] Jammer has provided a critical description of the effect on thought in the Middle Ages:

> *This identification of space and matter ... had a great influence on physical thought during the Middle Ages. ... Plato's* Timaeus *was succeeded by Aristotle's* Physics *only in the middle of the twelfth century. It is perhaps not wrong to assume that the obscure and vague language of the* Timaeus *contributed to preventing the concept of space from becoming a subject of strict mathematical research. Greek mathematics disregards the geometry of space.*[11]

As has been intimated in a preceding chapter, Aristotle repudiated any idea of space being infinitely extended in his conceptualization of the universe. He used the term 'topos' (place) exclusively, rejecting the concept of general, or empty space. Place is that part of space occupied by a body defined by the boundary of that body, while space is the sum total of all individual places occupied by bodies. Thus physical space was finite and there was no void.

Mathematics in the Middle Ages

During the first milenium of the common era, Greek scholarship was little known to Europeans. It was preserved and expanded, however, by the Islamic world whose

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[8] Jammer (1969), p. 14.
[9] Gray (1989), p. 38.
[10] *The Republic of Plato*, p. 246.
[11] Jammer (1969), p. 16.

mathematicians have given us the terms algebra and algorithm.[12] Islamic scholars were energetic in their search for books and they diligently translated the Greek classics into Arabic. In many cases, we only know of Greek works because of the survival of Arabic translations of them. During the caliphate of al-Mamun (809-833), founder of the House of Wisdom, Islamic scholars translated into Arabic all the Greek works they could obtain including Ptolemy's *Almagest* and Euclid's *Elementa*.[13] When Europeans again became interested in classical Greek thought and in the study of mathematics, they re-discovered geometry from Arabic versions of the ancient Greek texts which they then urgently translated into Latin.

That medieval mathematicians had difficulty grasping Greek texts such as Plato's *Timaeus* and the concepts conveyed by them is noted by modern scholars such as Max Jammer, Michael S. Mahoney and Bruno Latour. Mahoney comments that theoretical geometry was foreign to them "in more than a linguistic sense,"[14] and that scholars struggled to appreciate the sophisticated content of Euclid's *Elements*. Elizabeth L. Eisenstein comments:

> *'To discover the truth of a proposition in Euclid,' wrote John Locke 'there is little need or use of revelation, God having furnished us with a natural and surer means to arrive at knowledge of them.' In the eleventh century, however, God had not furnished Western scholars with a natural and sure means of grasping a Euclidean theorem. Instead the most learned men in Christendom engaged in a fruitless search to discover what Euclid meant when referring to interior angles.*[15]

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[12] The word *algorithm* is a corruption of the name of the great astronomer and mathematical scholar, al-Khwarizmi (d. ca. 850) while algebra is a derivation of the title of his most important book: *Al-jabr wa'l muqabalah*. See Carl B. Boyer, *A History of Mathematics (1968)* revised by Uta C. Merzbach (1989), p. 255-256. Al-Khwarizmi is also the author of a world map on the Ptolemaic model (Gray (1989), p. 41).
[13] Boyer (1989), p. 255. See also Gray (1989), p. 41.
[14] Michael S. Mahoney, "Mathematics," in *Science in the Middle Ages*, ed. David C. Lindberg (1978), p. 153.

Reading Latour, the difficulty which medieval scholars had with theoretical geometry can be seen to have a clear explanation. They could not *see* it, geometry had not been made visible to them through the use of graphical figures and their comprehension of the concepts of geometry was mediated only through the texts. It wasn't until the invention of the printing press in the fifteenth century made possible the exact duplication of geometric figures which could then be embedded in printed books that a 'natural and sure means to arrive at knowledge' of geometry was finally furnished. The use of duplicated visual 'inscriptions' produced an "optical consistency" which allowed thinkers to see contradictions as well as inherent logic.[16] It is not surprising that without graphical representation, the geometry of space was poorly grasped. The cognition of spatial concepts relies on a perception of space, and the adoption of Euclidean geometry as the basis for the new cosmology waited until the geometry of space became visible.

The earliest significant advances in European scholarship in mathematics and science date from the thirteenth century. Leonardo Fibonacci of Pisa and Jordanus de Nemore are two mathematicians who did original work, pursuing the Greco-Arabic tradition.[17] In the fourteenth century, Thomas Bradwardine and Nicole Oresme of England and France respectively also left a heritage of books and treatises. There was not a great deal of creative work in mathematics, however, until the fifteenth century and, indeed, the lack of successors to Fibonacci and Jordanus demonstrates how little original mathematics was done in medieval Europe.[18] Mathematics was something to be examined and debated, rather than initiated. The thirteenth and fourteenth were centuries of isolated work and slow accretion of

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[15] Elizabeth L. Eisenstein, *The Printing Press as an Agent of Change* (1979), p. 698-699.
[16] Latour (1986), p. 11-16.
[17] Mahoney (1978), p. 162.
[18] Mahoney (1978), p. 159.

knowledge; books had very limited circulation prior to the invention of the printing press. It was to be a century after Oresme (d. 1382) before anyone to rank with him appears in the history of mathematics and by that time, the scholarly environment had greatly changed.[19] It was the beginning of the Renaissance; Mahoney describes it as: "another age, one in which Apollonius, Diophantus, and Pappas began to circulate in Greek, one in which increased interest in technical astronomy fostered the development of trigonometry out of Greek and Arabic sources, and one in which the arithmetical and algebraic practices of merchants and engineers, long part of an oral tradition, were being set down in textbooks for wide circulation."[20]

Euclidean geometry

During the two centuries between the work of the Pythagorean school and the founding of an academy at Alexandria by Ptolemy I, which came to count amongst its cadre of elite scholars one known only as Euclid, much work was done in mathematics: in arithmetic, geometry and algebra.[21] Euclid's *Elements*, written in ca. 300 B.C., was an exposition of the fundamentals of the mathematics of his time intended for use in teaching. It became the best-selling textbook of all time, surviving for many centuries.

Euclid's *Elements* set out the various assumptions upon which the theories rested. Greek geometrical proofs worked because of certain assumptions about the underlying nature of space which lead to the concepts of congruence, similarity and parallelism. Congruence is a

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[19] Mahoney (1978), p. 169.
[20] Mahoney (1978), p. 169.
[21] Although the word *algebra* is derived from the title of al-Khwarizmi's book on the theory of quadratic equations (see note 12), the Greeks had themselves studied equations. Gray notes that this

fundamental idea. Two figures may be said to be congruent if they can be aligned such that they coincide perfectly with each other – they are therefore deemed to be exactly equal. This conceptual alignment is possible only in space in which movement is allowable in principle without distortion of the figure, or where two or more figures may be constructed with the same properties. Such space is necessarily homogeneous and isotropic. Parallelism follows from the notion of movement of figures: "Rather than discuss translations directly," Gray explains, "the Greeks preferred to work with parallel lines, which, one might say, are the tracks along which a translation is performed."[22] And for similarity, imagine two figures with equal angles and equal ratios of corresponding sides; thus, similarity implies the existence or possibility of scale copies.

Euclid's *Elements* deals with solid geometry to a small extent but the limited treatment of it demonstrates how little it had been developed by the Greeks. Although the idea of plane co-ordinates is ancient,[23] there is no reference in Greek mathematics to spatial co-ordinates (x,y,z).[24] Classical geographers certainly used latitude and longitude as co-ordinates for the earth or the celestial spheres, but a network of parallels and meridians was not adopted as a grid system until its use on Ptolemaic maps engendered a new geometrized consciousness of space.[25] Euclidean space with its homogeneous quality and the infinity of lines could not symbolize the finite, discontinuous, anisotropic Aristotelian universe so that Greek

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study was derivative of Babylonian ideas in solving simultaneous equations in two unknowns and quadratics. Gray (1989), p. 20.
[22] Gray (1989), p. 27.
[23] Jammer (1969), p. 25, suggests that the idea precedes Greek geometry, citing an Egyptian hieroglyphic symbol in the form of a grid which stood for 'district.'
[24] Jammer (1969), p. 25.
[25] See note 3, Chapter 1.

mathematicians disregarded the idea of abstract 3D space.[26] With the dissolution of faith in Aristotelian cosmology in the early Renaissance, Euclidean extension became a surrogate for absolute space, but the use of a 3D rectangular co-ordinate system was not considered reasonable until the 17th century when common sense ideas of space had changed drastically.[27]

The first printed edition of *Elements* appeared in 1482 – it was Johannes Campanus' Latin translation of 1255. From the beginning of the Renaissance until about 1800, mathematicians believed and sought to show that Euclidean geometry was the only possible geometry. A geometry with no parallels was a possibility disproved by Saccheri, Legendre and others.[28] This was despite the fact that spherical geometry was a geometry with no parallels and this obvious example of non-Euclidean geometry was well-known. Euclidean geometry was perceived as intuitively true, as was famously expounded by Immanuel Kant (1724-1804).[29] Kant's analysis went no deeper than the assumption of an intuitive *a priori* sense of space for which he pondered no cognitive or perceptual basis. After reading a paper by Leonhard Euler in 1768 in support of Newton's conception of absolute space, Kant concluded that 'space' existed independently of the mode of perception.[30] Gray describes Kant's analysis:

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[26] The Greek use of co-ordinates was always particular to the figure in the case, never as a general principle. See Cassirer (1963), p. 186.
[27] Jammer (1969), p. 26.
[28] This required a proof of the so-called 'parallel postulate' of Euclid, a task which ultimately proved fruitless. An interesting historical account is given by Jeremy Gray (1989).
[29] Kant lectured on physical geography at the University of Königsberg from 1756-96. His general approach detached geography from theology. See David N. Livingstone, "Geography," in *Companion to the History of Modern Science*, eds. R.C. Olby, G.N. Cantor, J.R.R. Christie, and M.J.S. Hodge (1990), p. 747: "Since God was now banished to the outer fringes of the shadowy noumena, it was futile to search for him along the course of river beds, behind the laws of mountain-building or in the ebb and flow of tides."
[30] Bronowski (1978), p. 5.

> *[T]he mathematician can call upon* a priori *intuitions which accord with the basic concepts [of mathematics]; conceptual analysis alone cannot avail, because that would make mathematical truths analytic (i.e. merely logical) which Kant says they are not. The synthetic* a priori *properties that belong to the concept and which do the trick cannot be empirical, because that would make geometry into a purely empirical science and not the universal science that for Kant it is. So they derive from our intuition, specifically our intuition of space and time. Our intuitions are not hunches ... the way in which we connect a mode of knowledge to its objects ... [Kant] wrote [in* Critique of Pure Reason, *1787], "The mathematics of space (geometry) is based upon this successive synthesis of the productive imagination in the generation of figures. This is the basis of the axioms which formulate the conditions of sensible* a priori *intuition (from which it can be shown) that two straight lines cannot enclose a space."*[31]

Gray concludes that Kant's belief in the intuitive necessity of Euclidean geometry is "either unintelligible or wrong."[32] From a perspective of knowing that the logic of non-Euclidean geometry is equally valid, Kant's explanation seems naïve. Boyer likens the effect of the theorizing of non-Euclidean geometry on Kantian philosophy to that on Pythagorean thought of the discovery of incommensurable magnitudes.[33] Devastating. But Kant's was a concept which rationalized what appeared to be indubitable in the 18th century. It wasn't until the fundamentals for other geometries were theorized by Riemann that the contradiction was resolved. Riemann was able to argue that lines might be unbounded without also being infinite and theorized spherical geometry as the geometry of geodesics on a surface of constant positive curvature.[34] Thus plane, or Euclidean, geometry is only a special case of Riemannian geometry where the surface has a constant zero curvature.[35] Jeremy Gray says of the explication of projective (elliptical) geometry:

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[31] Gray (1989), p. 84.
[32] Gray (1989), p. 85.
[33] Boyer (1989), p. 581.
[34] Gray (1989), p. 155.
[35] Curved space may have a curvature < 0 when it is hyperbolic, or a curvature > 0 when it is elliptical. On a sphere, the curvature = 1.

> *The historically interesting thing is that this simple description of a non-Euclidean geometry was not made very much earlier. That it could not be was due to the prevailing grasp of fundamentals; only on a Riemannian programme could the arguments ... be seen to be geometric and this simple model be understood as illustrative of anything comparable with Euclid's geometry of space. Lambert, Kant, and Taurinus rejected spherical geometry as a possible geometry for space on the grounds that in it two lines can enclose an area.*[36]

It was the incompatibility of flat and curved space which created the difficulty and it was only when this incompatibility was resolved and the distinction de-emphasized that non-Euclidean geometries became all the rage. Einstein's notion of 'spherical space' in relativistic cosmology is based not on Euclidean geometry but on Riemannian mathematics, and Jammer has suggested a comparison between this concept and Aristotle's idea of finite physical space.[37]

Euclidean space in mathematics has its counterpart in physics.[38] Newtonian space took geometry as the basis of universal mechanics and insisted upon the uniqueness of Euclidean geometry as God's sole ordering principle for the cosmos.[39] Because of complete confidence in the properties of Euclidean space, Descartes, Newton and Kant regarded these properties as also characterizing astronomical space or what Gray terms 'operational' space.[40] Thus, operational space took on the properties of abstract space. To be more specific, it took on the properties of the abstract space of plane geometry, two of which are homogeneity and

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[36] Gray (1989), p. 156.
[37] Jammer (1969), p. 22.
[38] Gray (1989), p. 177.
[39] Gray (1989), p. 178.
[40] Gray (1989), p. 178. It should be noted that Newton believed that geometry was practical, rather than an abstract ideal of perfection. See Preface to First Edition of the *Principia* in *The Leibniz-Clarke Correspondence, Appendix A: Extracts from Newton's Principia and Opticks*, (ed.) H.G. Alexander (1956), p. 143.

isotropism. Operational or physical space is neither homogeneous nor isotropic because of the objects in it but, as absolute space regards objects as being *in* a substanceless extension which is God's 'sensorium' or stage, this consideration is beside the point. The point is that the stage is bare.[41] The absolute space of Newton held sway just so long as the belief in the non-existence of other than Euclidean geometries, and from Newton's era until approximately eighty years ago, the idea that there was a limit to the space of the universe was an absurdity.

Maps and the geometry of space

Graphical representations of all kinds are often grouped together and it is not always obvious that maps and pictures are distinctively different. Both pictures, or paintings, and maps may be analogue representations of three-dimensional space on a two-dimensional plane and both certainly depend upon the viewer understanding and accepting conventions in the representation, or the graphic language. The picture, however, is in one sense a truer and more direct transformation because it represents the infinite space of the Void, the volumetric reality of ordinary, everyday experience, the air through which we move, on the infinite space of the plane. It is thus a true transformation of three-dimensional space into a two-dimensional representation. It accomplishes this through the convention of, for example, linear perspective. It has been asserted by Edgerton, Cassirer, Panofsky, and Ivins that the modern idea of space was anticipated by artists of the Renaissance who began to use the technique of linear perspective demonstrated by Brunelleschi in the early fifteenth century. This has been touched upon already in chapter 1 and I refer the reader back to the remark by Ivins. Ivins felt that the history of the development of the technique of linear perspective

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[41] Gray (1989), p. 178.

in art warranted attention precisely because of its logical treatment of the 'problem of space,' an issue which was central to the "subsequent history of physics and philosophy."[42]

Although both picture and map represent space as continuous, only the picture, particularly if it is a perspective drawing, is continuous in the sense of being full and complete. The continuous nature of the cartographic drawing should be treated with more caution as the putative surface may be more notional than perceived – a scheme for showing topological relationships amongst objects.[43] Furthermore, only in certain cases do maps even attempt to represent the volumetric reality of space by portraying the irregular third dimension of the earth's surface. The conventional depictions of relief are extremely contrived and require experience to interpret. Such conventions are also used to represent imaginary statistical volumes such as precipitation or population where these phenomena occur over geographical areas.

While a perspective drawing is a representation of a viewer's perception of the arrangement of objects in space, a projected map's relationship to reality is purely mathematical. The modern map is related to the earth in two ways: it has a defined dimensional relationship which is its *scale*, and a defined geometrical relationship which is its *projection*. Both scale and projection require the concepts of homogeneity of space, where one point resembles any other, and isotropism, where space has no preferred direction and extends uniformly, as necessary conditions. A map is essentially unconcerned with the character of objects existing in space, which is what preoccupies artists; its task is only to show connectivity and adjacency relations (topology) between these objects.

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[42] See note 12, Chapter 1.

Much has been written on the problems and the theory of map projections, of their characteristics, the patterns of distortion inherent in them and how to obtain a measure of distortion, and what happens to the properties of size, shape, distance and direction when transformed from the curved surface of the spherical earth into a representation on the two-dimensional surface of the plane. The third dimension of the earth's surface is essentially ignored for the purposes of map projection;[44] in any case, its range is extremely small when compared to the overall dimensions of the globe. While the sphere is undoubtedly a solid, its interior space is of no moment either. It is only the shell which is represented graphically on the map, a surface of no third dimension. The projection process is one of representing that area in the Cartesian plane, and here we arrive at the crucial paradox between the infinitude of the plane and the finite area of the earth's surface. The geometry of the plane is Euclidean with the essential properties of the infinity of lines and the existence of parallels while the geometry of the surface of a sphere is an example of a non-Euclidean geometry. It is easy to see that the properties of the infinite length of lines and the existence of parallels do not hold on the sphere where lines are finite in length and parallel lines do not exist – all great circles (which are analogous to straight lines in Euclidean geometry) intersect in two places. That they meet in two places is considered a problem by mathematicians, one that was resolved by considering not the whole sphere but half of it. Thus a geometry of half great circles, or meridians, is obtained and a metric is defined on the surface. The geometry of the surface then becomes the geometry of geodesics on a surface of constant positive curvature.[45]

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[43] John Keates, *Understanding Maps (1982)*, 2nd edition (1996), p. 84.
[44] It is sacrificed for the sake of preservation of surface angles and lengths. (Keates (1996), p. 86).
[45] Gray (1989), p. 155.

Clearly the geometry of the globe is non-Euclidean, elliptical (spherical) geometry in which lines are finite in extent[46] and lines intersect, cannot be parallel. A map, however, is a two-dimensional analogue of the space of the surface of the globe and in this form, the surface of the globe having been projected onto a plane, the geometry of the space in the plane is Euclidean: infinite, continuous, homogeneous and isotropic. So also, therefore, is the geometry of the surface of the earth *as it is made visible inscription*. Map projection, then, nullifies not only the spherical nature of the globe but its concomitant geometry. It reinforces the intuitive way in which we tend to situate ourselves intrinsically in a horizontal plane. Because the plane of the map is infinite, so by extension the universe is conceived as infinite. Now, as stated above, theoretical physics, no longer dependent upon Euclidean geometry for an explanation of space, can postulate that the space of the universe may be curved and therefore finite in extent, but this was not possible before Riemannian mathematics.

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[46] Lines must be $< \pi R$, where R is the radius of the sphere. Gray (1989), p. 155.

4

Cartography

We have seen in the previous two chapters how cosmology and geometry were understood in medieval Europe, and how they evolved during the Renaissance. I concluded Chapter 3 with observations on the paradoxical use of plane geometry to represent the earth, a solid body almost a perfect sphere and a real object, not an abstract conception. The use of plane, or Euclidean, geometry to depict the real earth, at a small or global scale was unheard of in medieval Europe. It was introduced to Europeans by the Ptolemaic opus, the *Geographia*, and its growing dominance coincides, as we shall see in this chapter, with those changing concepts of space already discussed.

Mapping in medieval Europe

The transmutation of the dominant form of mapping from the medieval representational paradigm for geographical ideas and knowledge to the modern normative depiction of absolute space took place in "a moment of unprecedented hermeneutic instability in the *imago mundi*."[1] While Gillies describes this mutation as a 'moment' it was far from instantaneous and, even after the new mapping model of the Renaissance came into circulation, the ancient forms persisted long after the general acceptance of the new cartography and the new cosmology which each took Euclidean geometry as axiomatic.[2]

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[1] Gillies (1994), p. 36.
[2] John Gillies has provided an elaboration of the ways in which the ancient iconography persisted in the decorative frame of the map. He states (1994), p. 164: "As long as the new geography could be

While the 'new cartography' came to be the dominant model, and to be perceived as mathematically legitimated, the medieval mapping conventions endured. In a way, their logic is still understood to this day, is available and in use as one of the mapping conventions with which we are familiar. I think it is safe to say, however, that most people in the Western world would point to the tradition of map projection and location of places by means of geographical co-ordinates as the standard, scientifically authoritative model. It was certainly the basis of the topographical surveys of the eighteenth and nineteenth centuries, and became a hallmark of Enlightenment thought.

Very few maps were created in the medieval period in Europe, however, as is evidenced by the paucity of examples surviving. Historians have argued that it was an age in which there was not a 'map consciousness' in the modern mode.[3] P.D.A. Harvey starts his *Medieval Maps* with the sentence: "Maps were practically unknown in the Middle Ages."[4] The statement is intended to be provocative and, of course, he does not mean that maps did not exist but that they were few, and that a general awareness of maps as a unique form was yet to come into being. As Denis Wood declares with his usual style:

> *[T]he Middle Ages were mapless for exactly the same reasons that prior ages had been, namely that the historical conditions that call maps into being had yet to develop. It was changed circumstances that fanned the spot fires of sketch mapping, and world-diagram drawing, and landscape painting of the Middle Ages, into the conflagration of map consciousness so evident in Europe in the sixteenth century.*[5]

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so inscribed within the familiar cosmography, it was controllable; or would be until the 'new philosophy' of Copernicus, Galileo, Brahe and Kepler had penetrated deep enough into general cartographic culture for the ancient cosmographic scheme to be abandoned." This formulation of the sequence of the process disregards the possibility of a more complex interaction between the new cartography and the new philosophy.

[3] For example, P.D.A. Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 464.

[4] P.D.A. Harvey, *Medieval Maps* (1991).

[5] Wood, *Cartographica* 31, 3 (1994), p. 57.

What is generally understood about the conventions of pre-Renaissance mapping is that representative practices were split into separate spheres – the practical, exemplified by the portolan charts which were navigational aids for mariners, and the theoretical, exemplified by the *mappaemundi*, sacred maps symbolizing the medieval concept of the world. The earliest surviving portolan chart dates from about 1300 though there are prior references to them while the *mappaemundi* are believed to have been produced from the eighth century on.[6] Though the latter depicted world geography in crude form, they were not intended to be judged for positional accuracy or used for orientation or other such ordinary purposes. They were, rather, designed as a visual representation of Christian cosmology, centred usually on the holy city of Jerusalem and symbolizing the stable, circular, harmonious universal order. Their purpose was didactic and formalistic for, as David Woodward relates, they were designed: "to instruct the faithful about the significant events in Christian history rather than to record their precise locations."[7] They illustrated the accepted cosmology of the Middle Ages, the Aristotelian world picture reconciled with Christianity.

The medieval *mappaemundi* did not employ a mathematical transformation of the surface of the earth (generally known to be spherical since antiquity) onto a plane surface as is deliberately used in modern map construction. Likewise, the idea of locating places by means of co-ordinates was unknown in Europe until the thirteenth century when Roger

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[6] P.D.A. Harvey, "Medieval Maps: An Introduction," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 283.
[7] David Woodward, "Medieval *Mappaemundi*," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 286.

Bacon described such a system in his *Opus maius* (1267-68).[8] Exact geographical location was irrelevant to the main purpose of a *mappaemundi* in any case, although it was not unknown for these maps to include content such as place-names that appeared to be an attempt to provide useful general geographical information in addition to Christian exegesis.[9]

The tradition of the *mappaemundi* appears, from the historical record, to have been one of long-term continuity but Harley and Woodward point out that the evidence of the continuity of this tradition indicates "little more than its fossilized preservation by copyists."[10] No signs of a changing, adapting, active cartography have yet been discovered by historians of the tradition, medieval mapmakers being apparently content to follow the standard model in ritualistic fashion. Bacon's suggestions for the use of a systematic geometry had no contemporary ramifications.[11] They followed on the heels of the new translation of Euclid's *Elements* by Campanus in 1255 but, as we have seen, the medieval Schoolmen had difficulty comprehending the concepts of Greek geometry. They were also pre-occupied with the teachings of Aristotle despite the church bans and the condemnations of Etienne Tempier. This century also saw translations of Ptolemy's *Almagest*, of Archimedes and Al-Khwarizmi; the first classification systems in natural history; and ideas on optics by Pecham, Grosseteste, and Witelo. The Aristotelian foundation for understanding the world, however,

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[8] Bacon argued in favour of a mathematical basis for understanding the world, and spoke of location of places on the earth by means of 'latitude' and 'longitude.' His ideas had apparently no influence on his contemporaries. Woodward and Howe (1997), p. 203-204.
[9] Woodward (1987), p. 288.
[10] J.B. Harley and David Woodward, "Concluding Remarks," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 503.
[11] David Woodward suggests that the idea of using co-ordinates to locate places, as Bacon advocated, may have remained latent simply because of the lack of accurate positional data at the time. See Woodward, "Medieval *Mappaemundi*" (1987), p. 323.

was not seriously questioned for nearly two more centuries and Aristotle's ideas about place precluded uniform treatment of space.

The portolan charts, in contrast to the *mappaemundi* – these comments also apply to the regional, itinerary maps of the day – were used for navigation and thus geographical position was important. The itinerary map – the Gough map is the prime example (1325-30) – was used for travel on land and thus emphasized interconnections between places with notes on the distances to be travelled. In modern parlance, they would probably be termed topological – they do not exhibit correct scale and direction, only relative emplacement. Likewise, the portolan charts were used for navigation at sea and attempted to show details of coastlines with sailing distances between points on the coasts and directional information. Again, however, absolute positioning is absent. The charts show the relevant geographical relations between points without locating them in a global framework. Similarly, sketches produced in the Middle Ages that we would now classify as maps, of which a few examples survive (plans of drains, or of property lines, e.g.), were localized, practical, pictorial drawings displaying only relative, local spatial relations.

These separate mapping traditions of medieval Europe were effective for the purposes for which they were designed. What is important to note, for our proposition, is the concept of space they display. Like the artistic conventions of the Middle Ages, the cartographic conventions show no sense of unified space. The concept of space was heterogeneous, a "space of emplacement characterised by localisation."[12] The Aristotelian concept of spatial

[12] Gillies (1994), p. 62. Gillies refers to Michel Foucault's formulation of the history of space in "Of Other Spaces", *Diacritics*, Spring (1986). Foucault blames Galileo for opening up the medieval 'space of emplacement.' "For the real scandal of Galileo's work," he says (p. 23), "lay not so much in his

relations, that is, that spatial relations are relations between specific places not in any sense absolute, is evident in both mapping cultures: that of the *mappaemundi*, and that of the portolan charts. In fact, on the *mappaemundi*, places of particular meaning or importance are often treated with an emphasis which deforms the character of spatial relations as understood in terms of physical scale.[13]

Thus the two main cartographic paradigms of the Middle Ages did not manifest a perceived absolute sense of space. The *mappaemundi* manifested a heterogeneous and relative sense of space in both topology and scale. It was a fluid concept, needing neither continuity in extension, congruence, nor similarity – it was clearly not Euclidean as both congruence and similarity are assumptions of Euclidean space. The portolan charts did show a concern with relative distances, i.e., congruence, and directions, but not with other geometrical relationships. This conforms perfectly with our sense of the medieval perception of spatial relations as discontinuous and unsystematic. The imagined sacred world portrayed by the *mappaemundi* and the tactile, experiential world portrayed by the portolan charts, and the itinerary and sketch maps, conformed with the artistic conventions of the Middle Ages and mirrored the unstructured, overlapping characteristics of feudal social space with its plurality of particular worlds and its chaotic tangle of allegiances and hierarchies. David Harvey summarizes this period:

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discovery, or rediscovery, that the earth revolved around the sun, but in his constitution of an infinite, and infinitely open space. In such a space the place of the Middle Ages turned out to be dissolved, as it were; a thing's place was no longer anything but a point in its movement, just as the stability of a thing was only its movement indefinitely slowed down. In other words, starting with Galileo and the seventeenth century, extension was substituted for localization." Foucault's characterization of the alteration in conception of space is eloquent even if his attribution of both the heliocentric cosmological model and the theory of the infinite universe to Galileo is inaccurate.

[13] See Harley and Woodward, "Concluding Remarks" (1987), p. 505.

> *Within each knowable world, spatial organization reflected a confused overlapping of economic, political, and legal obligations and rights. External space was weakly grasped and generally conceptualized as a mysterious cosmology ... Medieval parochialism and superstition were paralleled by an 'easy and hedonistic psycho-physiological' approach to spatial representation.*[14]

Ptolemy's map

During the fifteenth century, the separate and unique cartographic traditions of the *mappaemundi* and the portolan charts began to be overshadowed by a new paradigm which incorporated attributes of both. The usefulness of the portolan charts and the redemptive quality of the *mappaemundi* were both superceded and subsumed to an extent by a new world image which gradually came to dominate the medieval traditions even though both continued to be published and used into the seventeenth century. As well, the period is noted as one in which maps began to be recognized as objects of interest in their own right, rather than only as additions to written text and they began, therefore, to be published for their own sake.[15]

This new cartographic mode evolved from a practice which began in the fourteenth century when European sea charts began to be extended, by the Catalan school most notably, to include discoveries in Africa and Asia.[16] The new world image was a cartographic image in which every place was located with respect to every other place by means of an abstract, geographic grid.[17] The pivotal event in this transition, as noted in Chapter 1, was the

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[14] David Harvey, *The Conditions of Postmodernity* (1989), p. 240-41.
[15] David Woodward, "Medieval *Mappaemundi*" (1987), p. 314.
[16] Helen Wallis and A.H. Robinson (eds.), *Cartographical Innovations: An International Handbook of Mapping Terms to 1900*, p. 82. See also Woodward, "Medieval *Mappaemundi*" (1987), p. 358.
[17] The geographic grid is correctly termed the 'graticule' but I use the word grid intentionally as a more generic term which can imply a variety of types of ordering systems. It is not meant to refer only to a rectangular arrangement of parallels and meridians, but to include curved lines as well.

rediscovery, by a group of Florentine scholars, of Claudius Ptolemy's second century geography text in ca. 1400 and its subsequent wide distribution and enthusiastic acceptance.

Ptolemy's *Geographia* commanded considerable interest. The text included a description of the geography of the known world, and instructions on how to construct a map of the world. It was entirely practical, but its underlying concept was completely different from that of the map models in use in the fourteenth century. Scholars quickly realized that Ptolemy's geography was incomplete and, in some aspects, obsolete, but this fact did not distract them. The methodology and the implied philosophical approach to the world were so novel that they seem to have held a fascination. The world map of Ptolemy was an entirely different genre from either the portolan charts or the *mappaemundi*, the two types of maplike depictions with which Florentine society was familiar. As soon as it had been translated (1406-07),[18] the *Geographia* began to be copied and a 'scriptorium' was established in Florence specifically for the reproduction of the Ptolemaic opus, so strong was the demand for copies.[19] The earliest example of a map showing the influence of Ptolemy's new cartography is the world map by Pirrus de Noha[20] which is contained within a manuscript by Pomponius Mela, 1417. Subsequently, maps drawn on the Ptolemaic model began to proliferate, though the first map to project the globe as a whole did not appear until ca. 1508.[21]

On a Ptolemaic map, no place was privileged over any other place – every point on the map had identical weight and, moreover, there was no centre, no starting point. The medieval

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[18] See note 2, Introduction.
[19] See Edgerton (1975), p. 98.
[20] See Woodward, "Medieval *Mappaemundi*" (1987), fig. 18.79, p. 357.

cartographic tradition of world mapping had reflected the cosmology of the classical world and was firmly rooted in notions of centres and borders. Gillies says that the 18th century philosopher, Giambattista Vico, "derives the archaic origins of the 'world' from that of the city. The walls of the city and the borders of its territory are both telescoped into the edges of the ancient map."[22]

The opposition between the centre and the extreme edges of the world led to the 'outlandish' belief that the edges of the world were inhabited by 'barbarians' and characterized by moral exorbitance.[23] The 'edges' of the Greek world were less physical and more metaphysical and the Pythagorean fixation on the perfection of the circle as mathematical form meant that the margins were circular. Imagination populated the outer margins of the earth's sphere with mythological monsters and they were tainted with moral depravity because they had no definite form. Huizinga reminds us that "supernatural fear results from unbridled imagination, from the possibility that something new and dreadful could suddenly appear." And he continues, "As soon as the image becomes clearly drawn and defined it arouses a feeling of security and familiarity."[24] Before the appearance of maps and pictures in print, the world beyond experience could be only vaguely imagined by ordinary Europeans. The medieval *mappaemundi* had reflected the same preoccupation with centrality, generally locating Jerusalem at the privileged centre of the map, as it was the

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[21] See note 8, Introduction.
[22] Gillies (1994), p. 6. Gaston Bachelard in *The Poetics of Space* (1969) magnifies this idea to personal scale where the individual is sheltered in the space of his house (p. 5): " the sheltered being gives perceptible limits to his shelter. He experiences the house in its reality and in its virtuality, by means of thought and dreams." The walls of the house ultimately telescope out to the edges of the map.
[23] Gillies (1994), p. 18.
[24] Huizinga (1996), p. 193.

centre of the Christian world, and populating the outer margins with mythical and barbaric imaginings.[25]

Ptolemy specified several projection systems in the *Geographia* which enabled the plotting of a system of parallels and meridians, the abstract grid system which could be described on a sphere, on a plane in a way that attempted to account for the deformation mathematically.[26] On a Ptolemaic map, there would be no controlling centre, only an arbitrary reference meridian. Aspects of the distortion could be seen immediately and assimilated. It made possible a continuous, logical framework to which everything could be referenced. This allowed spatial relationships to be comprehended in entirety and accorded the world a mathematical unity which was previously unknown. The new world map was to be in direct opposition to the Aristotelian concept of space as disjoint, a "discontinuous residue between objects,"[27] in its assumption, manifested by the grid, that space had to be continuous and isotropic. The cartographic grid system depicted space as abstract extension, as container and as rationally ordered. It was nearly three hundred years before this concept was completely theorized but by that time, the notion was so thoroughly natural that it was accepted as axiomatic.

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[25] See Cosgrove, ""Mapping New Worlds: Culture and Cartography in Sixteenth-century Venice," *Imago Mundi* 44 (1992), p. 68, on the continuation of a 'moralized geography' in the syncretist cosmology associated with Ptolemaic cartography.
[26] Ptolemy's first projection was characterized by straight, converging meridians and equidistant, concentric circular arcs for parallels; the second projection modified the first by introducing curved meridians which were circular arcs; the third projection was a perspective view of the inhabited world from a distant point in space with true scale along the central meridian and parallels and meridians curved about the straight central meridian and straight central parallel. See Snyder (1993), p. 10-14; and O.A.W. Dilke (with additional material supplied by the editors), "The Culmination of Greek Cartography in Ptolemy," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 185-189.
[27] Edgerton (1975), p. 158.

Samuel Edgerton ascribes to Ptolemy's third projection the inspiration for the discovery of linear perspective in fifteenth century Florence, a discovery which radically altered artistic pictorial conventions. What was so different about the use of linear perspective was the illusion of depth created. The viewer was drawn into the painting by the illusionistic erasure of the surface of the painting, an essential ingredient of which was the detachment of the observer from the scene being depicted. The viewer no longer participated in the scene, the painting was no longer about the experience of walking about; the viewer had become a static voyeur, outside the frame which began to function like a theatrical frame.

Ptolemy's third projective method for envisioning the world involved map projection in which a viewpoint at a distance from the globe is postulated. Brunelleschi, the first painter to demonstrate the use of linear perspective (1425), and Alberti, the first to describe the construction of drawings on the principle (1435), applied Ptolemy's map projection method to create the illusion of depth but shifted the view from vertical to horizontal.[28] The pictorial space of a perspective drawing took on the characteristics of homogeneity and isotropism which were attributable to Ptolemy's map. It could also be said that the space within the drawing was infinite since the vanishing point is theoretically at infinity. It can be perceived that the newly conceived space of the perspective painting was similar to the cosmological space of a Ptolemaic world map and the increasing use of linear perspective, and of projected maps and the evolving theory of the infinity of the universe were closely linked through the fifteenth and sixteenth centuries.

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[28] See Ken Hillis, "The Power of Disembodied Imagination: Perspective's Role in Cartography," *Cartographica* 31, 3 (1994).

It has already been observed that the organizing principle of the medieval map was circular and that the space framed by it was a circular space with a controlling centre. Very often the map was contained within an inscribed bounding circle, though this was not necessarily the case. In many instances, the map was fitted to the size and shape of the page or other medium on which it was drawn. The idea of putting a rectangle around the map area was not, however simple and obvious it now seems, a routine concept prior to the Renaissance. To the modern map reader, the rectangular frame is a fundamental sign: it indicates that the cartographic space contained within the frame is continuous, that it is an excerpt of universal space which in reality continues beyond the border of the map fragment, and that it is independent of the page on which it appears. This framing concept is an essential feature of the depiction of absolute space. Such a framing, though not unique to Ptolemy, was a basic principle in the system of map construction described by Ptolemy in the *Geographia* and, with the popularity and proliferation of the Ptolemaic map, became a basic and emblematic mark of modern cartography.

With the change from the medieval, Aristotelian (qualitative), notion of space to the Renaissance (quantitative) model came also the change from an earth or human perspective to a heavenly perspective. Not only is the observer detached from the scene, an onlooker viewing a portion of the universe through the rectangular window defined by the neatline, but the observer is separated from the Earth itself. Ptolemy's system depicted the earth from an elevated, God-like viewpoint. The construction of the world map according to his projection system involved the shift from side view, used commonly on *mappaemundi*, to an orthogonal view. "At that imaginary point," comments Lestringant, "the eye of the cosmographer ideally coincided with that of the Creator."[29] Ken Hillis describes this shift in point-of-view as a shift in power to the "viewer-as-subject" which participates in the rise of individualism in the quattrocento. A shift to the geometrized universe, as we have seen in Chapter 2, simultaneously reduced the power of God as arbiter of all things while elevating humans as agents of change and control. Hillis writes,

> *[T]he re-orientation of medieval embodied vision to a cartographic optical overview offers a bridging symbolism that helps to position the Renaissance body within a world of visually ordered cartographized space. With Jacopo [de'Barbari], we are almost ready to be guided downward through the image, and, passing through the vanishing point, to discover the infinite space of the cartographic map.*[30]

Maps that use the Ptolemaic system and treat space homogeneously, do not privilege particular places within them. There is no centre or *omphalos* and therefore no radiating hierarchy of meaning or importance. Neither is there the stasis of balance around such a centre and the Ptolemaic *oikumene* assumes a dynamism, a potential for expansion in any direction. In the years following the rediscovery of Ptolemy's *Geographia* and his projections of the spherical world onto a plane surface, Europeans journeyed outward searching for knowledge of lands beyond the *oikumene*, treasures, fabled creatures and the fountain of youth. In many instances, maps of the fifteenth and early sixteenth centuries display a jumble of fact and superstition, geographies real and invented, but the dynamic of exploration and discovery is very evident. The Ptolemaic world map brought the cosmographical into the realm of the practical and made the exploration of the world beyond the edges of the *oikumene* a no-longer-threatening possibility. The idea of the world

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[29] Frank Lestringant, *Mapping the Renaissance World: The Geographical Imagination in the Age of Discovery (1991)* (1994), p. 5.
[30] Hillis (1994), p. 10.

had been made visible, had been brought out of the realm of mythical thought, and made familiar. The new image of the world impelled the urge towards direct observation and systematic description. Lestringant comments:

> *[C]racks were appearing all over the ancient representation of the cosmos. Cosmography gained a dynamism and mobility from the unstable borders that frayed out to an uncertain joining up of parts of the world, and that varied according to the whims of the map-makers and the progress of long-distance voyaging. Out of this play of earthly parts was born a moving space, as the cartographical imagination and a conquering pragmatism embraced each other tightly in the invention of new territories.*[31]

After 1492, remarks Gillies, "the Renaissance figure of the voyageur is born. Exorbitance and audacity are celebrated rather than damned."[32]

The grid

The key to the normalisation of the periphery of the world and thence to the geometrization of space was the Ptolemaic grid. Until the eighteenth century when a reliable method for determining longitude at sea was found, world maps were not used for actual navigation. Mariners continued to use portolan charts, maps which charted coasts with respect to distance, adjacency and approximate sailing directions but without an overall locational framework. The world maps of Ptolemy and other non-seafaring intellectuals were studied in

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[31] Lestringant (1994), p. 115.
[32] Gillies (1994), p. 171. See also Jonathan Z. Smith, *Map is Not Territory: Studies in the History of Religions* (1978), p. 131: "The difference between these two standpoints – horror in the face of the vast, and enthusiasm for expanse and openness – is not merely a matter of aesthetic sensibility. A total world-view is implied and involved in assuming these postures, one that has to do with a culture's or an individual's symbolization of the cosmos and their place within it."

the libraries and observatories of the scientific establishment where, as Edgerton suggests, they "gave powerful impetus to the Renaissance rationalization of the world."[33]

The cartographic grid of Ptolemy, new to fifteenth-century Europe, reinvented an ordering system which had been widely used by both the Greeks and the Romans. Alexandria, a city personally identified with Alexander the Great, was designed on an orthogonal plan while the Roman system of land survey, called centuriation and based on division of land parcels into equal portions, has been continuously in use up to the present day. As well, some regional maps surviving from the late Middle Ages do show a localized rectangular grid, though the idea was not adopted as a general principle.[34] The grid has innate appeal. It rhymes, and humans are subject to an aesthetic enjoyment of rhyme which has been defined as something which embodies both agreement and disagreement, unity and contrast.[35] The grid has a rhyming nature because it has parallel elements and like intersecting elements. Ptolemy introduced this appealing classification system to imagery of the world where it had been previously unknown to the contemporary culture in Europe and just at a time when visual imagery was coming into intellectual ascendancy. Edgerton comments that the society of sixteenth century Italy, in common with all human societies, "believed that geometric patterns formed in orthogonal relationships not only pleased the

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[33] Edgerton (1975), p. 113.

[34] For example, the early 14th century map of Palestine by Vesconte had such a grid. The idea was not taken up by others. See P.D.A. Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 476.

[35] The subject of the enjoyment of rhyme in nature has been elegantly treated by the philosopher, Nicholas Humphrey, in an essay entitled "The illusion of beauty," in *Consciousness Regained* (1983). Humphrey states, p. 126: "Considered as a biological phenomenon, aesthetic preferences stem from a predisposition among animals and men to seek out experiences through which they may *learn to classify* the objects in the world around them. Beautiful 'structures' in nature or in art are those which facilitate the task of classification by presenting evidence of the 'taxonomic' relations between things in a way which is informative and easy to grasp."

eye aesthetically but possessed talismanic power."[36] In any event, the acceptance of the Ptolemaic grid was assured by its appropriation by the Holy Church as a symbol of moral authority and Christian unity.[37]

Once the new cartographic grid system was accepted as normative, it was not long before it began to be used as a tool by theologians and secular politicians alike. The Treaty of Tordesillas of 1494 fixed a meridional line as a territorial demarcation, chosen to settle a question of jurisdictional conflict between Spain and Portugal. This line was a pure abstraction – it demonstrates a grasp of homogeneity of space where the space is the earth's surface and that surface can be imagined as subdivided by the imposition of an abstract geometric grid. The meridional line fell in the Ocean but it fell nowhere in particular. It was a construction, an artifact, of the cartographic grid, creating bounded space on either side of the line. The use of a mathematically-defined line to create a political reality was evidence of a sense of space as homogeneous, as able to be rationally-ordered, and as separate from physical matter. The sense in which the meridional line was used as a boundary for territorial space was expressed most clearly by Samuel Clarke, for Newton, in the great battle over the definition of space between Newton and Leibniz two hundred years later:

> *Bounded spaces are not properties of bounded substances; they are only parts of the infinite space in which the bounded substances exist.*[38]

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[36] Edgerton (1987), p. 11.
[37] Edgerton (1987), p. 11.
[38] See "Clarke's 5th Reply," *The Leibniz-Clarke Correspondence*, ed. H.G. Alexander (1956), p. 103. However, the phraseology used here is that used by Koyré (1957), p. 270, which is preferred. Koyré cites *A Collection of papers, which passed between the late learned Mr. Leibnitz and Dr. Clarke. In*

The classificatory function of the grid was an extremely significant aspect of its ready acceptance. Everything could be located with respect to the grid and therefore everything became potentially determinate and countable. Its invention, or discovery, marked the beginning of the transmutation of the sacred text, which was the discourse inherent in the medieval world map, into a scientific discourse, one in which surveillance was the paramount imperative, subject was separated from object, and humanity became spectator of instead of participant in the life of the world. A map was an efficient way of organizing sensory experience and the new cosmology of the Renaissance emphasized the interpretation of experience over deduction from theoretical principles. Francis Bacon (1561-1626) was "firmly convinced that the voyages of discovery had coincided with the beginnings of the new natural history,"[39] and used the voyages of discovery as a metaphor for the new empirical principles of science for which he was the prophet:

> *[B]y the distant voyages and travels which have become frequent in our times, many things in nature have been laid open and discovered which may let in new light upon philosophy. And surely it would be disgraceful if, while the regions of the material globe – that is, of the earth, of the sea, and of the stars – have been in our times laid widely open and revealed, the intellectual globe should remain shut up within the narrow limits of old discoveries.*[40]

The many new discoveries and revelations of the voyages were known from their descriptions by the explorers, but also from the fascinating new maps which located these discoveries in their global context providing a revolutionary new world picture, one which had a geometric framework. It influenced not only the disciplines of geography and

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the years 1715 and 1716 Relating to the Principles of Natural Philosophy and Religion. With an Appendix. (London: 1715).
[39] R. Hooykaas, *BJHS* 20 (1987), p. 470.

cartography, but all of the sciences because it changed the epistemological basis of natural philosophy itself. A century or more before the new science of Kepler, Galileo, Boyle, or Newton, the new cartography had inaugurated the twin conceptions: knowledge based on experience, and explanation based on mathematics.

The representation of the earth and the newly discovered lands captured in a geometrical net also neatly satisfied and symbolized the surging European passion for domination. "The abstract graticule," says David Woodward, "thus becomes a symbol in its own right, representing not the content of the physical earth but the expression of power over it, a separation of geometry from geography."[41] Because the grid as talismanic symbol had been appropriated by the Church, an active domination of nature took on the character of divine ordination and reinforced credence in Christianity's missionary authority throughout the world. To quote Edgerton again, "the cartographic grid in the Renaissance was believed to exude moral power, as expressing nothing less than the will of the Almighty to bring all human beings to the worship of Christ under European cultural domination."[42]

By the eighteenth century when the large-scale topographic surveys were launched, measurement had become the underpinning of the cartographic enterprise and of science in general. Maps and map-making are a paradigmatic expression of the practice of science, models of rational thought and measurement; cartography came to be almost an avatar of scientific and technical discourse, symbolizing – instead of the divine plan – the imperative of mastery over space. This imperative was to drive European civilization in the age of the

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[40] Francis Bacon, "Aphorisms I: 84," *The New Organon and Related Writings*, ed. Fulton H. Anderson (1960), p. 81.
[41] David Woodward, "The Meanings of Cartographic Co-ordinates", unpublished paper.

Enlightenment. Factual properties of space such as distance, direction, area, shape, pattern and volume implied the possibility of a rational ordering of space through the imposition of abstract grids of all kinds as a sense was created that space could be divided into mathematical units, unrelated to the physical, or mystical, characteristics of place, and that it could be redefined at will. Such grids could thus be notional, not physical, and might be mathematical, cultural or economic ordering systems (also termed taxonomic systems). They may be conceived of as *map* rather than *territory*.[43] That the distinction between map as ordering system and map as representation becomes largely uncomprehended is a significant factor in the argument that cartography actively creates a view of the world. The bounded spaces of the map are constructions - artifacts - of the imposed mathematical or cultural grid. Woodward adds that "the comforting mathematical precision offered by the graticule was only one aspect of its power. By its abstracted simplicity, it also homogenized space and offered to the explorer and his patron an iconic framework for exploitation."[44]

The actual geography of the earth was enmeshed in this framework by celestial determination as co-ordinate tables were produced with increasingly accurate measurements. Latitude sailing became more common in the Renaissance with oceanic voyaging so that the rutters and portolan charts used for navigation needed to be paired with world maps graduated in steps of equal latitude.[45] These pairings became more prevalent in the 16th century until the Mercator projection of 1569 merged the two traditions, and the

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[42] Edgerton (1987), p. 12.
[43] "The map is not the territory" was a maxim of the General Semantics movement begun in the 1930's by Alfred Korzybski. See the website of the ISGS (http://www.general-semantics.org/).
[44] David Woodward, "The Image of the Spherical Earth," *Perspecta (Yale Architectural Journal)* 25 (1989), p.13.
[45] David Woodward, "The Coordination of the World," paper presented April 24, 1991.

transfer of the gridded world map to a practical setting may have helped to popularize its image.

The grid was the 'trademark' of the Italian Renaissance, as Edgerton states, and its lattice was used to structure everything from pictures to agricultural fields. It became synonymous with and is symbolic of abstract space, that space which the grid, itself, in a practical sense, fractures into identical finite, determinate absolute spaces, spaces with no defining particularities or character, but which are only substanceless parts of infinite space. The grid therefore transcended its function as a locational framework on maps and became the primary rhetorical device of the new cosmology. This iconicity has survived. It replaced the religious symbolism of the *mappaemundi* with an icon more transparent and more insidious as the harmony and closure of the medieval world view was replaced by the somewhat unstable mobility of the universalistic modern world view. There is no more powerful symbol of universalism than the abstract grid; from the trademark of the Renaissance, it became the principal icon of modernity.

5

Mathematics and cartography – contextualizing links

As I stated in the Introduction, mathematics and astronomy have long been coupled with cartographic activities, especially with the development of map projections. It is notable that many individual thinkers who have been important contributors to the history of science and mathematics have been fascinated by maps, as can be shown by their work in the cartographic field as well. Can cartographic images have had unconscious effects on these individuals and, particularly, on their cognition of space? While this cannot be proven definitively, it is certainly a possibility. The conjunction of cartographic and mathematical work in the lives of many of the key individuals in the history of ideas is suggestive, at the very least. In this chapter, I will establish that the linkages between the fields of mathematics, cosmology, and cartography as manifested by the activities of individuals have historically been strong. This chapter is organized chronologically; I will give an overview of linkages known from the time of Pythagoras up to the time of Isaac Newton with a more detailed look at two individuals of the fifteenth century who I feel may have been particularly important in linking Ptolemaic cartography with geometry and cosmology.

Ancient Greece to the Middle Ages

The legacy of classical Greece to cartographic history is generally agreed to be one of theoretical mathematical concepts although the documentary record for the period is scanty and largely second-hand.[1] In Ancient Greece, cartography was bound up with the study of mathematics and the graphical representation of the world was a problem of legitimate interest for mathematical theoreticians. The era of most significant achievements in Greek cartography was the period from the sixth century B.C. up to Ptolemy's work in the second century A.D. with the early credit for map construction given to the astronomer, Anaximander (ca. 610-546 B.C.), a disciple of the philosopher-mathematician, Thales of Miletus.[2]

The first description of the spherical nature of the earth is attributed either to Pythagoras or to Parmenides (ca. 530 B.C.), author of a philosophical poem entitled *Concerning Nature*, and the description is believed to have been a hypothetical idea. The Pythagorean infatuation with the perfection of the circle as mathematical form influenced beliefs about the nature of reality and was probably the root of this orthodoxy. The fifth century B.C. atomist philosopher, Democritus (ca. 460-370 B.C.), is thought to have conceived of the world as oblong and to have produced a map in this shape in a work entitled *Geography*.[3] This work was one of three books considered mathematical works though none of them survive.[4] Democritus was reputed as a geometer and wrote other noted mathematical texts now

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[1] Editors, from materials supplied by Germaine Aujac, "The Foundations of Theoretical Cartography in Archaic and Classical Greece," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 130.
[2] Thales is described by Carl B. Boyer in *A History of Mathematics (1968)*, 2nd edition (1989), p. 54, as the "first true mathematician." According to Snyder (1993), p. 18-19, Thales used the gnomonic projection, a perspective azimuthal, in oblique form. This projection was rarely used prior to 1600 and an early European example of its use is by Kepler (1571-1630) for a star map in 1606.
[3] Aujac, "The Foundations of Theoretical Cartography in Archaic and Classical Greece" (1987), p. 137.
[4] The others were *Uranography*, and *Polography*, and a fourth book, considered a work in physics, was entitled, *Cosmography*. See Aujac, "The Foundations of Theoretical Cartography in Archaic and Classical Greece" (1987), p. 137.

known only by their titles: for example, *On Numbers*, *On Geometry*, *On Tangencies*, *On Mappings*, and *On Irrationals*.[5]

Greek mathematicians were mainly concerned with the geometry of the heavens and the earth and cartographic exercises were studies in the application of geometrical ideas. Eudoxus (ca. 408-355 B.C.), celebrated for defining the equality of ratios and for his celestial globes, and Eratosthenes (ca. 275-194 B.C.) whose work is adjudged fundamental in geography, cartography, and geodesy[6] are cases in point. Eudoxus has been called the father of scientific astronomy and the "most capable mathematician of the Hellenic Age."[7] Eratosthenes' famously brilliant calculation of the circumference of the earth was based on an understanding of the geometry of a sphere and the theory of a geocentric universe. Eratosthenes was a contemporary of the influential mathematician, Archimedes (287-212 B.C.), with whom he conducted a correspondence. Archimedes himseif made globes and is supposed to have written a treatise, now lost, on the construction of a representation of the universe which improved upon the existing models.[8]

The growth of the Roman Empire in political and economic spheres did not mean an end to the influence of Greek intellectual output. In fact, Greek cartography came under the patronage of Rome as is seen in the writings of Polybius (ca. 220 to post 118 B.C.) and the

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[5] Boyer (1989), p. 90.
[6] Editors, from materials supplied by Germaine Aujac, "The Growth of an Empirical Cartography in Hellenistic Greece," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 154.
[7] Boyer (1989), p. 105.
[8] These representations were known as *sphairopoiia*, see Aujac, "The Growth of an Empirical Cartography in Hellenistic Greece" (1987), p. 159.

Stoic, Crates of Mallos (150 B.C.), creator of a large terrestrial globe.[9] However, it is probably the astronomer and mathematician, Hipparchus (ca. 180 to ca. 125 B.C.), who contributed the most to geodesy and cartography during this period though the originality of his map projections is questioned by some. Germaine Aujac comments:

> *While some authors have claimed that [Hipparchus'] description of the* oikoumene *as a trapezium indicates an attempt to improve on the rectangular system of Eratosthenes, there is no direct evidence that this is not simply a reference to the chlamys-shaped inhabited world, as Dicks has pointed out. Conversely, although some authors have questioned whether Hipparchus was aware of the stereographic projection (for terrestrial or celestial use) or the astrolabe, the weight of opinion now points to his invention of both.*[10]

Hipparchus is considered to have originated trigonometric tables, making systematic use of the 360° circle which was adopted by Ptolemy who further subdivided a degree into 60 parts, or minutes, and a minute into 60 seconds.[11] He may also have been the originator of the trapezoidal (Donis) projection used by Conrad of Dyffenbach in 1426 and Nicholas Germanus in his printed editions of Ptolemy from 1466 on.[12]

In the second century A.D., the Greek tradition in mathematics and cartography came to a remarkable culmination in the synthesis of Greek and Roman influences achieved by Marinus of Tyre (fl. A.D. 100) and Claudius Ptolemy (ca. A.D. 90-168). The lasting influence of Ptolemy on astronomy, geography, and cartography is indisputable; his two major works: the *Almagest*, a treatise on mathematics and astronomy comprising thirteen books, and the

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[9] Editors, from materials supplied by Germaine Aujac, "Greek Cartography in the Early Roman World," *The History of Cartography* I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, eds. J.B. Harley and David Woodward (1987), p. 161-164.
[10] Aujac, "Greek Cartography in the Early Roman World" (1987), p. 167. Chlamys is described as the shape of a Macedonian cloak (see figure 9.5), p. 156.
[11] *Partes minutae primae*, and *partes minutae secundae*: these measurements were needed by Ptolemy for constructing his astronomical tables (Boyer, p. 188).
[12] Snyder (1993), p. 8-9.

Geographia, comprising eight books, were considered authoritative in European and Islamic intellectual circles for some fourteen centuries. While his books were little known or used in medieval Europe, they remained of substantial influence in the Arab world, in astronomy and cartography, and upon the re-discovery of the *Geographia* in ca. 1400, powerfully re-structured European cartography as we have already seen.

Both Ptolemy and Marinus of Tyre are celebrated for their foundational work in the mathematics of map projections,[13] a subject which continued to fascinate mathematicians for centuries. Marinus' work on the problem of the representation of the globe, or a portion of it, on a plane resulted in the devising of a rectangular arrangement of parallels and meridians.[14] Inaccuracies in geographical data, in measurement, and in copying, led Ptolemy to reject Marinus' work and, as we have seen, he produced his own systematic treatment of the problem. In the *Geographia*, Ptolemy describes in detail four map projections, one of which is the rectangular projection of Marinus with a modification to make scale true along the central parallel of the region shown on the map. (He recommended using this projection for smaller regions). Ptolemy's first two projections introduced curved parallels and meridians though they were not true conic projections but only 'coniclike'. These projections were widely used for maps included in editions of the *Geographia* in the fifteenth and sixteenth centuries. Ptolemy's third projection, described in Book 7 of the *Geographia*, details construction of a perspective view of the globe with true scale along the central meridian.

Greek traditions in mathematics and cartography continued to be pursued in Europe approximately until the sixth century A.D. After this, although they endured in Islamic

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[13] O.A.W. Dilke, "The Culmination of Greek Cartography in Ptolemy" (1987), p. 177-180.

scholarship, they died out in Christian Europe, and the Middle Ages were aware of only the most elementary mathematical ideas. The Christian Church was of the view, expressed by Tertullian, that since the word of God had been revealed in Christ, science was "superfluous."[15] No dissent from this outlook occurred for many centuries. In the twelfth century, however, a trickle of translations started and universities began to be established, e.g., at Paris, Oxford, and Bologna. In the thirteenth and following centuries this trickle developed into a strong tradition of translation and was augmented by commentaries on the translated works. Of links between mathematics and cartography in the medieval era, there are almost none to speak of, the only persons of particular note being Roger Bacon and Nicole Oresme.

Roger Bacon (ca. 1214-94) is regarded as not much of a mathematician[16] though Bacon strongly championed the study of mathematics, geometry and astronomy, emphasizing its role in spiritual understanding for the Christian theologian. As touched upon in chapter 3, his insistence on the necessity for a systematic way of positioning places on earth set him apart from his contemporaries. He described such a system in his *Opus Maius* in a section on geography and alluded to a map where places were located by their distance from the equator (which he termed latitude), and their distance east or west (longitude). He explains that he uses a 'device' to create the map which is the drawing of a pair of intersecting straight lines analogous to a parallel of latitude and a meridian of longitude.[17] A method of positioning, this 'device' did not imply a graticule as Bacon makes no mention of a network

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[14] This projection is usually called equirectangular or plate carrée. See Snyder (1993), p. 5.
[15] Boyer (1989), p. 281.
[16] Boyer (1989), p. 291. Also Woodward and Howe cite Thorndike's low opinion of Bacon's mathematical powers in "Roger Bacon on Geography and Cartography" (1997), p. 215.
[17] Woodward and Howe, "Roger Bacon on Geography and Cartography" (1997), p. 208-209.

of parallels and meridians added to the map.[18] The co-ordinate system he described generated little interest in his own time and did not come into use until the fifteenth century with the acceptance of the Ptolemaic cartographic model.

Probably born in Normandy, Nicole Oresme (ca. 1320-1382) was a noted mathematician amongst the Schoolmen, responsible for the suggestion of special notation for fractional exponents and the idea of irrational powers.[19] The latter theory developed from his use of the clock metaphor for the universe and the suggestion of the irrationality of celestial motion.[20] He does not have a direct link with cartography, but he was the first to use two-dimensional graphs to represent hypothetical distributions and velocites geometrically.[21] For the modern terms ordinate and abscissa, he used the terms latitude and longitude,[22] and the use of two-dimensional figures to represent mathematical functions became known as the 'latitude of forms.' It remained popular up to the time of Galileo.[23]

The Renaissance milieu

The onset of the European Renaissance is not precisely dated, but it is clear that most scholars regard the fifteenth century as the transitional period between the Middle Ages and what is known as the Renaissance. How the Renaissance was different from the medieval

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[18] David Woodward, "Roger Bacon's Terrestrial Coordinate System," *Annals of the Association of American Geographers* 80, 1 (1990), p. 116.
[19] Boyer (1989), p. 294.
[20] Marshall Clagett, "Oresme, Nicole," *The Dictionary of Scientific Biography* X, p. 224.
[21] In the *Tractatus de configurationibus qualitatum et motuum* of the 1350's. Clagett, *The Dictionary of Scientific Biography* X, p. 226.
[22] Greek mathematicians had, of course, used co-ordinate systems, most notably the geometer, Appollonius, author of *Conics*, a work on analytic geometry very similar in method to the modern approach (Boyer, p. 175).
[23] See Boyer (1989), p. 297.

period has been the subject of much debate and selecting particular political, social, or cultural characteristics which exhibit marked differences is problematic. It is generally agreed, however, that there was, as Bondanella and Musa phrase it, "a perceptible shift in vision that caused even thinkers of the time to conceive of a rebirth."[24] As we have seen, there resulted from this shift a fundamental change in the conception of the world, a turning away from the contemplative, mystical, and non-subjective poetic of the medieval period to an active, subjective, secular consciousness which led to the scientific and philosophical revolution of the seventeenth century.

Italy was the geographical centre of the reorientation in thought from that which was typical of the Middle Ages to a consciousness typical of modernity, but Italy was not politically unified. It consisted of independently powerful city states which occupied the political vacuum left by the decline of the feudal system and the Holy Roman Empire in the late Middle Ages. The economy of the region recovered from the Empire's decline despite the scourge of the Black Death and the agricultural crisis of the fourteenth century. Italy's seaports of Genoa and Venice controlled trade in the Mediterranean "bringing into Europe new goods, peoples, and ideas, from the precious spices of the Orient to the scientific knowledge that flourished in the Middle East."[25] A newly emerging mercantilism combined with a rising humanist movement to create the possibility, in the minds of individuals, of power to shape personal circumstances. There was a high mortality rate as human life was dangerously susceptible to epidemic disease, and few persons lived into old age.[26] There was

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[24] Julia Conway Bondanella and Mark Musa (eds.), *The Italian Renaissance Reader* (1987), p. ix.
[25] Bondanella and Musa (1987), p. xi.
[26] Bondanella and Musa (1987), p. xiii.

thus potential for more rapid cultural change without an older generation to adhere to cherished tradition.

The project of the humanists was to revive the art, literature, and learning of the classical world and, for this purpose, they promoted the study of languages, history, ancient literature, rhetoric, grammar, and moral philosophy.[27] These subjects were opposed to the medieval university curriculum which stressed theology and natural philosophy. The humanists also believed in the importance of a critical approach to the study of literature, and in the value of the individual. In its early phase, the enthusiasm for the new ideas engendered a heady optimism, but later the destruction of the secure system of belief of the medieval period brought about anxiety and doubt. Towards the end of the sixteenth century, it clearly became less acceptable and more dangerous to express views which did not conform to established tenets. So, for example, it would appear that Nicholas of Cusa benefited from the relaxed attitude prevalent in the first half of the fifteenth century while Giordano Bruno and Galileo Galilei suffered from the backlash against the freedom and flowering of intellectual enquiry which occurred in the late sixteenth.

Rose has provided an extensive study of the collaborative relationships between the Italian humanists and mathematicians of the Renaissance.[28] The revival of classical literature undertaken by the humanists was accompanied by a renewed interest on the part of mathematicians and philosophers in the traditions of Greek mathematics. These were the traditions of Archimedes, Euclid, Proclus, Pappas, and Apollonius in geometry, and of

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[27] Bondanella and Musa, p.xiv-xv. Rose (1976), p. 3, describes a humanist as a scholar, teacher, or patron of the humanities.
[28] Paul Lawrence Rose, *The Italian Renaissance of Mathematics* (1976).

Ptolemy in astronomy, among others.[29] The humanists were very involved in the collection and translation of Greek manuscripts in mathematics, and were connected to Renaissance mathematical scholars through personal networks. Mathematical scholars were those who pursued the subjects of the *quadrivium*: arithmetic, geometry, astronomy, and music – and were interested also in optics and mechanics.[30] They would also have included scholars and professionals of various kinds who, directly or indirectly, drew on or were curious about mathematical ideas – such persons as Paolo Toscanelli, physician, mathematician, geographer; Nicholas of Cusa, ecclesiastic, philosopher, cartographer; and many other individuals – artists like Leonardo da Vinci, architects such as Brunelleschi, astronomers such as Copernicus and Regiomontanus, and the Neoplatonist philosophers.

During the fifteenth century, a popular revival of Platonism occurred, especially in Florence, through the translation of Plato's dialogues. Marsilio Ficino's Platonic Academy, established near Florence in 1462, was dedicated to the interpretation and teaching of Platonic philosophy.[31] Ficino himself was a foremost translator of Plato and Plotinus, and wrote commentaries on their works.[32] Through his commentaries were propagated his ideas on the soul's reality which later influenced Giordano Bruno.[33] Though Ficino was more interested in theology, and in reconciling Platonic philosophy with Christianity, than in mathematics and physics, the flowering of Neoplatonist thought led inevitably to interest in Plato's belief that the highest reality could be realized only through mathematical philosophy. Ficino had friendly relations with mathematicians and philosophers of the time, a number of whom

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[29] Rose (1976), p. 2.
[30] Rose (1976), p. 3.
[31] Paul Oskar Kristeller, *The Philosophy of Marsilio Ficino* (1964), p. 16.
[32] Kristeller (1964), p. 3.
[33] Kristeller (1964), p. 19.

visited his Academy. These included Pier Leone, Paolo Toscanelli, Nicholas of Cusa, Piero della Francesca, and Leon Battista Alberti.[34] That the universe was structured mathematically, and that this mathematical structure was an expression of the rationality of God, was an attractive and popular philosophy and garnered a wide audience amongst the intelligentsia of fifteenth century Italy. Paul Lawrence Rose remarks: "Even on the simplest technical level of geometry and arithmetic, mathematics was remarkable for its certainty and the immutable truth of its propositions. But on the philosophical level of mathematics, it gave intimations of the eternal truths of God, man and the universe, and it could suggest new pathways for the contemplation of reality."[35]

Rose argues that there was thus an intense interest in mathematics which arose during the fifteenth century partly as a result of the humanist and Neoplatonist movements, and partly the consequence of the waning of Aristotelian thought. It was accorded importance in the curricula of the universities where the mathematical sciences – the *quadrivium* – had been an important component of medieval education. Despite a new emphasis on the humanist subjects of grammar, rhetoric, and logic (the *trivium*), mathematics remained significant[36] and, of special note, mathematical astronomy was thought by influential humanists such as Salutati, to lead to knowledge of the divine Creator.[37] Rose notes that in a manuscript sent by the noted humanist and jurist, Jacopo Sadoleto, to the mathematician, Pietro Bembo, in 1532, Sadoleto describes arithmetic and geometry as "affording wonderful pleasure to the mind in the contemplation of eternal and unchanging truth."[38] Geometry and astronomy

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[34] Rose (1976), p. 9; Crosby (1997), p. 179.
[35] Rose (1976), p. 6.
[36] Rose (1976), p. 12.
[37] Rose (1976), p. 12.
[38] Quoted by Rose (1976), p. 12. Rose cites Sadoleto, *De Pueris recte Instituendis*, Venice, 1533.

were constantly linked together, being two of the four disciplines making up the *quadrivium* (the other two were arithmetic and music).

Since Florence was at the forefront of the revival of interest in Greek scholarship, a circle of Florentine scholars began to assemble collections of classical manuscripts and these libraries included a good representation of mathematical treatises. At the end of the fourteenth century and during the fifteenth, Italy was a preferred destination for Byzantine immigrants abandoning the endangered empire.[39] (Constantinople was to fall to the Ottoman Turks in 1453). Among them were notable Greek scholars such as Manuel Chrysoloras for whom a chair of Greek was established in 1396 in Florence, and whose students were among the most diligent and brilliant manuscript collectors.[40] For example, the aforementioned Coluccio Salutati, interested in mathematical manuscripts, had copies of Euclid, Ptolemy's *Almagest*, and Pecham's *Perspectiva* and was familiar with the *Geographia*. Salutati had urged Jacobus Angelus, the translator of the *Geographia* to study Greek.[41] Other pupils of Chrysoloras, Corbinelli and Strozzi, had Latin versions of Euclid, the *Almagest*, and Fibonacci's *Liber abaci* and *Practica geometriae*, among their manuscript possessions.[42] Rose says that Strozzi and Vespasiano considered that "the great treasure of the collection was the Greek codex of Ptolemy's *Geographia* which Chrysoloras brought to Florence" and that the *Geographia* "aroused the translating ardour of the humanists."[43] This has already been noted in chapter 4; it paved the way for the *Geographia*'s wide circulation, and set an

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[39] Jonathan Harris, "Byzantium: Byzantines in Renaissance Italy," *ORB: The Online Reference Book for Medieval Studies* (http://orb.rhodes.edu/encyclop/late/laterbyz/harris-ren.html) (1996).
[40] Rose (1976), p. 26.
[41] Rose (1976), p. 27.
[42] Rose (1976), p. 27.
[43] Rose (1976), p. 27.

example which inspired the translation of other Greek texts in mathematics, physics, and other sciences.

Toscanelli, Renaissance man

Mathematics and cartography are intertwined through the conjunction of interests in individuals as well as through the collections of humanist translators. Paolo dal Pozzo Toscanelli (1397-1482) is described by Rose as Florence's "best-known mathematician."[44] Toscanelli was a central figure with far-reaching interests and a wide circle of acquaintances. According to Vespasiano, he was "learned in astronomy above all others ... skilled in geometry ... A friend to all the lettered men of his time."[45] In 1425, Toscanelli had been introduced to the architect Filippo Brunelleschi and this was to result in a fruitful collaboration, as previously noted. Toscanelli instructed Brunelleschi in geometry and Brunelleschi's 're-discovery' of linear perspective, Edgerton concludes, led to a "perceptual revolution."[46] Leon Battista Alberti, also a friend of Toscanelli, was to document the geometric principles of linear perspective for painters in *Della pittura*, 1435.

Toscanelli received his university training at Padua where the study of medicine required mathematics. He also studied geography and optics as a foundation for astrology which was an aspect of medicine.[47] As Toscanelli finished his studies in 1424, he would have been acquainted with Ptolemy's *Geographia*, popular at the time. In fact, we know that he was very much interested in Ptolemaic geography and Samuel Edgerton has sought to show how

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[44] Rose (1976), p. 29.
[45] Quoted by Rose, p. 29. Rose cites Vespasiano, *Vite*, p. 355-356.
[46] Edgerton (1975), p. 4.
[47] Edgerton (1975), p. 61.

Brunelleschi's experiments in perspective were probably inspired by Ptolemy's projective theory, introduced to him by Toscanelli. Edgerton suggests that Ptolemy's map projection was the 'missing link' between the architectural expertise of Brunelleschi and the geometry of linear perspective. In any case, Toscanelli, the mathematician, was very much interested in cartography and is known to have possessed maps and a co-ordinate framework for the construction of a geographic map.[48] He tried his hand at drawing maps as well; in a letter dated June 25, 1474 he tells the canon of Lisbon:

> [Your] *Most Serene King requests of me some statement, or preferably a graphic sketch, whereby that route might become understandable and comprehensible ... Although I know this can be shown in spherical form like that of the earth, I have nevertheless decided ... to represent* [that route] *in the manner that charts of navigation do. Accordingly I am sending His Majesty a chart done with my own hands ... the straight lines ... drawn vertically in the chart, indicate distance from east to west, but those drawn horizontally indicate the spaces from south to north.*[49]

It can be seen from this fragment of prose that Toscanelli had a thorough grasp of the difference between the map types: the navigational or portolan chart, and the Ptolemaic projected map. From his description of his chart, it seems that he was not satisfied to show navigational rhumb lines as the medieval charts did, but added an abstract grid like the parallels and meridians of Ptolemy's map. Toscanelli's original chart has not survived but a copy, together with the letter, is said to have been obtained by Christopher Columbus several years later[50] and Toscanelli apparently corresponded with Columbus directly, encouraging him to carry through with his oceanic vogages. Toscanelli was also well-

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[48] Dana Bennett Durand, *The Vienna-Klosterneuburg Map Corpus of the Fifteenth Century: A Study in the Transition from Medieval to Modern Science* (1952), p. 258.
[49] Quoted in Edgerton (1975), p. 121. Edgerton does not cite a source for the letter. However, Woodward (1989) cites Columbus' copy of Cardinal Piccolomini's *Historia Rerum Ubique Gestarum* in the Biblioteca Columbina, Seville for a copy of Toscanelli's letter.

acquainted with the astronomer, Regiomontanus, the Byzantine humanist scholar, Bessarion, and the mathematician, cartographer, and ecclesiastic, Nicholas of Cusa.

Nicholas of Cusa

Nicholas of Cusa, or Cusanus in Latin, (1401-1464) was the son of a fisherman from Kues in the Moselle region of Germany. His early education was with the Brothers of the Common Life community at Deventer where he would have had an early exposure to the mystical *devotio moderna* movement.[51] He entered the University of Heidelberg, possibly taking introductory philosophy, and then studied canon law and mathematics at the University of Padua.[52] In Padua, he met Paolo Toscanelli with whom he had a life-long friendship and who soon introduced him to the geographical, cosmological, and physical problems pre-occupying intellectuals at that time.[53] Cassirer states further that Cusanus "received from Toscanelli an impulse that he later passed on to the German mathematicians and astronomers *Georg Peurbach* and *Regiomontanus*, and which influenced even *Copernicus*."[54] In 1426, Cusanus was in Cologne, in the service of the archbishop of Trier, and began his research into Germanic law, discovering lost Latin manuscripts of Pliny the Elder and others. He became interested in the philosophy of medieval Scholasticism after being introduced to Proclus' commentary on Plato's *Parmenides* and the writings of Ramón Lull.[55] Lull's work may have inspired Nicholas with the desire to use mathematics to find a

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[50] A number of writers mention this. Among them are Edgerton (1975), p. 122 and note 19, p. 182; and Woodward (1989), p. 12.
[51] J.E. Hoffman, "Cusa, Nicholas," *The Dictionary of Scientific Biography* III (1971), p. 512; Cassirer (1963), p. 33.
[52] Hoffman (1971), p. 512; Cassirer (1963), p. 35.
[53] Cassirer (1963), p. 35.
[54] Cassirer (1963), p. 35.
[55] Hoffman (1971), p. 513.

deeper mystical meaning of philosophical significance and, as a Neoplatonist too, he thought mathematics led to the highest reality and subscribed to the hierarchy God-angel-soul-body.[56]

Nicholas was ordained a priest, and acted as emissary for the archbishop of Trier to the Council of Basel in 1431. Here, he met the humanist, Piccolomini, who eventually became Pope Pius II.[57] In 1437, Nicholas took part in an embassy to Constantinople to unite the Eastern and Western churches and, through this and later diplomatic missions, became a skilled and respected negotiator at ecclesiastical councils.[58] Durand suggests that during the abortive embassy to Constantinople, "maps were among the incidental topics of discussion," and that it is "reasonable to suppose that Nicholas of Cusa, who was present at the council, should have been inspired by these geographical discussions."[59] His major philosophical work, *De docta ignorantia*, which appeared in 1440, was written while travelling on these diplomatic missions. In it, he presented his concept of the coincidence of opposites by which he hoped to resolve all problems of contradictory logic. He made extensive use of geometrical examples such as the opposites of straight line and circle being resolved in infinity since a circle with radius of infinite length will have a straight line for its circumference.[60] In a circle infinitely large, or one infinitely small, the centre also loses its

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[56] Ramón Lull (ca. 1233-1315) wrote about the science of navigation and was the first to describe the navigational chart. See Woodward, "Medieval *Mappaemundi*" (1987), p. 305. He was not a mathematician but used mathematics to probe for mystical meaning (Claudia Kren, *Medieval Science and Technology: A Selected Annotated Bibliography* (1985), # 871, p. 178).
[57] Hoffman (1971), p. 513.
[58] Hoffman (1971), p. 513.
[59] Durand (1952), p. 259.
[60] Hoffman (1971), p. 514.

determining position and coincides with the circumference. It is therefore everywhere and nowhere at once.[61]

Nicholas applied his concept of *coincidentia oppositorum* to cosmology, and determined that there could be no centre to the universe because it would coincide with the circumference and, therefore, any centre point would, of necessity, include the whole of the universe. This 'centrum' which contains the universe is a metaphysical entity: the Absolute Being or God, in Whom all is contained. He pursues his ideas further, reversing the Aristotelian argument defending the limitation of the world:[62]

> *Since, therefore, it is impossible to enclose the world between a corporeal centrum and a circumference, it is [impossible for] our reason to have a full understanding of the world, as it implies the comprehension of God who is the center and the circumference of it.*

And, therefore,

> *[T]hough the world is not infinite, yet it cannot be conceived as finite, since it has no limits between which it is confined.*[63]

Nicholas of Cusa goes on to derive the motion of the earth, thereby, as Jammer notes, "anticipating certain ideas of the Copernican theory."[64] His conjoining of the physical limits of the world with God is possibly the origin of the Judeo-Christian idea of connection between space and God which developed in Italy during the Renaissance. Jammer argues that the spread of cabalistic ideas contributed to new concepts of space and links Newton's

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[61] Koyré (1957), p. 9.
[62] Koyré (1957), p. 11.
[63] As translated by Koyré (1957), p. 11. The passage is from *De docta ignorantia, Book II, Ch. 11*. See *Of Learned Ignorance*, ed. Dr. W. Stark (1954), p. 107.
[64] Jammer (1969), p. 82.

theory of space to Pierre Gassendi, a personal friend of Campanella. In *De sensu rerum et magia*, 1620, Campanella declares that space is "in God," but "God is not limited by space,"[65] language which resonates strongly with that of Nicholas of Cusa in *De docta ignorantia*.

Although they were cloaked in theological language, Nicholas of Cusa "[placed] mathematics and experimental science at the service of philosophy,"[66] rejected the medieval cosmological conception, and adduced a theory of relativity of place and motion. His speculations could hardly have been more at variance with the received ideas of the time and they left him open to attack. He was pre-occupied for some years after publication of *De docta ignorantia* with defending himself against detractors and political enemies. Nevertheless, he continued to be interested in mathematical and cosmological problems. He became "enthused with admiration" for Archimedes, and wrote several works on the quadrature of the circle and other problems in applied geometry dedicating *De geometricis transmutationibus* and *De arithmeticis complementis* of 1445 to Toscanelli.[67] His attempts on the ancient problem of squaring the circle met with some ridicule, particularly, later, from Regiomontanus but his efforts stimulated other attempts of a similar nature.[68] Kepler, who had as equally mystical a turn of mind as Nicholas, was influenced by him, as was the unfortunate hermeticist, Giordano Bruno.

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[65] Jammer (1969), p. 35. Jammer cites *De sensu rerum et magia*, libri quatuor (Frankfort, 1620) I, c. 26.
[66] Hoffman (1971), p. 513.
[67] Rose (1976), p. 30. See also Hoffman (1971), p. 515.
[68] Boyer (1989), p. 306; Rose (1976), p. 30 and 103.

Nicholas of Cusa's diplomatic services to the Curia led to his appointment as a cardinal and, in 1450, as legate for Germany, he undertook a journey through Germany and the Low Countries. It was during these travels that he probably compiled at least one of two maps attributed to him.[69] The two maps are of Germany and are known from versions issued long after the cardinal had died. They may possibly be derived from one compilation.[70] One is in manuscript form and accompanies the maps of Ptolemy in a 1490 atlas by Henricus Martellus Germanus, while the other was engraved in copper at Eichstätt in Bavaria in 1491 for an edition of Ptolemy which was unfinished.[71] These two maps are unusual, and not just because of their authorship. Though they are similar to itinerary maps, they are geometrically based and drawn to scale – the Martellus manuscript uses the modified Marinus rectangular projection, and the Eichstätt plate the scientific trapezoidal (Donis) projection[72] – and are compiled from detailed measurements. This is not surprising given that Nicholas believed that knowledge was based on measurement, *mens* being, he thought, related etymologically to *mensura*.[73] That he put his belief into practice, compiling one or two maps during his travels as papal legate and using the Ptolemaic cartographic model applied to larger-scale mapping marks him as one of the most vigorous and original minds of his time. He had undoubtedly come into contact with a school of science responsible for

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[69] Durand (1952), p. 264.
[70] P.D.A. Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 497.
[71] See Durand (1952), p. 253 and 265. Durand also advises, p. 256, that the date of 1491 for the Eichstätt plate may have been a transcription error, an actual date of 1451 being equally plausible. The latter would have been the year following the compilation of the map.
[72] See Durand (1952), p. 252-259. The trapezoidal projection may have been used for star maps by Hipparchus in ca. 150 B.C., but was definitely used by Conrad of Dyffenbach for his 1426 sketch maps. Later it was dubbed the Donis because erroneously attributed to Donnus Nicolaus Germanus who claimed to have created it in 1482. See Snyder (1993), p. 8-9.
[73] Boyer (1989), p. 305. Cassirer (1963) also notes, p. 176, that Nicholas of Cusa considers "all knowledge to be merely a particular case of the universal function of measuring." For Nicholas, measurement required the selection of fixed points as bench marks and the fixed points are not predetermined by objective reality, but arbitrarily chosen by the observer. One point may be as good as any other; all possible points have identical weight.

what is called the Vienna-Klosterneuburg Corpus which flourished from early in the 1420's until 1442 and left a legacy of manuscripts and treatises in astronomy, mathematics, and cartography.[74] The maps and co-ordinate tables of the Vienna-Klosterneuburg Corpus clearly demonstrate the influence of Ptolemaic cartography, the earliest surviving example being two 1426 sketches with latitude and longitude plotted which are attributed to Conrad of Dyffenbach.[75]

Nicholas of Cusa's map or maps, compiled in 1450-51, demonstrate the use of measurement of angles as well as distances, which supposes the use of a magnetic compass on land, a usage which widened in the fifteenth century.[76] The maps were probably composites as they also embodied material from the Vienna-Klosterneuburg school and the Trier-Koblenz maps[77] and we can also imagine him displaying and discussing them with the visitors he hosted in his house in Rome during the Papal Jubilee celebrations of 1450. Or perhaps it was only the idea of compiling a map of central Europe, a *Tabula moderna* to supplement Ptolemy, that was born at this gathering.[78] "But whatever the methods and authorship of these maps," states P.D.A. Harvey, "they are of great importance: their accuracy argues a high degree of technical accomplishment, and they mark the beginning of a real tradition of topographical maps drawn to scale in Europe."[79]

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[74] Durand (1952), p. 260-266, establishes the influence of the Vienna Corpus on the Cusan compilation.
[75] Woodward, "Medieval *Mappaemundi*" (1987), p. 316-317.
[76] P.D.A. Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 497.
[77] These maps were owned by Nicholas of Cusa. See Durand (1952), p. 264.
[78] Durand (1952) suggests this view. See p. 263.
[79] P.D.A. Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 497.

During the same year – 1450 – Nicholas wrote a treatise on squaring the circle (*De circuli quadratura*) and the *Idiota* dialogues in which an 'unlettered' man instructs a scholar in Cusan philosophy and solutions to practical problems. After 1452, when he had read Jacob of Cremona's translation of Archimedes, he wrote several more treatises containing refinements of ideas expressed earlier with few new insights except some variations contained in *Perfectio mathematica* (1458) which included a note on the method of *visio intellectualis* which J.E. Hoffman says "anticipated an infinitesimal concept of great significance" even though, as he further remarks, it was "inadequately expressed."[80]

Another scholar appointed cardinal by Pope Nicholas V was the Greek monk, Bessarion, a figure of importance in Rome after 1440, whose house was frequented by humanist intellectuals. Bessarion had a very large collection of Greek manuscripts which included maps; his collection was "a major landmark in the transmission of classical Greek culture to Renaissance Italy."[81] As a Platonist, Bessarion was interested in mathematics and had many mathematical works in his library. He became a close friend of Nicholas of Cusa and, while visiting Nuremberg and Vienna, met the astronomers Peurbach and Regiomontanus. He brought the latter back to Rome after the death of his teacher.[82] In 1461-62, Bessarion, Nicholas of Cusa, Toscanelli, Regiomontanus, and Alberti were all in Rome at the same time. It is difficult to know exactly what fruit this conjunction of minds may have borne, but we can imagine the cross-fertilization of ideas in philosophy, geometry, astronomy, and geography that may have taken place. Rose makes the following remarks:

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[80] Hoffman (1971), p. 515.
[81] Rose (1976), p. 44.
[82] Rose (1976), p. 44. Cassirer (1963, p. 15) also mentions the association between Cusanus and Bessarion.

> *Although there is no definite proof that Alberti and Cusanus knew each other, their mutual friendship with Toscanelli, their common ideas, and their proximity on several occasions suggest that they were personally acquainted.*

and,

> *There is little direct evidence of the acquaintance of Toscanelli and Bessarion, but Regiomontanus was a mutual friend when they were all at Rome in 1461. Moreover, Bessarion owned a map which was probably the joint work of Regiomontanus and Toscanelli.* [83]

Nicholas of Cusa wrote a further treatise at this time on the characteristics of the earth and motion, *De figura mundi* (1462), which is unfortunately lost. We can only speculate as to how his mathematical philosophical ideas, his interest in Archimedes and in cartography, and his contacts with the geographer-physician, Toscanelli, and the astronomer, Regiomontanus, might have blended in a theory of the earth. That it would have been theologically based is certain. Although he was among the earliest scholars to reject Aristotelian cosmology and the rationalisms of the Schoolmen, and the first to declare that the universe was unbounded, he was a theologian and concerned with understanding the divine. As a personality, he was temperamental and often expressed himself in flamboyant language.[84] Some of his more foolish ideas left him open to ridicule. Thus his work was flawed by inadequate expression and odd flights of fancy which made both his contemporaries and later scholars fail to appreciate the importance of his thought. In the investigation of the links between geometry, cosmology, and cartography in the era immediately after the re-discovery of Ptolemy's *Geographia*, he can be seen as an individual of stature, bringing together as he does all these disciplines, and producing ideas which

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[83] Rose (1976), p. 61, note 51.
[84] Hoffman (1971), p. 516.

provided a conceptual basis for the discoveries and hypotheses of Copernicus, Kepler, and, eventually, Newton.

Mathematicians and cartography in the 16^{th} and 17^{th} centuries

From the fifteenth century re-discovery and popularizing of Ptolemy's *Geographia*, cartography has been in the domain of scientists and, more particularly, of mathematicians, as it was in ancient Greece. The analytical challenge of mapping the spherical earth on the plane so as to account for the deformation mathematically has exercised a fascination, and may have helped to so imprint the cartographic image of the world that its taken-for-granted ideation of space has gone unregarded.

The astronomer, Regiomontanus (1436-1476),[85] has been mentioned already. He was the first mathematician to restore astronomy to the status of a serious intellectual discipline in Europe. Melanchthon wrote, in 1545, that astronomy "had lain for centuries without honour, but has recently flowered again in Germany, restored by those two great men Peurbach and Regiomontanus."[86] After moving to Rome in 1461, Regiomontanus had the freedom to pursue mathematics and astronomy. He wrote an important critique of the *Almagest* and a commentary on the *Geographia* and intended to provide new translations of both.[87] He attributed the flaws in the Jacobus Angelus version of the latter to the author's inadequate knowledge of both Greek and mathematics and his new translation was to draw

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[85] Johann Müller, born in Königsberg, for which the name Regiomontanus is a Latin form (Boyer, p. 278).
[86] Quoted by Rose (1976), p. 90. Rose cites: Philip Melancthon, Preface to Joannes de Sacrobosco, *Libellus de Sphaera*, Wittenberg, 1545, sig. a4.
[87] Rose (1976), p. 104-107.

in part on Paolo Toscanelli's erudition.[88] Regiomontanus died suddenly in 1476, possibly of the plague, and was not able to carry out his full plan of work but his new translation of the *Geographia* is probably one later owned by Pirkheimer who published his own new edition in 1525 (Strasbourg) along with Regiomontanus' critique of the Angelus version.[89]

Although he did not make any significant contribution to mathematics, this history would not be complete without mention of that extraordinary artistic and scientific genius, Leonardo da Vinci (1452-1519). He is well-known for his enthusiastic application of mathematics to engineering concepts and the theory of perspective. Edgerton makes the assertion: "Leonardo's combination of the human figure with a perfect geometric square and circle is closely connected with cartography, especially the employment of the Ptolemaic grid."[90] It was the use of geometry as ordering principle for the universe which attracted Leonardo. He owned a printed copy of Ptolemy's *Geographia* which he greatly admired, making many references to it in his notebooks.[91] Leonardo was a visual thinker, loved to draw his ideas, and probably preceded Galileo in his ability to merge geometry and physics. He was interested in cartography and created a spherical octant map of the world, one of the earliest uses of this novel projection.[92]

Like Regiomontanus before him, Nicolas Copernicus (b. 1473) benefited from the study of mathematics and Greek, and exposure to Italian humanist thought influenced his decision to

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[88] Rose (1976), p. 105.
[89] Rose (1976), p. 106-107. Wilibald Pirkheimer (1470-1530) was a humanist-mathematician, one of the Nuremberg group (Rose, p. 107).
[90] Edgerton (1987), p. 11.
[91] Edgerton (1987), p. 12.
[92] Snyder (1993), p. 40.

study in Italy.[93] In 1503, after seven years in Italy, he returned to Poland to spend the rest of his life there. Copernicus' dissatisfaction with the Ptolemaic planetary system inspired him to theorize a different model, the heliocentric cosmology considered such a landmark achievement and the beginning of modern science. Waldemar Voisé has pointed to the fact that Copernicus had a 'visual imagination' and suggested that the new geographical maps in the Ptolemaic mode had a seminal effect on his thinking.

> *[T]he Ptolemaic model of the Universe was based on Aristotelian physics which assumed that the Earth, being the heaviest element, is in the center of the concentric system of spheres ... the Earth, it was alleged, was as an island in an ocean, bound on all sides by water. This system of spheres, as Thomas Goldstein points out, was presented by the traditional cosmographies which continued to appear as late as the 16th century. Notable among these was the* Cosmography *of Apianus, published for the first time in 1524. But at the same time, other maps that began to appear with the beginning of that century represented a different picture of the world. They took account of other continents ... With this picture of the universe in mind, Copernicus was able to imagine the Earth as a homogeneous spherical heavenly body composed of two fundamental elements (water and land). ... The heliocentric model of the Universe, which was also the product of his reflection upon the new map of the world, not only repudiated the Ptolemaic geocentrism but also subverted Aristotelian physics.*[94]

Copernicus' astronomy was not based on observation and experiment and was not 'modern' in the Baconian sense even though Kepler calls him "the first to dare to yearn for a restoration of astronomy and [who] sought to achieve it by the most valuable observations and inventions."[95] His habit of thought was that of the Ancients. He was pleased that his ideas could be supported by reference to Greek learning (he adduces Aristarchus of Samos' theory of the revolution of the earth around the sun to give his concept credibility) and that they accorded with Pythagorean notions of a sun-centred universe and a mobile earth, and

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[93] Rose (1976), p. 119.
[94] Waldemar Voisé, *The Scientific World of Copernicus*, (ed.) Barbara Bienkowska (1973), p. 89.

ideals of harmony and proportion.[96] Of Copernican cosmology, the model which was to form the basis for the imaginative work of Kepler and Galileo, Koestler says: "The Copernican universe is not only *expanded* towards the infinite, but at the same time *decentralized*, perplexing, anarchic ... There are no longer any absolute directions in space."[97]

Koestler also remarks that, like the famous astronomer Ptolemy, both Copernicus and Kepler were map-makers. Although Koestler asserts that Ptolemy copied the principles of map projection from Hipparchus, he agrees that the re-discovery of the *Geography* initiated 'scientific geography.'

Galileo became a mathematician in the 1580's, joining the flourishing renaissance of mathematics and physics.[98] He was passionate about the application of geometry to physical problems, and, apparently, he gave lectures at the Academia Fiorentina which, Rose states, "consisted of a geometrical analysis of the topography of Dante's *Inferno*, all based on Euclidean and Archimedean principles."[99] One of Galileo's teachers, the Sicilian mathematician Giuseppe Moleto (1531-1588), had written a discourse for the 1561 edition of Ptolemy by Girolamo Ruscelli. Cosgrove states that Moleto's theme was: "the universal application of mathematical sciences to practical interventions in the world."[100] Here, we can see a direct link between Ptolemy's cartography and Galileo, himself, and his passion for geometry.

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[95] Quoted in Rose (1976), p. 133. See note 136, p. 141. Rose cites Kepler, *Astronomia nova* (Werke III, 8).
[96] For Copernicus, see Rose (1976), p. 118-133, and Hooykaas (1987), p. 463-467.
[97] Koestler (1959), p. 217.
[98] Rose (1976), p. 280.
[99] Rose (1976), p. 280.

To Galileo we can attribute the credit, or the infamy, of having first formulated the mechanistic view of the universe, divorcing the material world from the spiritual.[101] Galileo, like Copernicus, possessed a visual imagination and used graphical images extensively. I have cited, in a previous chapter, Latour's extension of Drake's comparison between Galileo's diagrams and commentaries and those of Stevin and Jordan, two of his predecessors. Of Galileo's ability to connect geometry with mechanics, Latour comments, "Drake explains the efficiency of Galileo's connection in terms of his creation of a geometrical medium in which geometry and physics merge."[102] That Galileo extended the merging of geometry and physics into the realm of cosmography can be clearly seen in the star maps which constituted the most important element of his influential *Sidereus Nuncius* of 1610.[103]

Three other mathematicians of the period who combined interests in mathematics, astronomy, and map-making are Johannes Werner (1468-1522), Albrecht Dürer (1471-1528), and Peter Apian (1495-1552). Werner and Dürer knew each other in Nuremberg and Apian was a professor of mathematics at Ingolstadt near Nuremberg. Apian published maps based on Ptolemy's method and his modifications of the globular projection of Roger Bacon were presented in 1524.[104] Werner belonged to the Nuremberg group which also included

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[100] Cosgrove (1992), p. 81.
[101] Rowland (1999), p. 153-159, p. 166.
[102] Latour (1986), p. 24.
[103] Koyré (1957), p. 88, writes: "I have already mentioned the *Sidereus Nuncius* of Galileo Galilei, a work of which the influence – and the importance – cannot be overestimated, a work which announced a series of discoveries more strange and more significant than any that had ever been made before."
[104] Snyder (1993), p. 14.

Regiomontanus and Pirkheimer;[105] he studied astronomy, mathematics, and geography and, in addition to his work on projections, published treatises on spherical triangles and conic sections.[106] His 22 books on *Elements of Conics* was printed in 1522,[107] while a revision of Ptolemy's *Geographia* with four 'new' projections had appeared in 1514.[108] Three of the four projections were heart-shaped (cordiform); the fourth was the oblique stereographic used in the fourth century. Albrecht Dürer was influenced by Werner;[109] he combined mathematical and artistic interests and is known for the application of geometry to art, especially in perspective and proportion. He also produced a celebrated star chart on the polar stereographic projection in 1515[110] and proposed projections onto solid figures: the tetrahedron, dodecahedron, and icosahedron in 1538, but did not construct maps based on them.[111]

Possibly the most famous cartographer-mathematician of the sixteenth century is the one whose projection bears his name, Gerardus Mercator (1512-1594). Born in Flanders, he attended the University of Louvain where he quickly became interested in philosophy, theology, mathematics, geography, astronomy, engraving, and calligraphy,[112] combining abstract academics with technical arts. He moved to Germany in 1552 and presented his cylindrical projection in 1569, about twenty years before Galileo became interested in mathematics. Mercator produced numerous maps and globes, both terrestrial and celestial,

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[105] Rose (1976), p. 109. Rose notes, p. 107, that Werner was a good friend of Pirkheimer who criticized Werner's version of the *Geographia* in the dedication of his own 1525 translation.
[106] Snyder (1993), p. 34
[107] Boyer (1989), p. 328.
[108] Snyder (1993), p. 23-24, 34-37; see also O.A.W. Dilke, "The Culmination of Greek Cartography in Ptolemy" (1987), p. 187.
[109] Rose (1976), p. 108.
[110] Snyder (1993), p. 22-23.
[111] Snyder (1993), p. 43.

during his lifetime. The maps were bound together and published in book form as an 'Atlas' after his death and his famous 1569 projection appears as a world map of eighteen sheets.[113]

Erhardt Etzlaub (ca. 1460-1532), also of Nuremberg, a surveyor and compass-maker, had produced a regional map of central Europe similar to that attributed to Nicholas of Cusa. Etzlaub's map was dated to the last decade of the 15th century and it was designed for practical use.[114] He had also created two sundial maps in 1511 and 1513 with latitude scales showing increasing distances between parallels, the principal feature of the later Mercator projection.[115] Thus we can detect traces of the subtle influences of Ptolemy in the developing mathematics of map projection, perhaps, in this case, via the Vienna Corpus.[116]

The mathematics of the Mercator projection were not well understood by its acknowledged author who is reputed to have devised it graphically.[117] They were described by Edward Wright (1561-1615) in 1599 while Giordano Bruno was in prison refusing to recant his heretical theories of an infinite universe and the non-existence of absolute truth. Edward Wright was considered "one of the most eminent mathematicians of the time"[118] and is known to have assisted Gilbert with his magnetically-based concept of the solar system, and to have translated Napier's work on logarithms (1614) into English.

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[112] Snyder (1993), p. 43.
[113] Snyder (1993), p. 45.
[114] Durand (1952), p. 267; Harvey, "Local and Regional Cartography in Medieval Europe" (1987), p. 497.
[115] Durand (1952), p. 269; Snyder (1993), p. 49.
[116] Durand (1952), p. 269-70. See also note 74.
[117] Snyder (1993), p. 47.
[118] Snyder (1993), p. 47.

The sinusoidal projection was the last of the major projections devised during the Renaissance and was used most notably by Nicolas Sanson of Abbeville, and John Flamsteed.[119] Flamsteed was the first astronomer royal of England, and acquainted with Isaac Newton to whom he wrote that he used the sinusoidal: "for drawing the charts of the constellations, ... as the appearance to the naked eye is less distorted than by any projection I have yet seen."[120]

Although Sir Isaac Newton does not have a direct link with the theory and practice of cartography, other than to devise the polar co-ordinate locational system, this fragment of correspondence demonstrates an assumption of interest in and familiarity with maps and mapping. It is clear that mathematics, astronomy, and cartography have intimately shared a context of thought for many centuries of history, and may have had intangible, multiple mutual effects on the development of ideas including the ideas of Newton. Newton, himself, famously declared that if he had accomplished anything of importance, it was because he had stood on 'the shoulders of giants.' This was not false modesty, but the essential truth. So many of the thinkers who were of crucial importance in the developing ideas about the cosmos, and who prepared the way for the synthesis of Newton, were engaged by mathematics and cartography and influenced by the Ptolemaic opus. These individuals were also instrumental in the diffusion of the new ideas into the wider culture such that they became so basic that they could be taken for granted in subsequent centuries.

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[119] Snyder (1993), p. 50.
[120] Snyder (1993), p. 50.

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Reflections and extrapolations

Reflections

This brief study of the history of the conceptualization of space provides a first take on the complex interactions between mathematics, cosmology, and cartography. I have focussed on a period which exhibits relative discontinuities in both cartography and cosmology, positing an instrumental effect for cartography on the latter. The new cartographic image which took Euclidean geometry as axiomatic may have adventitiously suggested ideas about the infinity of the universe leading, ultimately, to the Newtonian conception of the cosmos. The new cartography of the Renaissance was of great interest to the humanist and mathematical scholars of the time, as cartography had been and was to continue to be to scholars of all historical ages in Europe. It was thus opportunely positioned to provide a perceptual pre-requisite for the developing philosophy of the cosmos which replaced the Aristotelian conception.

Inherent in the Ptolemaic map model was a concept of space – that it was homogeneous, continuous, isotropic, and infinitely extended. These are the properties of Euclidean space and they became the properties of space in modern physics. Because this cartographic mode literally depicted a world-view in which the earth was shown as flat inscription, the perception of the world as a planar object became so imprinted that it assumed the character of intuitive knowledge. Two-dimensional space is governed by the laws of Euclidean geometry and thus the cartographic mode of depiction, the Ptolemaic model, was instrumental in the perception of the space of the universe as infinite and absolute.

Cartography achieves its epistemological importance because it is, in a sense, virtual geometry and the geometrization of time and space, and the separation of humanity from nature followed from the concept that reality is essentially mathematical. Galileo and Kepler made the distinction between the objective, absolute, and mathematical and the sensible, fluctuating, and qualitative. This distinction is central to the character of modern science. Cartography occupies an ambivalent position which gives it a unique potential for unconscious impact. Although the new Ptolemaic mapping model with its abstract grid was purely a mathematical construction, a map has a visible form and a representational purpose. This was especially significant, as I have pointed out, in the late-medieval-early-Renaissance period as the dominance of the visual was a relatively recent and growing phenomenon and its novelty gave it power over the imagination. The written word, mechanical clocks, musical notation, arabic numerals, and, of course, pictures and maps all contributed to this phenomenon. The Renaissance idea that knowledge involved "the recognition that even things that seem formless participate in form,"[1] was a pivotal concept for Cassirer who also stressed the creative act of giving form: "[T]he focal point of intellectual life must lie in the place where the 'idea' is embodied," he says, "i.e., where the non-sensible form present in the mind of the artist breaks forth into the world of the visible and becomes realized in it.[2]

[1] Cassirer (1963), p. 66.
[2] Cassirer (1963), p. 67.

Maps have always been, are defined as, realizations of an *idea* of the world, whether that idea is primarily mythological or solely physical, qualitative or quantitative. As well, maps are graphic images which provide an aesthetic experience; they are perceived directly through the senses and have an immediate hermeneutic value. The primacy of the perceptual experience is likely to lead to unconscious absorption of basic elements of the form. The inherent conceptualization of space is one of these basic elements.

Simultaneously with the 'shift to the visual,' the new philosophy of the Renaissance began to advocate rejection of the evidence of the senses as unreliable and unmeasureable, and viewed mathematics as providing the only certifiable truths. The Neoplatonic movement fostered by Ficino and others elevated the idea of mathematics as having the highest value. Cosgrove has remarked upon the conjunction between mathematics and visual form in the Neoplatonic tradition of the fifteenth and sixteenth centuries:

> *One of [Neplatonism's] consistent themes was that of 'seeing' mathematics, and through them God's immanence in the Universe. This placed considerable significance on graphic representation, whether in the visual arts or in mapping and survey which depended on visual measurement: the most powerful and immediate graphic representation of the Universe or nature* (natura) *and the proportional harmony running through them. Mapping at the descending scales of the Ptolemaic hierarchy served to reveal proportion and harmony at all levels of creation.*[3]

The Neoplatonist movement of the early Renaissance also engendered a movement from valuing the particular and the local to valuing the general and the universal which came to full fruition with Descartes.[4] Though imperfect, a geometrical map at world scale represented a conception of an ideal form of the cosmos. It was a universalistic, mathematical conception

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[3] Cosgrove (1992), p. 76.
[4] See note 44, Chapter 2.

made visible by the application of the principles of Ptolemaic cartography. In the act of making concrete this expression of an idea of the world as geometry, cartographers had perforce to suggest an image of the real, physical world. Even the medieval *mappaemundi*, though vague about the physical world, presented quasi-realistic depictions of geography. A map's notionality, is easily forgotten. It often happens that the metaphor is confused with its object of reference and, in the case of a cartographic image, that image comes to be seen as not just a sign for, but a realistic depiction of the physical world. Before the appearance of printed maps (1470's on), Europeans had only vaguely imagined ideas of the world beyond their experience. When printed maps began to circulate, they changed or re-formed people's conceptions of places and of geographical features too large to be comprehended in their entirety on the ground, and they formed people's conceptions of the continuity of physical space. The Ptolemaic mathematical cartographic model became an archetype which was endlessly repeated.

Why does the conceptualization of space matter? It matters because it is fundamental to metaphysical assumptions which underpin the modern scientific, mechanical explanations of the universe. With the change in conceptualization of space for Europeans came a change in their sense of relationship to the things in the world around them. Where once they had felt that they participated in the world, they learned to regard the world as their *environment*, a word evocative of the separation of humanity from nature. Nature became objectified as humanity shifted from involvement to detachment, from a transactional relation to a controlling one; the vitality of existence which was a given for medieval Europeans was downgraded to spectacle. E.A. Burtt in a book whose significance may still not be recognized, described the difference between the medieval worldview and that of the 17^{th} century:

> *The scholastic scientist looked out upon the world of nature and it appeared to him a quite sociable and human world. It was finite in extent. It was made to serve his needs. It was clearly and fully intelligible, being immediately present to the rational powers of his mind; it was composed fundamentally of, and was intelligible through, those qualities which were most vivid and intense in his own immediate experience – colour, sound, beauty, joy, heat, cold, fragrance, and its plasticity to purpose and ideal. Now [post-Galileo and post-Descartes], the world is an infinite and monotonous mathematical machine. Not only is his high place in a cosmic teleology lost, but all these things which were the very substance of the physical world to the scholastic – the things that made it alive and lovely and spiritual – are lumped together and crowded into the small fluctuating and temporary positions of extension which we call human nervous and circulatory systems.*[5]

Burtt's radical critique was first published in 1924 and in it he provided a careful examination of the underlying metaphysical implications of the achievements of Galileo and Kepler. The qualities which made the medieval world so beautiful and confusing and alive were those things which could not be measured and as quantification was the new imperative, so "the only domain of true reality has to be the world outside man."[6] This conception ushered in the destructiveness of Cartesian dualism: the world as mathematical machine extended in space v. the world of unextended thought and spirit.[7] As Burtt says, "Man begins to appear for the first time in the history of thought as an irrelevant spectator and insignificant effect of the great mathematical system which is the substance of reality."[8]

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[5] Burtt (1932), p. 116.
[6] Cohen, p. 94.
[7] Burtt (1932), p. 113.
[8] Burtt (1932), p. 80.

Extrapolations

Am I justified in adducing an early idea of spatial conceptualization which is both idealist and mathematical to what is generally deemed an essentially technical activity, and suggesting that it had a formative effect on the new philosophy of the cosmos which evolved in early modern Europe? As long as I don't ascribe sole responsibility for determination of a mode of perception, I believe I am justified in claiming that it may have had an unrecognized effect in grounding the theoretical work of scientists and natural philosophers over the approximately three hundred years between the onset of the Renaissance and the acceptance of Newtonian physics and cosmology.

As Bronowski has said, what we think about the world is not what it is but what we perceive it to be, and thus our mode of perception structures our understanding.[9] "Consciousness appears as the quality of perceiving order in things," Collins states. "And the history of consciousness," he continues, "is concomitantly a history of changes in modes of perception; it is thus a history of different 'orders.'"[10]

The mode of perception which developed in the early modern era, formed in part, I contend, by the effect of the Ptolemaic cartographic model survives in current common sense perceptions of space despite the development of new ideas in mathematics. Cartography continued to have a role in the evolution of ideas about space at least until the end of the nineteenth century. By the close of the Renaissance, mathematical cartography had evolved into an established model and more than sixteen map projections had been

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[9] Bronowski (1978), p. 4-5.
[10] Collins (1989), p. 5.

devised for depiction of global geography. About half of them continued in use, including the simple conic which dated to Ptolemy and was continually refined. Until the late 17th century, the analytical tools of mathematics were too elementary to permit the construction of projections to exhibit certain pre-determined characteristics. In the late seventeenth century, Newton and Leibniz almost simultaneously invented calculus but it was not until Johann Heinrich Lambert that it was applied to the development of map projections. Lambert (1728-1777), one of the leading mathematicians of the 18th century, is known for his work on the parallel postulate of Euclid which was foundational in the geometry of space. Boyer comments that: "No one else came so close to the truth without actually discovering non-Euclidean geometry,"[11] and Gray considered Lambert's work on the geometry of the sphere inspired: "To enter the land which Lambert's vision was the first to descry," he writes, "was to take mathematics another hundred years."[12] Lambert presented the first proof that π is an irrational number, and his writings include ideas in mathematical philosophy, cosmography, descriptive geometry, logic, probability, optics, and cartography.[13] In a single paper in 1772, he presented seven original projections of which three are among those most commonly used in the twentieth century.[14] He was the first to apply calculus to a general study of the theory of map projections and their properties, especially the properties of conformality and equivalence.

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[11] Boyer (1989), p. 515.
[12] Gray (1989), p. 75.
[13] Boyer (1989), p. 515.
[14] The contemporary names of the three are: the Lambert Conformal Conic, the Transverse Mercator, and the Lambert Azimuthal Equal-Area. See Snyder (1993), p. 76.

Carl Friedrich Gauss (1777-1855), a prodigious mathematician, is considered the first to adopt the view that a non-Euclidean geometry was logically possible.[15] He was fearful of publishing such a radical idea and exposing himself to ridicule, however, especially as he was unable to formulate it systematically. This would be accomplished later almost simultaneously by Nicolai Lobachevski and Janos Bolyai and, more definitively, by Riemann, but Gauss certainly claimed to have thought of it first. Gauss had an interest in geodesy and cartography and applied his formidable powers to a consideration of the intrinsic properties of curved surfaces.[16] His employment by the Hanoverian government to oversee a geodetic survey of Hanover focussed his attention even more particularly on geodetic problems and he published two papers which broke new ground on the theory of surfaces and established the foundations of differential geometry.[17] These were to provide Riemann with the analytical tools to theorize non-Euclidean geometry in general:

> *These papers ... became through the work of Riemann the foundation of modern mathematical investigations into the structure of space. Once again we see that, historically viewed, abstract theories of space owe their existence to the practice of geodetic work, just as ancient geometry originated in the practical need of land surveying.*[18]

It is, therefore, demonstrable and notable that mathematics, astronomy, and cartography have been closely linked through these many centuries of history in both theory and practice. Philosophers and geometers from Democritus, Archimedes, and Ptolemy to Copernicus, da Vinci, and Galileo were engaged by and with maps and the principles of mapping. After the acceptance of Newtonian cosmology, mathematicians continued to

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[15] See Gray (1989), p. 86, and Jammer (1969), p. 148.
[16] Gray (1989), p. 133-137.
[17] Jammer (1969), p. 152-153.

pursue interest in geodesy and cartography. These many individual thinkers were important contributors to the history of science and philosophy as currently understood, and, additionally, their contributions clearly affected, even determined, the trajectory of cartography. It is less clear, because so far almost completely unexamined, how the theory and practice of cartography may have affected their ideas in science and philosophy, but it is a reasonable claim that it was seminal in the development of the theory of non-Euclidean geometry through the work of Lambert and Gauss.

It was mentioned in Chapter 3 that Max Jammer thought the spherical space of Einsteinian relativistic cosmology reminiscent of Aristotle's belief in the finiteness of the material world and, therefore, of space.[19] "The idea of a finite physical space," he writes, "is not as absurd today as it must have appeared fifty years ago, when physics acknowledged solely the conception of an infinite Euclidean space and when a finite material universe could but be conceived as an island, so to speak, in the infinite ocean of space."[20] But, as we have seen, Aristotle did not so much have a concept of 'space' as a definition of 'place' (*topos*). Einstein, himself, describes this idea as follows:

> *Now as to the concept of space, it seems that this was preceded by the psychologically simpler concept of place. Place is first of all a (small) portion of the earth's surface identified by a name. The thing whose "place" is being specified is a "material object" or body. Simple analysis shows "place" also to be a group of material objects. ... If the concept of space is formed and limited in this fashion, then to speak of empty space has no meaning.*[21]

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[18] Jammer (1969), p. 152.
[19] See note 37, Chapter 3.
[20] Jammer (1969), p. 22.
[21] Foreword to Jammer (1969), p.xiii.

This idea of space is re-created in the digital era by the ubiquitous 'icon map' to be found most often in cyberspace, and in the discrete digital boundary file used in Geographic Information Systems. In the icon map, and the GIS file, a small portion of the earth's surface is detached from its surrounding geography, identified by name, and given status as an independent object. Seldom does this type of map carry a graticule or a frame functioning in the traditional way as a signal that it is interrupting the continuous geometry of the earth which, by implication, continues beyond. The icon map and the GIS file thus re-create the Aristotelian sense of the discontinuity of space and of the inapplicability of the concept of empty space. The space of the map is a quality of the region being mapped, rather than a slice of absolute, continuous space. Thus in the digital era, it is possible that cartography will again have an effect on the way we perceive the space of the world. While the Ptolemaic cartographic model was perpetuated in the modern era by the continuing emphasis on topographical survey and mapping, common sense perceptions of space continued to rely on an intuition of Euclidean geometry and infinity. With the shift to cyberspace and the use of the Geographical Information System, it is interesting to speculate that commonsense notions of space may finally abandon their habitual reliance on plane geometry, one hundred and fifty years after the mathematics of non-Euclidean geometries was theorized, and eighty years after the physics of relativity, and return to the spatial intuitions of Aristotelianism. ❑

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