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ISBN: 0-612-84014-X
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ISBN: 0-612-84014-X

THE STUDENT EXPERIENCE OF LEARNING ADVANCED PLACEMENT CALCULUS AB IN A WEB-BASED ENVIRONMENT

by

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A thesis submitted to the

School of Graduate Studies

in partial fulfilment of the

requirements for the degree of

Master of Education

Faculty of Education
Memorial University of Newfoundland

October 2002

St. John's, Newfoundland

Abstract

The primary purpose of this study was to identify impediments to learning calculus concepts in a web-based, Advanced Placement (AP) Calculus AB course and to make recommendations for the improvement of learning in that environment. The subjects of the study were 15, level III students from 3 high schools in Newfoundland and Labrador, Canada who enrolled in Mathematics 4225, a provincially approved, first calculus course which prepared students to write The College Board AP Calculus AB examination for university credit. In this study, students and an instructor in 4 geographically distant sites in 1 school district were linked via a digital Intranet to create a virtual classroom where instruction occurred synchronously, using Microsoft NetMeeting and Meeting Point computer software, and asynchronously, through a website developed specifically for Mathematics 4225 using WebCT and Microsoft Front Page 98. Students were surveyed to determine their experience of learning calculus in a web-based environment and to identify features of the course that needed to be added, modified or removed. The course textbook was analyzed to determine the appropriateness of the epistemological messages conveyed to students. Calculus AB College Board examinations were also compared in terms of the changes in structure and difficulty level enabled by the recent authorized use of graphing calculators. On the basis of the gathered data and the text and examination analyses, recommendations are made for the improvement of learning.

Acknowledgements

The names of four very important people come to mind as I write this page. I want to thank my co-supervisors, Dr. Ken Stevens of Memorial University of Newfoundland and Dr. David Reid of Acadia University. The flexibility, guidance, and encouragement of these gentlemen have influenced my growth as a person and as a teacher and made my experience as a graduate student rewarding and memorable.

The understanding and sacrifices of my wife Tracy have not gone unnoticed. She has offered unwavering support through a very demanding and tumultuous time in our lives and allowed me to work and study untethered. I am very fortunate to be able to share my life with someone so beautiful.

In the midst of my work on this thesis came a wonderful child who has forever changed our lives. I had not realized the depth of love for a child until Grant became the first addition to our family. He fills our days with joy and laughter and love. This thesis is dedicated to him.

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INTRODUCTION AND STATEMENT OF RESEARCH QUESTION

Chapter One: Introduction

Research into the learning of calculus is varied and extensive, especially with respect to the learning that takes place in the traditional classroom setting. As well, much research has been carried out into learning at a distance. However, there currently exists a lack of research into student learning in general, and into the learning of calculus in particular, in a web-based environment. This thesis addresses the need for research in this area by describing recent research into the learning that occurs in such an environment.

The thesis will first examine the learning experience of students, located at three geographically distant sites within a school district, who participated in a web-based, Advanced Placement (AP) Calculus AB course that was delivered using both synchronous and asynchronous instruction. This will be accomplished through a student survey derived from a review of relevant literature on calculus reform, learning theory, advanced mathematical thinking, and learning at a distance. It will then compare the current textbook with another text which has received support as a more suitable first calculus course text in terms of the appropriateness of the epistemological messages it contains. Finally, the thesis will compare official AP Calculus AB examinations from 1988 and 1997 to determine the changes to the examination enabled by use of graphing calculators. It will then make recommendations for improvements in learning in the general context of a web-based environment and in the particular context of an AP calculus course.

The thesis consists of twelve chapters. The remainder of chapter one presents the research question, which establishes the overall focus of the thesis. Chapter one also identifies the significance and limitations of the study. The literature review consists of four chapters. Chapter two examines the constructivist theory of learning. The student survey results are interpreted in light of this theory of learning. Chapter three examines calculus reform. This chapter identifies the problems associated with calculus courses taught in the traditional manner, presents the visions and goals of a new, revitalized calculus course, and examines the role of technology in calculus reform. Chapter four examines advanced mathematical thinking in terms of the sources of obstacles to learning calculus concepts, those concepts which students typically have difficulty in understanding, and how conceptual understanding can be attained. Chapter five examines learning at a distance and identifies the challenges in learning at a distance as well as how distance learning can be enhanced. Chapter six (research procedures) describes the context in which the research takes place, justifies the selection of a qualitative research methodology, and lists the actual methods of data collection. The results are contained in four chapters. Chapter seven categorizes students as completers or noncompleters and provides a description of each student as compiled from the student surveys. Chapter eight presents the results of part one of the student survey. This part of the survey examined the features of the Mathematics 4255 website, course text, online classes, evaluation, collaboration and social interaction. Chapter nine presents the results of part two of the student survey. This part of the survey elicited students' opinions on factors which research literature indicated was important in learning calculus concepts and in learning at a distance. The quantity of results presented in chapters eight and nine necessitated that the discussion of the results for each feature occur immediately after the results themselves were presented, instead of being delayed to a later chapter. Chapter ten contains a comparison of the current text for the course and an alternate text in terms of the features and epistemological messages contained in each. This chapter also compares College Board Calculus AB examinations from 1988 and 1997 in terms of the changes enabled by the use of graphing calculators. As much of the discussion of results occurred in earlier chapters, the analysis chapter (chapter eleven) contains only a succinct general description of students' experience in learning calculus in the context of this study and identifies some of the major contributors to the obstacles to learning in the course. Chapter twelve contains the conclusions and recommendations arising from the study.

Statement of the Research Question

The purpose of this study is to investigate, and to recommend strategies for the improvement of, student learning of calculus in a web-based environment. Hence the research question is:

What are the impediments to learning calculus concepts in a web-based, Advanced Placement Calculus AB course which is delivered via a district Intranet, and how can this learning be improved?

Definition of Terms

Student refers to fifteen, level III high school students in one school district in Newfoundland and Labrador.

Advanced Placement Calculus AB Course refers to a first calculus course based on the topical outline (available:

http://apcentral.collegeboard.com/repository/ap01.cd_cal_4313.pdf} developed by The College Board, USA. The web-base version of this course refers to the online course developed in the Centre for TeleLearning and Rural Education, Memorial University of Newfoundland in 1998 (available: *http://www.stemnet.nf.ca/~vsddi/*).

District Intranet refers to the digital connection of 9 high schools in a school district for the purpose of delivering Advanced Placement courses.

Significance of the Study

This study is significant because of its focus on learning early calculus in a web-based environment. All aspects of learning calculus in this context should be investigated, and any recommendations for improved learning implemented, to ensure that AP calculus distance education is an attractive and real alternative to traditional face-to-face calculus instruction. The availability of viable web-based courses is especially important in rural areas of Newfoundland and Labrador where student enrolment and teacher expertise may prevent student participation in an Advanced Placement Calculus course taught in the traditional manner. Linking schools electronically to create virtual classrooms provides gifted mathematics students from rural areas with the opportunity to participate in AP courses and hence contributes to the equality of educational opportunity for students in Newfoundland and Labrador, irrespective of their geographical location. As well, the further refinement of this web-based model of course delivery will create a more powerful learning environment for high school students who currently enroll in non-AP web-based distance education courses which are delivered using the Internet. The study is also significant in that it investigates learning in an environment which provides for synchronous instruction. The majority of the current research into web-based learning focuses primarily on asynchronous instruction. Hence, along with addressing the need for research into learning in a web-based environment in general, this study also addresses the particular lack of research into the synchronous aspect of web-based learning.

Limitations

1. The findings of this study are specific to the context of this study and may not apply to other web-based, first calculus courses
2. Data gathering and participant observations occurred at three AP sites in one school district of Newfoundland and Labrador. However, two of these sites were larger high schools in the district and one was a smaller, more rural all-grade school. Therefore, some of the findings arising from gathered data may be applicable only to specific sites and not generally to all sites in the study.
3. The participants in this study were teenagers in their final year of secondary school. Therefore, findings of this study may not be applicable to more junior students or to the adult learner.

LITERATURE REVIEW

Introduction

The literature review for this thesis consists of four chapters. The purpose of chapter three is to identify the inadequacies of calculus courses taught in the traditional manner and to provide an overall vision of a revitalized, first calculus course. This chapter on calculus reform will provide the benchmark for identifying problems and solutions in the web-based, AP calculus course under study in this thesis. Chapters two, four, and five are more theoretical in nature and provide a solid theoretical basis for the conclusions and recommendations of chapter twelve. The chapter on constructivist learning theory examines an alternative to the behaviorist view of learning characteristic of the majority of first calculus courses. The chapter on advanced mathematical thinking investigates cognitive theories of conceptual understanding. This chapter concentrates on Ed Dubinsky's Action Process Object Schema (APOS) Theory and the procept notion of David Tall. The final chapter of the literature review examines distance learning and in particular how a deep approach to learning can be promoted in such a context.

Chapter Two: Learning Theory

Skemp (1987) notes that models of learning which see learning as a passive, reproductive process have been unsuccessful in both explaining and bringing about "the higher forms of learning … of which mathematics is a clear example" (p. 134). As a result, a theory of knowledge known as constructivism has emerged in current mathematical research. This view of learning has been supported in publications by a variety of influential educational organizations including the National Research Council (1989), the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics (1989), the Commission on Teaching Standards for School Mathematics of the National Council of Teachers of Mathematics (1991) and the Mathematical Sciences Education Board (1990).

According to Jeremy Kilpatrick, former editor of the *Journal for Research in Mathematics Education*, the constructivist view involves two principles: (1) knowledge is actively constructed, not passively received, and (2) coming to know is an adaptive process of organizing one's experiences and does not involve discovering an independent, pre-existing world outside the mind of the knower (cited in Selden & Selden, 1990). Similarly, von Glasersfeld (1983) asserts that meaning is built or constructed by the individual using his or her own experiences and previous knowledge as a guide and thus, concepts reside within each person and have subjective representations. However, Selden and Selden (1990) note that this does not necessarily imply that a concept can mean whatever the learner wishes. Its meaning must be consistent with experience, the meaning of related concepts and the meaning constructed by others. According to the authors, this implies that the emphasis in mathematics teaching should shift from ensuring that a student can correctly replicate what the teacher has demonstrated to helping students organize and modify their mental schemata (i.e. their internal conceptual networks, hierarchies and processes). Von Glasersfeld contends that mathematical knowledge is the product of conscious reflection on the part of the student and that the mathematics teacher should be a facilitator of student thought processes. Specifically, a teacher's role is to suggest appropriate critical examples or counterexamples for student reflection; the actual cognitive conflict resolution must be left to the student.

Young and Marks-Maran (1998) describe learning in a constructivist framework. Rather than describing learning as practicing, performing a task correctly, or as a passive endeavor, the authors characterize quality learning as (a) personal understanding and meaning; (b) interpretative; (c) active; (d) learning to learn; (e) constructive; (f) reviewing, redefining and integrative; and (g) inquiring, exploring and inquisitive.

Constructivism itself has a variety of forms. The acceptance of the first of Kilpatrick's principles is known as simple constructivism while acceptance of both principles is referred to as radical constructivism. While constructivism itself may defy a strict definition, Robler, Edwards and Havriluk (1997) have identified some instructional characteristics that are commonly attributed to the influence of constructivist theories:

1. Problem-oriented learning activities that are relevant to student interests, require some time, a variety of skills, and several people working together.

2. Highly visual formats such as those made possible with videodiscs and multimedia materials.
3. "Rich" learning environments that use a variety of resources, such as computers and construction kits, as opposed to minimalist environments which rely primarily on the teacher, textbook, and prepared material.
4. Collaborative and cooperative group work.
5. Learning through exploration, with an emphasis on problem solving rather than getting the right answer.
6. Authentic assessment methods with qualitative (e.g. portfolios, teacher narratives, and performance measures) rather than quantitative (e.g. objective paper-and-pencil tests) strategies.

This thesis will assume a constructivist model in the analysis of learning in the AP Calculus AB course. This model views learning as a constructive process where the person who is learning seeks and builds information on the basis of previous knowledge and past experience. The assumption underlying this view is that human beings act upon their environment; they do not simply respond to it (Schuell, 1986). This challenges the behaviorist view that the learner passively receives information. Instead, the learner is seen as "a purposeful agent, extracting and imposing meaning" (Lauder, 1996).

Chapter Three: Calculus Reform

Introduction

The process of calculus reform had its official beginning at the Tulane Conference, a workshop organized by Ronald G. Douglas and held in January 1986. Subsequent work on reform led to the publication of *Calculus for a New Century: A Pump, Not a Filter,* by the Mathematical Association of America (MAA) in 1988, a result of a national colloquium held on October 28 and 29, 1987, in Washington, D.C.. At this conference, over six hundred mathematicians, scientists and educators presented 75 background papers, presentations, and responses on the issue of calculus reform. These submissions highlighted the perceived shortcomings of traditional calculus courses, offered an overall vision for a reformed calculus course, and included specific suggestions as to how this vision might be achieved.

Problems with the Traditional Calculus Course

In her introduction to *Calculus for a New Century: A Pump, Not a Filter,* editor Lynn Arthur Steen states that less than twenty five percent of students who study calculus survive to enter the science and engineering pipeline. Calculus thus becomes a barrier to these professional careers for a vast number of students. As well, many of those who do survive calculus are too poorly motivated to fill graduate schools, too few in number to sustain the needs of business, academe and industry, and too ill-suited to meet the mathematical challenges of the next century (Steen, 1988a).

Problems in the traditional calculus course are numerous and serious. Current courses are highly structured and automated (Moskowitz, 1988). There is little or no emphasis on conceptualization (Moskowitz, 1988; Reed, 1988), or modeling (Moskowitz, 1988). Classes are too large (Moskowitz, 1988; Fulton, 1988), and courses contain too much material and are too technical (Reed, 1988). There is a lack of word problems (Reed, 1988), and those that are posed are template problems which only require straightforward calculations (Hallett, 1996; Steen, 1988b; Carr, 1992) and no justification of answers (Priestley, 1988). Problems, dealing with the theory of calculus and requiring rigorous calculus, have vanished (Hallett, 1996; Steen, 1988b).

In the context of our present calculus courses, the mathematical power and beauty of calculus that mathematicians know and relish cannot be illustrated to students (Curtis & Northcutt, 1988). Students see calculus as a collection of rules and problems which they must memorize before being allowed to proceed (Curtis & Northcutt, 1988; Lovelock & Newell, 1988). Hence calculus is viewed as a filter, as a rite of passage (Curtis & Northcutt, 1988; Malcom & Treisman, 1988). Students are seldom helped to understand why they need calculus (White, 1988) – that calculus in its present and future forms will serve as a critical passageway into majors that lead to higher paying technical and professional careers (Newman & Poiani, 1988) or that calculus is part of the knowledge base of the liberally educated individual.

We no longer ask students to understand, but instead emphasize manipulation (Cipra, 1988; Kolata, 1988, J. Ryan, 1992). A large portion of the content of calculus courses is devoted to gaining skills in numeric and algebraic operations (Layton, 1988).

Unfortunately, skills such as differentiation and integration are quickly forgotten and must be reviewed (Moskowitz, 1988). Most students achieve by memorizing algorithmic procedures given in the text or by teachers, and working textbook problems following those procedures (Anderson, 1988; Steen, 1988c). Consequently, students learn much of their mathematics by memorization and often do not have much experience with being expected to understand concepts (Layton, 1988). There is also the expectation on the part of students that memorization of algorithmic procedures will be the primary focus of the course (Ostebee, 1993).

Traditional textbooks have also come under attack by those wishing to transform calculus. Archer and Armstrong (1988) state that current textbooks include too many topics, excessive highlighting and summarizing sections, too many template problems and too much 'plug and chug' (p. 61). Newman and Poiani (1988), discussing the increasing percentage of adult learners who study calculus, claim that these adults have an intellectual maturity and curiosity which demands more depth of understanding about the relationship between mathematics and the real world than most texts can provide. Kolata (1988) asserts that textbook writers have made calculus courses so instructor-proof that even Ph.D. mathematicians cannot introduce any concepts that might reflect the beauty and excitement of the field. Ostebee (1993) contends that standard textbooks have a strong emphasis on symbolic manipulation. Lyons (1988) notes that even with the long history of dissatisfaction with standard textbooks, there has been no radical change in those that have been recently published. The author notes that this is illustrated by the fact that many popular texts are now in their third, fourth, and fifth editions. Anderson

(1988) states that textbooks that are adopted are similar to past texts which have traditional values. Texts with radical changes seldom get adopted, thus they are not written and published. Kolata (1988), writing on textbook stagnation, asserts that traditional textbooks would be all but useless for a reformed calculus course but notes that developing new texts would be going against a very entrenched industry. Given this hurdle, Wilson and Albers (1988) note that it is generally agreed that calculus courses are unlikely to change unless new textbooks are produced.

Impeding the effort of calculus reform is the fact that there is little agreement as to the basic purpose of the reform. For example, Starr (1988) distinguishes proponents of calculus as a service course, as a course to provide techniques, methods and manipulation, and as a course which is a core part of the learning of an educated individual. Kolata (1988) notes that various client disciplines would prefer emphasis alternately on approximation theory and numerical solutions to differential equations, models and qualitative analysis, and breadth and problem solving.

Disagreement also arises on the issue of partitioning classes by discipline. Moskowitz (1988) and Carr (1992) claim that partitioning would allow the possibility of focusing more on applications that are relevant to students in their respective disciplines. However, Erdman and Malone (1988) note that there is a strong feeling that there should *not* be separate sequences and that the course and educational objectives would be better served with a variety of students in the course. Stevenson (1988) warns that "principles of mathematics taught in the context of specific application have the danger of not being recognized for their breadth of applicability" (p. 26). Bossert and Chinn (1988) also believe that partitioning is inappropriate, claiming that students taking a first calculus course have not yet set career directions, that the value of a mathematics course as part of a liberal education will be compromised if it is taught with a narrow disciplinary focus and that "there is a danger that specialization of a calculus course will obscure the generality of the mathematical way of thinking that separate disciplines have come to mathematics to gain" (p. 67).

Also retarding the reform effort is the significant group of scientists and engineers who wish to maintain the status quo. This group is uninterested in change, stating that the traditional calculus course is sufficient, and they do not want to expend the time and effort needed to radically change it (Kolata, 1988; Tucker & Leitzel, 1995; see also Anderson, 1988). This group also holds the view that any student who cannot pass the traditional calculus course should revise their plans to become a scientist (Kolata, 1988).

Vision/Goals of a Revitalized Calculus Course

The overall vision of calculus is that of a pump which can fill our scientific pipeline. Lynn Arthur Steen, in her introduction to *Calculus for a New Century: A Pump, Not a Filter,* states:

> To fill this pipeline, we must educate our youth for a mathematics of the future that will function in symbiosis with symbolic, graphical and scientific computation. We must interest our students in the fascination and power of mathematics – in its beauty and in its applications, in its history and in its future. [We must offer] a vision of the future of calculus, a future in which students and faculty are together involved in learning, in which calculus is once again a subject

> at the cutting edge – challenging, stimulating, and immensely attractive to inquisitive minds. (p. xii)

The purpose then, of calculus instruction, is to help students to succeed, not to weed them out to reduce the numbers in highly competitive programs (Malcom & Treisman, 1988).

More specifically, a vision of a reformed calculus consists, in part, of a ubiquitous emphasis on conceptual understanding and much less emphasis on facility in symbolic manipulation (Davis, 1988; Levin, 1988; Ralston, 1988; Ross, 1996; Tucker & Leitzel, 1995), which Ralston believes, humans typically do poorly and computers do well and whose mastery does not aid the ability to apply calculus or to proceed to advanced subject matter. Salamon (1988) advocates that the objective of instruction in calculus is to have students acquire the most thorough and functional understanding of calculus they can achieve. Davis (1988) supports the need for calculus courses to emphasize clear thinking and not merely symbolic manipulation. Hodgson (1988) is certain that, in a revitalized calculus course, a shift will take place from purely computational to more interpretative skills – in other words, from calculation to meaning. O'Meara (1988) announces that mathematics education at the college level will have to be stripped of rote and concepts emphasized. Wilson and Albers (1988) note the common agreement that concept building is a fundamental goal. Haines and Boutilier (1988) call for much more use of the conceptual approach, which lies between pure skills and pure theory. Young and Blumenthal (1988) state that fundamental understanding of underlying concepts is what is important for students. These views on the importance of conceptual understanding are also supported by a myriad of other authors, including Anderson

(1988), Armstrong, Garner and Wynn (1994), Bailey and Chambers (1996), Buccino and Rosenstein (1988), Cipra (1988), Curtis and Northcutt (1988), Flashman, (1996), Fulton (1988), Gehrke and Pengelley (1996), Harvey (1988), Kenelly (1996), Kenelly and Eslinger (1988), Kolata (1988), Lathrop (1988), Layton (1988), Levin (1988), Ostebee (1993), Schoenfeld (1995), Smith (1996), Starr (1988), Tucker and Leitzel (1995), Watnick (1993), and Zorn and Viktora (1988).

Another pervasive goal is to have calculus contribute to the broad aims of undergraduate, liberal education (Alberts, 1988; Buccino & Rosenstein, 1988; O'Meara, 1988; Starr, 1988; Steen, 1988b) – that is to help students think clearly (Davis, 1988; Starr, 1988; Steen, 1988b), to communicate effectively (Steen, 1988b), both mathematically and scientifically (Fulton, 1988), and to wrestle with complex problems (Hallett, 1996; Steen, 1988b; Tucker & Leitzel, 1995).

Other universal goals include cultivation of a fascination with mathematics and of an appreciation (Flashman, 1996; Harvey, 1988; Hodgson, 1988; Levin, 1988) for mathematics, including its power (Anderson, 1988; Buccino & Rosenstein, 1988; Miller, 1995; Salamon, 1988), beauty (Anderson, 1988; Archer & Armstrong, 1988; Barrett & Teles, 1988; Buccino & Rosenstein, 1988; Salamon, 1988), application (Cipra, 1988), and history (Barrett & Teles, 1988; International Commission on Mathematical Instruction [ICMI], 1998). What is needed is a calculus which provides a body of understanding that contributes to the flexibility and adaptability required of scientists, engineers and managers (White, 1988). Such a calculus course must focus on intellectual mastery of the subject (Ralston, 1988), must develop increased analytical ability (Egerer

& Cannon, 1988; Prichett, 1988; Tucker & Leitzel, 1995), develop mathematical maturity (Brito & Goldberg, 1988; Haines & Boutilier, 1988) reasoning and logic (Davis, 1988; Harvey, 1988; Prichett, 1988), and emphasize (and illustrate) the way mathematicians do (and think about) mathematics (Archer & Armstrong, 1988; Lathrop, 1988) – in other words, develop higher order thinking skills (Harvey, 1988). A revitalized calculus course should have students and faculty involved together in learning (Davis, 1988), based on an apprenticeship system, in which students have laboratory experiences (Alarcón & Stoudt 1997, Bressoud (1996), Dodge, 1988; Ross, 1996) and sustained, direct contact with real mathematics professors (Starr, 1988). Calculus must become more dynamic (Davis, 1988) and intuitive (Archer & Armstrong, 1988; Armstrong et al., 1994; Dodge, 1988) in approach, but in doing so must maintain a balance between the teaching of new skills and the development of mathematical and intuitive reasoning (Stevenson, 1988). Finally, it must return a sense of discovery and excitement to calculus classrooms (Buccino & Rosenstein, 1988; Cipra, 1988; Ferritor, 1988) and lead to the development of students who pose, and actively investigate, *what if* questions (Cipra, 1988; Moskowitz, 1988). According to Lovelock and Newell (1988):

> What is important is that we structure the curriculum so as to nurture the students' abilities to develop clear thinking, do word problems and be willing to use modern technology with confidence. Former students who have some work experience are far more apt to say that it is their ability to think and learn rather than their precise knowledge of a given topic which is important to them in later life. (p.163)

Hence it is clear that calculus reform must place strong emphasis on reasoning, logic, investigation, adaptability, flexible and higher order thinking skills, and real learning and understanding.

Another goal of a new calculus is emphasis on writing. Steen (1988b) notes that even in calculus courses which are very well done, it is probably the case that students can get a grade of B, or maybe even an A, without ever writing a complete sentence. Priestley (1988) states that the most overlooked shortcoming in the teaching of mathematics is the failure of teachers to insist that students justify their answers – if not in complete sentences, then at least with a few suggestive English phrases. Buccino and Rosenstein (1988) claim that such writing enhances students' higher-order thinking skills, forces students to consider their results, and increases students' appreciation for the problems of writing for the benefit of others. David Smith, of Duke University, states that "failure to read and analyze instructions prevents students from getting started on a problem, and their ability to understand a solution process is related to their ability to explain in English what they have done" (cited in Cipra, 1988, pp. 101-102). Prichett (1988) notes that the expository skills of most college students are dismal and that it is time to embrace writing as a primary initiative in calculus. Harvey (1988) sees a revitalized introductory calculus course as one in which students become more precise in written presentations. Others authors, such as Bressoud (1996), Carlson and Roberts (1996), Fulton (1988), Gehrke and Pengelley (1996), Kenelly (1996), Layton (1988), Ostebee (1993), Ross (1996), Schoenfeld (1995), Smith (1996), and Tucker and Leitzel (1995), agree with the importance of writing in calculus.

Another important goal is the provision of meaningful applications in calculus (Bossert & Chinn, 1988; Carlson & Gulick, 1988; Chrobak, 1988; Cipra, 1988; ICMI, 1998; Miller, 1995; Malcom & Treisman, 1988; Ralston, 1988; Ross, 1996; Young & Blumenthal, 1988; Watkins, 1988), provided these applications are nonroutine, troublesome to students, and require considerable discourse with knowledgeable teaching staff (Fulton, 1988). However, a lack of total agreement on the importance of meaningful applications in a reformed calculus is illustrated by Boyce (1988) who states:

> I am not particularly enthusiastic about 'realistic' applications. They must often be couched in terminology unfamiliar to many students and require too much time to describe the underlying problem. It is better to use simple problems and models (even if 'unrealistic') so that everyone can understand them. (p. 42)

Carr (1992) also notes that a difficulty with problem solving is that teachers must rely on students having a sufficiently good background in other non-mathematical subjects.

Encompassed in the goal of meaningful applications is the goal of relevance in mathematics. Levin (1988) notes that biologists, who take calculus as freshmen, have forgotten all they learned long before college graduation. Levin insists that our goal in teaching mathematics should be to demonstrate relevance through choice of examples and topics, method of presentation and emphasis on concepts. Other authors such as Bailey and Chambers (1996), Gehrke and Pengelley (1996), Kenelly and Eslinger (1988), Prichett (1988), Ralston (1988), Smith (1996), and Watnick (1993) support this view. Levin also asserts that students must be able to distinguish whether what is learned is of general relevance and applicability, or whether it is particular to the application presented.

Also related to the goal of meaningful applications is the goal of problem solving. Young and Blumenthal (1988) state that word problems from client disciplines which require students to think about the meaning of their solution are important in calculus. Barrett and Teles (1988) assert that students must be taught to think about new problems as well as solve template problems. However, Curtis and Northcutt (1988), and Tucker and Leitzel (1995) contend that these problems should be more realistic than has generally been the case and that the parameters of at least some of these problems not be so carefully chosen. This would preclude all solutions being given in a closed form. Zorn, (in Cipra, 1988) suggests that students should be eased into the practice of problem solving by giving progressively more open-ended problems. Ponzo (cited in Cipra, 1988, p.96) supports the idea of 'long winded' solutions, allowing the computer to handle calculations, leaving the student to concentrate on problem set up and the sequence of formal operations that will lead to a solution. Lathrop (1988) maintains that the course should contain periods in which experts are confronted with new problems and that these experts describe their thinking as they attempt a solution. Lovelock and Newell (1988) assert that it should be emphasized that mathematics is an experimental subject and that most real problems are not solved correctly the first time. They believe teachers should emphasize that the answer alone is not good enough – students need to explain their logic, and the meaning of their solution. They should also be encouraged to examine the reasonableness of their answers (Boyce, 1988; Haines & Boutilier, 1988; Kolata, 1988;

Lovelock & Newell, 1988). Problems with no solution should also be given since, according to Lovelock and Newell and Alarcón and Stoudt (1997), most students have the impression that all problems have solutions. Erdman and Malone (1988), Armstrong et al. (1994) and Watnick (1993) also support problem solving in calculus (see also Carr, 1992).

There is also considerable support for mathematical modeling, especially among biologists. Brito and Goldberg (1988) note a general agreement that it is essential for students to represent particular problems in the language of mathematics. Egerer and Cannon (1988) state that a mathematics course should provide students with the opportunity to construct, articulate and interpret formal models in their own discipline. Levin (1988) asserts that the most important mathematical problems facing biologists are conceptual and involve the proper formulation of a model. He also contends that a goal of calculus should be to provide biologists with a better understanding of mathematical models. (see also Boyce, 1988; Bossert & Chinn, 1988; Bucchino & Rosenstein, 1988; Carlson & Gulick, 1988; ICMI, 1998; Ross, 1996; Tucker & Leitzel, 1995; Young, 1988; Young & Blumenthal, 1988)

Homework, consisting of nonstandard questions, and regular and extensive feedback constitute two other important goals of a nontraditional calculus course. Fulton (1988) states that there should be early and frequent assessment of each student's grasp of concepts as well as regular, often extensive, conferences with students. Strang (1988) notes that, in many calculus courses, students get no feedback to homework assignments and he asserts that we may expect too much of students to work well without recognition.

Buccino and Rosenstein (1988) claim that graded homework provides valuable information to the student about course objectives and teacher expectations and provides the teacher with information about class and individual progress. Curtis and Northcutt (1988) contend that understanding and communication of ideas on the part of the student will not be achieved unless the student receives high quality feedback from the instructor. Brito and Goldberg (1988), Bressoud (1996), Kolata (1988), Schoenfeld (1995), and Watkins (1988) also support the need for frequent, high quality feedback in calculus courses.

Another important goal is that of interactive approaches to teaching and learning in calculus. Kolata (1998) claims that students need the personal involvement of a professor or teaching assistant to do well in calculus. Finkel and Monk claim that "collective work is a key ingredient to intellectual growth" (cited in Cipra, 1988, p. 101). Douglas asserts that calculus cannot be taught in an impersonal environment (in Kolata, 1988). Salamon (1988) claims that help sessions and drop-in mathematics labs would go a long way to giving students control over their learning, as would 20 minutes for questions before and after each lecture. Malcom and Treisman (1988) report that a significant number of Chinese students achieved proficiency in mathematics through work in informal study groups, which provided feedback about the quality of their work and aided them in constructing concepts. The two authors also state that group work in which the instructor enters into conversation with students is important in that it provides important insights into the student's thought processes. Alarcón and Stoudt (1997), Armstrong et al. (1994), Bailey and Chambers (1996), Carr (1992), Dodge (1988), Fulton

(1988), Gehrke and Pengelley (1996), Gillman (1988), Newman and Poiani (1988), Ross (1996), Schoenfeld (1995), Smith (1996), Tucker and Leitzel (1995), and Watnick (1993) also support interaction in calculus courses.

Time is also deemed to be important. Haines and Boutilier (1988) state that calculus courses may be improved by giving students time to linger over concepts and consider the consequences of their thoughts and actions. Bradburn (1988), commenting on how the use of computers and symbolic manipulation packages can facilitate the teaching and learning of calculus, asserts that students will still need time to think. J. Ryan (1992), who recommends calculus teachers take time with early concepts, notes that allowing time for gradual development may produce long-term dividends in student performance and satisfaction. In supporting Bradburn's view, Ryan notes that technology can blur underlying ideas just as much as traditional delivery techniques if time for reflection is not built into the process. Gillman (1988) and Watnick (1993) advocate the inclusion of more time in calculus for students to sit back and reflect, to mull over ideas and express them in their own words.

It is also purported that high standards and expectations should characterize a calculus course. Both the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) recommend that the calculus course offered in the 12th grade be treated as a college level course. Malcom and Treisman claim that we should have expectations of competence of calculus students, and note that student performance in the authors' workshop program justifies this assumption of competence. Small (1988) asserts that "offering a watered-down college level course with no expectation of students earning advanced placement [is] not considered to be [an] acceptable option" (p. 225). Newman and Poiani (1988) declare that the maintenance of standards of quality and high levels of expectations are essential for all students in mathematics courses. Bailey and Chambers (1996) support emphasis on learning which requires students to achieve at established proficiency levels relating to subject matter.

There is sharp disagreement of the nature, and even the inclusion, of proof in a first calculus course. It is argued that formal proofs are essential to attract strong students (see Wilson & Albers, 1988), while others think proofs should be done in calculus using either intuitive *or* formally constructed premises (Bradburn, 1988). Horn (1988) asserts that teachers must "develop and emphasize proofs, not from first principles, but from agreed-on intuitive principles" (p. 21). However, Bradburn suggests that the course should include convincing examples of cases where intuition led to incorrect conclusions. Lathrop (1988) contends that proof should be reserved for second courses in calculus, and even then should be examined in the context of examples relating the significance of the result obtained to practical applications. There is agreement however, that it is essential that statements of proofs be given in a clear, concise form and that changes in the hypotheses be explored to help motivate theorems (Wilson & Albers, 1988).

There is also universal agreement for an increased emphasis on numerical methods. Redish, (1988) states that:

> Practical numerical methods form the heart of computer approaches to real-world problems, yet these are consistently ignored in the traditional introductory sequence. The problems of numerical integration and differentiation are

suppressed in favor of extensive discussions of how to differentiate and integrate large numbers of special cases, despite the fact that the real-world problems most scientists face will almost certainly have to be treated numerically a large fraction of the time. (p. 111)

There is also support for numerical methods from Bossert and Chinn (1988), Carlson and Gulick (1988), Horn (1988), Kolata (1988), Levin (1988), Tucker and Leitzel (1995), Wilson and Albers (1988), and Young and Blumenthal (1988).

There are also numerous other goals for a revitalized calculus course. These include: (a) the provision of exercises requiring good judgement (Compton, 1988; Davis, 1988; Hodgson, 1988); (b) creating links to other disciplines (Bailey & Chambers, 1996; Bossert & Chinn, 1988; Carlson & Roberts, 1996; Carr 1992; Knight, 1988; Malcom & Treisman, 1988); (c) requiring estimation and approximation (Lovelock & Newell, 1988; Salamon, 1988; Watkins, 1988); (d) error estimation (Boyce, 1988); (e) class presentations by students (Bressoud (1996), Fulton, 1988; Kenelly, 1996; Ross, 1996); (f) qualitative analysis (Bossert & Chinn, 1988; Kolata, 1988; Redish,1988; Tucker & Leitzel, 1995; Wilson & Albers, 1988); (g) reading calculus (Buccino & Rosenstein, 1988; Gehrke & Pengelley, 1996; Haines & Boutilier, 1988; Smith, 1996); (h) smaller class size (Cipra, 1988; Hughes, 1988; Newman & Poiani, 1988); (i) projects (Bressoud, 1996; Gehrke & Pengelley, 1996; Tucker & Leitzel, 1995); (j) an appropriate mix of routine practice, graded homework exercises and exemplary applications (Haines & Boutilier, 1988); and (k) the development of intuition (Buccino & Rosenstein, 1988; Cipra, 1988; J. Ryan, 1992). There is also support for the introduction of elements of discrete mathematics (Carlson & Gulick, 1988; Davis, 1988; Erdman & Malone, 1988; Watkins, 1988), and for student training in the detection of anomalies (errors and invariants) during the processing of information (Lathrop, 1988).

Layton (1988) states that the goals of first-year calculus include:

> The ability to give a coherent mathematical argument and the ability to be able not only to give answers but also to justify them. In addition, calculus should teach students how to apply mathematics in different contexts, to abstract and generalize, to analyze quantitatively and qualitatively. Students should learn to read mathematics on their own. In calculus they must also learn mechanical skills, both by hand and by machine.
>
> As for things to know, students must understand the fundamental concepts of calculus: change and stasis, behavior at an instant and behavior in the average, and approximation and error. Students must know the vocabulary of calculus used to describe these concepts, and they should feel comfortable with that vocabulary when it is used in other disciplines (pp. 150-151).

According to Anderson (1988), what we seek in a revitalized calculus course is a balance between old and new. To achieve this we must design our educational practices to conform to students' future needs in the post-calculus learning environment and in the workplace.

Technology and Calculus Reform

A report from the National Science Foundation (1988) states that " the explosion of new applications of mathematics and the impact of computers require major change in undergraduate mathematics" (p. 221). According to Moskowitz (1988), these changes should include a greater focus on calculus concepts, modeling, and interpretation and a relegation of computation to computers. These technologies are important because they can enhance a student's understanding of the fundamental ideas of calculus (Dodge, 1988). Fulton (1988) claims that technology can be used to instill new calculus concepts such as approximation, estimation, error analysis, asymptotic behavior and goodness of fit, in addition to the current concepts of change, limit, and summation. Zorn and Viktora (1988) describe how technology may affect calculus curriculum and pedagogy. The authors state that easy access to technology may (a) allow more realistic applications, (b) cause some topics and methods to become obsolete and allow others to replace them, (c) create a more active experimental attitude in students, (d) lead to deeper, more flexible understandings, (e) develop student intuition, (f) support qualitative reasoning, and (g) foster more effective problems solving. Harvey (1988) contends that the use of technology (i.e. graphics calculator) can permit expansion of testing higher-order thinking and problem solving skills. Gillman (1988), commenting on student practice and feedback, writes that "a well-designed program, with thoughtful conditional branching, will offer guidance while at the same time allowing students to pick the topics they need practice on. The instructor is [then] freed to devote full time to the exchange of ideas" (p. 218). Many other authors support the prudent use of technology in calculus, including

Archer and Armstrong (1988), Bailey and Chambers (1996), Barrett and Teles (1988), Buck (1988), Carlson and Gulick (1988), Chrobak (1988), Cipra (1988), Curtis and Northcutt (1988), Davis (1988), Gillman (1988), Kolata (1988), Layton (1988), Lovelock and Newell (1988), Newman and Poiani (1988), Ostebee (1993), Ralston (1988), Ross (1996), Steen (1988c) and Wilson and Albers (1988).

Several authors urge caution in, and express concern about, the use of technology in calculus. Zorn and Viktora (1988) note that it would be a mistake to add more material to the already overcrowded syllabus of a course when time is saved through the use of technology. Bradburn (1988) states that care must be taken so that technology does not take on the role of performing "Mathemagic" (p. 155). White (in Cipra, 1988) notes that, instead of spending the time saved by the use of graphics calculators on understanding underlying concepts, the time is sometimes used simply to show students which buttons to push. Haines and Boutilier (1988), and Newman and Poiani (1988) advocate that the use of technology cannot be an add-on, it must be integrated properly into the course.

There is also a fear of the "black box syndrome"; the use of mechanical systems without understanding of the underlying concepts. Boyce (1988) questions if we should permit students to use symbolic manipulation packages without understanding what these packages are doing. Haines and Boutilier (1988) assert that concepts must be developed and understood before being used on a calculator or computer, which can then be used to enhance and expand the understanding of these concepts. Egerer and Cannon (1988) question whether "black box" technology is a suitable substitute for understanding concepts and procedures. There are those, however, who contend that such systems allow students to concentrate on higher-level reasoning, instead of on lower level manipulative skills.

Given the cautions and fears of some authors, technology can benefit calculus learning in many ways. According to Zorn (1988), technology can create better mathematical learning by (a) making undergraduate mathematics more like real mathematics, (b) better illustrating mathematical ideas, (c) helping students work examples, (d) aiding in the study of algorithms as opposed to the performance of algorithms, (e) supporting more varied, realistic and illuminating applications, (f) improving geometric intuition, (g) encouraging experiments, (h) facilitating analysis (i) teaching approximation, (j) preparing students to compute effectively, but skeptically, (k) showing the mathematical significance of the computer revolution, and (l) making higher-level mathematics accessible to students. Hence technology has the potential to aid in attaining many of the goals of a revitalized calculus course.

Chapter Four: Advanced Mathematical Thinking

Introduction

Tall (1995) asserts that advanced mathematical thinking involves using cognitive structures produced by a wide range of mathematical activities to construct new ideas that build on and extend an ever-growing system of established theorems. Dreyfus (1991) states that advanced mathematical thinking is extremely complex, requiring the intricate interaction of a large number of component processes. He notes no sharp distinction between many of the processes of elementary and advanced mathematical thinking. Processes such as abstraction, representation, analysis and visualization are present in elementary mathematical thinking as well as in advanced thinking. However, he identifies the management of complex concepts as a distinguishing feature of advanced mathematical thinking. Tall (1995) writes that "elementary mathematical thinking becomes advanced mathematical thinking when the concept images in the cognitive structure are reformulated as concept definitions [the mathematically accepted definition] and used to construct formal concepts that are part of a systematic body of shared mathematical knowledge" (p.61). Tall defines concept image as the term used to describe the "total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p. 7). Tall (1991, 1992) also identifies the possibility of formal definition and deduction as another factor which distinguishes advanced mathematical thinking. The move from elementary to advanced mathematical thinking involves a significant transition: that from describing to defining and from convincing to proving in a logical manner based on those definitions.

Sources of Obstacles in Learning Mathematics

Tall (1991) asserts that it is often the incongruity between concept image and concept definition which is the source of cognitive obstacles for learners. Learners form a personal concept image which may or may not be equivalent to the formal concept definition. It is the concept image instead of the concept definition which receives emphasis (Vinner, 1992), and this may result in a lack of conceptual understanding on the part of the student. According to Tall (1992), the move to more advanced mathematical thinking involves a difficult transition from the development of concepts on an intuitive basis, founded on experience, to one where concepts are specified by formal definitions and their properties reconstructed through logical deductions. The growing body of deductive knowledge will exist simultaneously with earlier experiences and properties and this can produce cognitive conflicts which act as obstacles to learning.

Students may also never experience the full range of thought processes commonly used by mathematicians and this may be the result of the instruction that students receive. Skemp states that students are typically taught "the products of mathematical thought instead of the process of mathematical thinking" (cited in Tall, 1991, p. 3). Instructors tend to present mathematics in its final, polished form, following the standard sequence of theorem – proof - application, even though they are aware that mathematics is created through processes such as intuition, trial and error, conjecturing, and testing. Such presentation of mathematical products enables a well-planned and scheduled course presentation but lacks flexibility in its adaptability to students. As well, this logical sequence of instruction may not be appropriate for the learner's cognitive development.

The result of this approach is that most students learn to carry out large numbers of standard procedures, cast in precisely defined formalisms, for obtaining answers to clearly delimited classes of exercises. Many "successful" students end up with considerable mathematical knowledge but without the ability to use their knowledge in a flexible manner and transfer it to unknown problem types; that is, they lack the methodology of the working mathematician (Dreyfus, 1991).

Language may also be a barrier to understanding in mathematics. For example, the connotations of the word "limit" in everyday life are often at odds with the mathematical idea of limit and may lead to unavoidable misconceptions of the limit concept. As well, terminology such as "gets close to" and "approaches" is often used in the development of the limit concept in mathematics and carries the implication that an expression can never equal the limit value. Wells (1999) has published a 200 page *Handbook of Mathematical Discourse* which is a compilation of mathematical usage with a focus on usage which causes problem for mathematics students - an example being the logical errors which are a result of the use of everyday English language and its meaning in a mathematical context. The author notes that mathematical English language may be foreign to students as it uses familiar words and grammatical constructions which may have meanings which are very different from those to which students are accustomed.

Concepts Which Cause Problems in Calculus

Research has identified a variety of calculus concepts which cause problems for students. Rassian and Vinner (1997), in a study of high school students, found that the "special case" approach to odd and even functions caused the formation of concept images which created serious difficulties in the formulation and the application of concept definitions. Davydov (in Binns, 1994) notes that this inductive or special to general approach has failed in Soviet schools. Bills (1996) and Meel (1997) offer an explanation, referring to psychological evidence produced by Tall and Vinner (1981) and Tall (1989), which indicates that irrelevant attributes of examples from which a concept has been abstracted are not forgotten once the concept is formed. These attributes are retained as part of the concept image and can form the basis for obstacles to the understanding of mathematical concepts.

Ursini and Trigueros (1997), in a study of starting college students, found that the majority of students were restricted to the action concept of variable and that this conception of variable precludes some procedures from being interiorized. A procedure is interiorized when the learner no longer has to perform the operation in order to think about the procedure. White and Mitchelmore (1996) noted that such students are still at the condensation phase of their development of the concept of variable and concluded that an abstract-general concept of variable at or near the point of reification is necessary for the successful study of calculus. Condensation refers to the stage where a complicated process is condensed into a form that becomes easier to cognitively manage. Reification is the stage where concepts are conceived as objects (For an expanded discussion of interiorization, condensation and reification, see the next section). Mamona-Downs (1997), in a study of first year university students, asserted that an understanding of variable is necessary for the objectification of the concept of function, which in turn is crucial for the understanding of calculus concepts. Trigueros, Ursini and Reyes (1996) studied starting college students and found that most did not have the understanding of variable required for advanced mathematical thinking.

Crawford, Gordon, Nicolas and Prosser (1994), studying first year mathematics students, found that students exhibited a wide range of awareness of the concept of function but that most focused on the representation of function rather than its essential meaning. DeMarois and Tall (1996), Dreyfus and Eisenberg (1992), Eisenberg (1991), Hollar (1996), Rassian and Vinner (1998), Vinner (1983) and Vinner and Dreyfus (1989) have identified other problems with functions.

Research has also revealed student problems with the concepts of: (a) proof (Finlow-Bates, 1994), (b) infinity (Shama & Movshovitz-Hadar, 1994; Tirosh, 1991; Tsamir, 1992), (c) rate of change (Martin, 1996; Orton, 1984; Porzio, 1997), (d) average rate and average value (Bezuidenhout, Human & Olivier, 1998), (e) tangent (Vinner, 1982; Tall, 1987), (f) differential equations (Esquinca, 1996; Mochon, 1996; Rasmussen, 1998), (g) the chain rule (Vidokovic & Czarnocha, 1996), (h) derivative (Ferrini-Mundy & Graham, 1994; Porzio, 1997; Risnes, 1997; Villarreal & Borba, 1998) and (i) limits (Azcarate & Espinoza, 1995; Cornu, 1991). For a general discussion of student difficulties and misconceptions see Thomas and Hong (1996) and Naidoo (1998).

Conceptual Understanding

Dubinsky, Czarnocha, Prabhu and Vidakovic (1999) identify five learning theories found in research literature. These include Sfard's operational/structural characterization (Sfard, 1991), the concept image/concept definition dichotomy of Tall and Vinner (Tall & Vinner, 1981), the didactical engineering view (Artigue, 1991), the procept notion of Gray and Tall (1991, 1994) and Dubinsky's own Action Process Object Schema (APOS) Theory (Dubinsky, 1991, 1992; Dubinsky et al., 1999). The sections which follow focus mainly on APOS theory, although at least partial descriptions are included of most of the other four theories. The section ends with a critique of APOS theory given by David Tall, in which he acknowledges the usefulness and success of APOS theory in the development of undergraduate mathematics curricula, but questions the universal applicability of the theory, especially in the learning of geometry.

Given that the concept image is often at odds with the formal concept definition, and that cognitive obstacles are frequently the result of this incongruency, how are the conceptual entities, that are the essence of advanced mathematics, formed? David Tall, in the preface of the book *Advanced Mathematical Thinking*, asserts that conceptual entities are formed through the process of reflective abstraction, which, according to Dubinsky (1991) is a Piagetian concept used to describe the construction of logico-mathematical structures by an individual during the course of cognitive development. Dubinsky (1999) identifies and briefly describes the main mental constructions that need to be made in the learning of mathematics. He describes an *action* as a transformation which is a reaction to stimuli which the subject perceives as external. The author notes that although the action conception is limited, it is an important basis for understanding a concept. An action becomes a *process* when the individual reflects on the action and interiorizes it. The individual can then establish control over it. When the individual reflects on the operations applied to a process, becomes aware of the process in its totality, realizes that the process can be transformed and is able to construct the transformations, the process is thought of as an *object*. Finally, Dubinsky defines a *schema* as a more or less coherent collection of processes, objects and other schema that is invoked to deal with a new mathematical problem situation. He maintains that learning involves applying reflective abstraction to existing schema in order to construct new schema for understanding concepts.

Dubinsky (1991), who emphasizes its constructive aspects, asserts that reflective abstraction is the construction of mental objects and of mental actions on these objects. He identifies five kinds of construction mechanisms which are important for advanced mathematical thinking; *interiorization, composition or coordination, encapsulation, generalization* and *reversal.*

Goodson-Espy (1998) describes Sfard's three stages of concept development, which are similar to that of Dubinsky's. He describes stage one (*interiorization*) as the stage where the learner performs operations on lower level mathematical objects. Eventually, the learner becomes familiar with these processes and can think about them without actually having to carry out the process. When this occurs, the process is said to have been interiorized. For example, a learner may initially find the sum of 5 and 3 by beginning with five fingers and counting three more fingers in succession to arrive at the sum of 8. Eventually, the learner can perform this operation without having to carry out the finger counting process and will arrive at the answer of 8. At this point the process of addition of natural numbers has been interiorized.

New processes in a schema can be constructed by the *coordination* of two or more processes (Dubinsky, 1992). For example, the process of finding $(x+3)^2$ can be viewed as the coordination of the process of adding 3 to a number and the process of squaring.

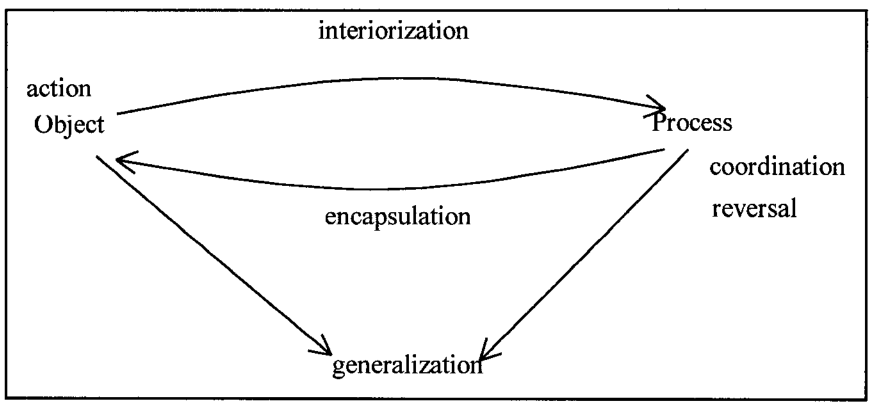
Dubinsky (1991, 1992) defines *encapsulation* as the conversion of a dynamic process into a static object. For example, the expression $7x+5$ can be viewed as the process of multiplying a number by 7 and then adding 5 or as the binomial $7x+5$ (an object). When $7x+5$ can be conceived of as an object as opposed to a process, encapsulation is said to have occurred. This construction is viewed as the most important, and the most difficult, for students of mathematics (see also Goodson-Espy, 1998). The ability to flexibly view a concept as process or object is what makes mathematical thinking powerful (Tall, 1992). The symbol $7x+5$ becomes a "procept" when $7x+5$ comes to be viewed flexibly as either a process or a concept (binomial) (Gray & Tall, 1994, p. 121).

Generalization occurs when the learner can apply an existing schema to a wider collection of phenomena than was previously the case. For example, when the schema for addition is applied to obtain multiplication, generalization of the schema for addition has occurred. Tall (1991) makes a distinction between the different types of generalization in accordance with the cognitive activities involved. An *expansive* generalization is one which extends the learner's existing cognitive structure without requiring changes in current ideas. A *reconstructive* generalization requires reconstruction of the existing cognitive structure.

When a learner has interiorized a process, they can think about the *reversal* of this process to create a new process. For example, the process of subtraction can be constructed by reversing the process of addition.

The construction of new objects and processes out of existing ones can be described in terms of the five constructive aspects of reflective abstraction (see Figure 1).

Figure 1. Construction of objects and processes.[1]



Note. From "A Learning Theory Approach to Calculus," by E. Dubinsky, 1992, Symbolic Computation in Undergraduate Mathematics Education, p. 47. Copyright 1992 by Mathematics Association of America. Reprinted with permission.

In the construction of new, higher level objects and processes, actions are performed on lower level objects, and these actions can become interiorized as a new process. This new process is eventually encapsulated as a higher-level object which then has other actions performed on it which are interiorized to form new higher-level processes. This cycle continues in an upward spiral of ever increasing sophistication (Dubinsky, 1992).

Dubinsky (1991) gives a more complete explanation of Figure 1. The essence of his explanation is that actions are performed on objects and these actions are interiorized to form new processes. However, working with existing processes can also form new processes. One way of making new processes is to compose or coordinate two or more existing processes. Another way is to reverse an existing interiorized process to create a new one. Processes can then be encapsulated as objects which can then be used for higher-level interiorization. Note that a schema can be extended by the generalization of both processes and objects.

Dubinsky, Czarnocha, Prabhu and Vidakovic (1999) acknowledge that this explanation is potentially misleading in that it may suggest a linear progression from action to process to object to schema. Dubinsky et al. explain that in fact, understanding mathematical concepts often appears more like a dialectic in which there is partial development at one level, passage to the next level, and movement between levels, with development at each level influencing that at both higher and lower levels.

A process similar to that of Dubinsky's is described by Goodson-Espy (1998) in her summary of Sfard's theory of reification. Stage one (*interiorization*) has been described earlier and is essentially the same as in Dubinsky's APOS theory. The second stage (*condensation*) is reached when a complicated process is condensed into a form that becomes easier to use and think about. At this stage the learner can combine processes, make comparisons, and generalize. The new concept is actually born at this stage but remains tied to an algorithmic process. An important facet of this phase is the ability to alternate between the different representations of the concept. The final stage (*reification*), which is similar to Dubinsky's encapsulation stage, has been reached when the learner can conceive of the mathematical concept as a complete object with characteristics of its own. For example, when the learner is aware of, and thoroughly understands, the different representations of a linear function and can compare linear functions with other functions, such as quadratics, the concept of linear functions has been reified. Sfard notes that reification usually occurs as a sudden shift and is frequently the hardest stage for learners to obtain. She also offers an explanation for this difficulty. There would be no reason for the reification of a process unless this resultant object was to be acted upon and this action interiorized to form a higher-level process. However, for this higher-level interiorization to occur, the existence of the object acted upon is required. Hence lower level reification and higher-level interiorization are prerequisites for each other.

Given this difficulty, Dubinsky (1991) does offer a general four step instructional approach to foster conceptual thinking in mathematics. It consists of:

1. Observing students in the process of learning a topic to see their developing concept images.

2. Developing a genetic decomposition of the topic. A genetic decomposition of a topic consists of a description of possible mental construction methods for developing a schema for that topic (Vidakovic, 1996).

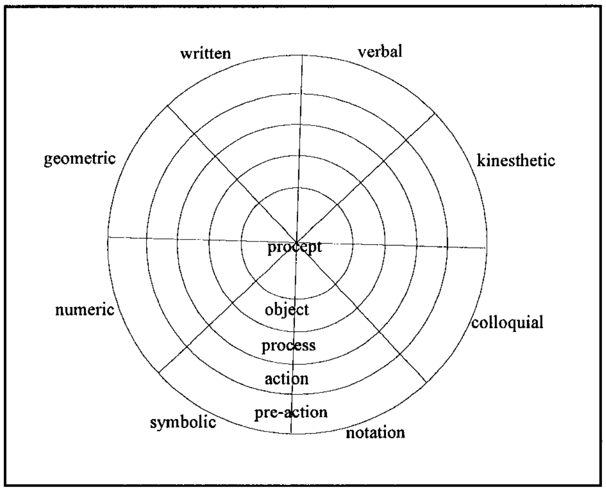
3. Designing instruction that will move students along the cognitive steps in the genetic decomposition (i.e. induce students to make the appropriate reflective abstractions).

4. Revising the genetic decomposition and instruction until stabilization occurs.

Dubinsky (1999) also describes a more specific pedagogical approach (called the *Activities, Class, Exercises (ACE) Teaching Cycle*), involving cooperative learning and computer programming, which is used to induce students to make the mental constructions necessary for understanding mathematics. It consists of group computer activities designed to foster the mental constructions proposed in the genetic composition, followed by more difficult tasks, with ideas reinforced by assigning relatively traditional exercises to be completed outside of class by teams of students.

DeMarois and Tall (1996), who studied community college students' conceptualization of function, offer a method to analyze the concept images of the learner in terms of facets and layers. The *facets* of a mathematical concept refer to the various ways of thinking about the concept and communicating it to others, including, but not limited to, verbal, written, kinesthetic, colloquial, notational conventions, numeric, symbolic and geometric aspects. Facets can be used to build up a description of the breadth dimension of a concept. *Layers* refer to the various levels in the depth dimension in the cognitive development of the concept from the pre-action layer to proceptual layer. A sample profile is given in Figure 2.

Figure 2. Facets and layers of a concept. [2]



Note. From "Facets and Layers of the Function Concept," by P. DeMarois and D. Tall, 1996, Proceedings of the Twentieth International Conference for the Psychology of Mathematics Education, Spain, 2, p. 298. Copyright 1996 by P. DeMarois and D. Tall. Reprinted with permission.

The depth of the learner's cognitive development for each facet of a concept can be indicated on the diagram to develop a student profile for that concept which could then be part of the first component of Dunbinsky's instructional approach to foster conceptual thinking in mathematics.

Tall (1995) offers an extension to the model of concept formation developed by Dubinsky. Tall describes the cognitive growth from elementary to more advanced mathematical thinking as two distinct, parallel developments. One visuo-spatial, starting with "perceptions of" objects, moving to verbal descriptions and deductions, and leading to proof. The other successive process-to-concept encapsulations, starting with "actions on" objects leading to the use of symbols both as processes and concepts. He notes that these developments can occur independently but that learners should be cognizant of the links which have been made between the two developments so they can develop a versatile approach which uses each to its best advantage. Tall (1999) also disputes the primacy of action over object in Dubinsky's APOS theory, especially in geometry and in the formal construction of knowledge from definition to deductions, which exemplifies advanced mathematical thinking. While acknowledging the success of APOS theory in the design of arithmetic, algebra and calculus curricula, Tall insists that at least a partial understanding of a concept can occur before the encapsulation of the mathematical processes involved in the concept. For example, Tall uses slope fields to help students build a visual solution to a first order differential equation and provide a skeletal cognitive schema for the solution process before they are introduced to specific numeric and symbolic solution processes.

Tall (1992) offers a more specific approach to alleviate some of the difficulty in dealing with the formal definitions to which learners are exposed in the transition to more advanced mathematical thinking. He suggests an approach that builds on concepts, called cognitive roots, which have the "dual role of being familiar to students and providing the basis for later mathematical development" (p. 497). For example, Tall argues that a concept such as limit is difficult to use as a basis for students' thinking and suggests that an idea such as "local linearity" (p. 497) may be more suitable as a cognitive root for calculus.

According to Tall (1992):

> True progress in making the transition to more advanced mathematical thinking can be achieved by helping students reflect on their own thinking processes and confront the conflicts that arise in moving to a richer context where old implicit beliefs no longer hold. Such intellectual growth is stimulated by flexible environments that furnish appropriate cognitive roots and help the student build a broader concept image. Oversimplified environments designed to protect students from confusion may only serve to provide implicit regularities that students abstract, causing serious conflict at a later stage.
>
> In taking students through the transition to advanced mathematical thinking, we should realize that the formalizing and systematizing of the mathematics is the final stage of mathematical thinking, not the total activity. (pp. 508-509)

Hence an appropriate learning environment is one which allows students time, to reflect on their thinking, and to face and resolve conflicts in their conceptual understanding. It must provide effective ways for students to develop a personal understanding of calculus and internal sources of conviction; that is, to become Connectors as opposed to Technicians and Collectors (Frid, 1992). Such an environment must also expose students to the full spectrum of mathematical activities from investigating through conjecturing to proving.

Chapter Five: Learning at a Distance

Introduction

Distance education has become much more prevalent as a result of the growing demand for educational opportunities designed for, and directed toward, specific target groups. As such, it has begun to move from a marginal to a more central role in education. Recent advances in educational technology are facilitating a shift from an industrial, mass production model of distance education to more personalized and individualized instruction with the potential for accommodating a variety of learning styles. However, designers and deliverers of distance education courses must overcome many hurdles before this shift can be fully achieved. This section identifies some of the challenges to learning at a distance, and provides some suggestions as to how the design and delivery of distance education courses can be enhanced.

Challenges to Learning at a Distance

According to Thorpe (1995a), one of the major challenges which distance education currently faces is its over-reliance on behaviorist approaches to teaching and learning. The author notes in particular that the view that course design can be applied irrespective of context, content and learners has not proved to be realistic or effective. She also notes that much of the current practice in the field is the product of the harnessing of information technologies and modern communication, and behaviorist and information processing models of teaching and learning. Courses consist of predetermined aims and behavioral objectives and the key measurement of success is the quantitative achievement of students at the end of the course. As well, the development of general learning abilities and self-awareness as a learner have typically not been made explicit goals of education courses. These goals become more crucial in distance education courses, which typically offer diminished contact with teachers, yet require proactive and independent learning for success.

Kasworm and Yao (1992) insist that students should engage in the development of meaningful and relevant learning with in-depth information processing and autonomous self-learning. In particular, they believe that learning should be an interactive, constructive process and learning strategies should reflect global, interactive and metacognitive actions. However, they maintain that most current distance education courses do not view learning from this perspective. Rumble (1992) states that most distance learning systems can be categorized as institution centered, as opposed to learner centered, and are designed to maximize the effectiveness of the educational process by treating learning as the processing, storage and retrieval of information. Schuemer (1993) states that many distance education courses are characterized by a high level of structuring, provide few incentives for the development of an active-constructive conception of learning and rather, lead to a passive-reproductive conception (see also Persons & Griffiths, 1992). This high level of structuring, along with the presentation of knowledge as a ready-made system, allows for a systems-oriented teaching method which runs the risk of degenerating into the 'spoon-feeding' of knowledge and may lead to a dependence on the teaching system instead of creating autonomous learners. This system-oriented method of presentation, coupled with great time pressure, leads learners to a kind of surface learning, aimed at reproduction, instead of leading them to a deeper learning, aimed at understanding. Bullen (1998) states that much of distance education is rooted in a transmission model of learning that inhibits the development of critical thinking. Learners passively assimilate knowledge rather than critically examine and construct it, based on their own experiences and previous knowledge.

In comparing some of the psychological differences between distance and conventional education, Schuemer (1993) notes that in distance education, the learner is usually isolated and does not benefit from the motivational factors arising from the contact or the competition with others in face-to-face classes, nor do they experience the immediate support and motivation of a teacher. This means that the learner must take over personal responsibility for his or her own learning. However, strong teacher control and a passive-reproductive learning conception may be serious obstacles to independent learning.

Uegama, Bullen and Neufeld (1992) identify two features of distance education which threaten the development of independent learners and highly developed thinking-reasoning skills. Uegama et al. note that most distance education courses are developed without regard for how they fit into programs of study or how they contribute to the broader goals of education. They also state that the most widely practiced models of course development are rooted in the behaviorist view of learning. They question whether it is possible to develop critical thinking abilities and learners who are self-directed and intellectually self-reliant in distance education courses with such a narrow focus and such a superficial view of learning.

Garland (1995), writing on the achievement of epistemological autonomy by students, notes that epistemological problems result from the incongruencies between the student's cognitive and affective perceptions of knowledge and the nature of the knowledge in the courses themselves. Epistemological autonomy involves the development of a personal sense of power over the creation and validation of knowledge. Many students assume that the source of knowledge is outside themselves, and that it is both created and sanctioned solely by external authorities. They believe that their role as individual learners is to passively receive knowledge and to reproduce it as accurately as possible. Epistemological autonomy is achieved when the learner becomes confident in their mathematical knowledge and skills, in their ability to apply their knowledge and skills in both routine and nonroutine mathematical tasks, and in their ability to understand mathematical ideas and concepts. Epistemological autonomy is also evidenced by a sense of self-efficacy (i.e. a confident self-image of themselves as successful in mathematics), and by a personal ownership of mathematics. Garland states that the student's epistemological stance is a screen through which new knowledge must be acquired and that this screen can become a barrier when the epistemological stance of a course's content or expectations is incompatible with that of the student. In some distance education courses, materials and behavioral objectives seem to suggest the content is something to be learned by rote, whereas assignments demand that higher level cognitive skills be applied in using the content knowledge as underpinnings for new personal understandings. Some students are able to make these epistemological adaptations, but for those who cannot, this epistemological gap becomes a significant factor in their decision to drop out.

If learning in the distance education context is to be successful, approaches must be used which develop general learning abilities, provide for and support interaction, encourage deep learning and help students achieve epistemological autonomy. These approaches must also be less generic and rigid in structure and be able to accommodate a variety of learning styles. The following section identifies and describes some methods through which distance learning can be improved.

The Enhancement of Learning in Distance Education

Deep versus Surface Approach to Learning

A description of deep and surface approaches to learning.

To improve the quality of the learner's educational experience, many studies illustrate the importance of encouraging students to adopt a deep approach to learning as opposed to a surface approach. According to Joughin, Lai and Cottman (1992), a deep approach to learning would generally mean that a student (a) is interested in the academic task, (b) searches for meaning in the task, (c) personalizes the task, (d) integrates aspects of the task into a whole, (e) sees relationships between this whole and previous knowledge and (f) tries to theorize about the task. A surface learning approach would mean that the student (a) sees the task as a demand to be met to reach some other goal, (b) sees the aspects of the task as discrete, (c) worries about the time it takes to compete the task, (d) avoids personal meanings in the task, and (e) relies on memorization (see also Kasworm, 1997). Thorpe (1995a), contends that a goal of distance education is to create courses in which learners are rewarded for demonstrating a deep approach to learning, in which they process new information actively and relate it to a developing structure of concepts and meanings in the domain.

Assessment and course design.

Studies show that assessment and course design are crucial for influencing students' approaches to learning. From her research, Thorpe (1995a), concluded that appropriately designed assessment (i.e. assessment which demands deep-level processing Holmberg (1995)) provides a powerful mechanism for stimulating and supporting reflection and deep learning. The opportunity for reflection, provided by assessment, allows further opportunities for the development of learning. Holmberg asserts that to support deep learning, instructors must direct students' attention towards the subject matter of the texts studied and away from the textual presentation as such. The author concludes that the mechanisms (besides assessment) which may enable this attention shift are far from obvious, but does suggest that the answer may lie in making learners conscious of their own learning and in using advance organizers and learning conversations.

Biggs (1994) presents four key aspects of course design which are likely to foster a deep approach to learning:

1. Motivational context: Students are likely to take a deep approach to learning if their motivation is intrinsic, so they should be involved in selecting and planning their learning so they can feel ownership of their work.

2. Learner activity: Students need to be active participants, as opposed to passive recipients, in learning and must be helped to reflect on their learning experiences, and to relate them to existing knowledge and understandings.

3. Interaction with others: Negotiations of meanings and learning through discussion are powerful ways to facilitate students to adopt a deep approach to learning.

4. A well structured knowledge base: New ideas and concepts must be related to students' existing knowledge and conceptual frameworks. As well, new material must be presented in an integrated whole structure.

Projects.

Morgan (1995) claims that project-based learning has the potential to embody the first three aspects of course design, and especially the first two - intrinsic motivation and learner activity. The author maintains that such projects should involve local issues where outcomes directly affect the student, should draw upon the experience and skills the learner already possesses, and should require that the learner develop new skills of inquiry, interpretation and advocacy. Thorpe (1995a) notes that project based courses and project assignments have been more generally successful at encouraging deep learning. Rowntree (1992) claims that an independent project may contain guidance about how to choose a topic, sources of data, techniques of analysis, forms of reporting, the role of the tutor, and so on, but should contain nothing of substance which has to be learned. This type of learning will counter tendencies towards transmission modes of teaching and help students take a deep approach to learning.

The use of metacognition.

Thorpe (1995b) asserts that the active use of metacognition will encourage a deep rather than a superficial approach to learning. If we become aware of our existing attitudes and knowledge on a particular topic, then we have prepared our minds to address more actively the new information we study on the same topic. In theory, this means we are more likely to search for the meaning of what is being said, and to make connections with our existing knowledge in order to develop a grasp of the topic. However, Schuemer (1993) notes that metacognitive monitoring processes do not have positive effects on learning and thinking in all situations. These processes can overburden the cognitive apparatus and therefore disturb the processing of information, as in cases where basic cognitive strategies and processes are not yet sufficiently mastered and additional metacognitive action leads to a split in attention. In support of Schuemer, Winne (1995) notes that in the early stages of skill development, students cannot time-share among the various self-regulatory skills and the learning process per se. The author warns that instructions to monitor one's learning in the early stages of skill development may hurt learning, particularly in low ability learners and learners with low prior knowledge.

Interaction in Distance Education

Kearsley (1995) states that "one of the most important elements of contemporary distance education is interaction. It is widely held that a high level of interaction is desirable and positively affects the effectiveness of any distance education course" (p. 366). Henri and Kaye (1993) and Lauzon (1992) note that without interaction among students and between students and instructors, distance education can only be an inferior imitation of the best face-to-face education because learners are unable to clarify and challenge assumptions and to construct meaning through dialogue. Scholdt, Zhang and Fulford (1995) assert that the types and quality of interaction provided in distance education is important because it affects learner satisfaction and perceived learning. Rodriguez (1995) found that personal contact between instructors and learners enhanced communications and improved teaching and learner interest in content matter. The author also found a slight desire for interaction between learners. This indicates the need to incorporate interpersonal interaction into distance education courses. Miller and Webster (1997) state that the variety of media currently available for distance education has led to the dividing of interaction into two categories: synchronous and asynchronous. Hillman, Willis and Gunawardena (1994) defined synchronous interaction as being "real-time, live and conversation-like during the instructional session" and asynchronous as "delayed, before or after the instructional session" (p. 46). In their study, Miller and Webster found that asynchronous learners did not perceive interaction to be as important as learners taking courses synchronously. However, the vast majority of those who did not complete the course were asynchronous learners. The authors hypothesize that even though asynchronous learners perceive interaction as less important, the lack of interaction may in fact be a contributing factor to their high attrition rate. This may indicate the importance of interaction in both synchronous and asynchronous learning.

Deekers, Cuskelly, Kemp and Phillips (1992) claim that introducing interactive communication into distance education offers several distinct advantages. First, students are able to test their knowledge and obtain immediate feedback. Students acknowledge the usefulness of hearing the questions and answers of other students as it not only provides a basis for comparison, but also stimulates interest and further learning. Second, students feel less isolated when they are able to see and hear their peers and instructor. These students frequently form study partnerships or groups, which increases their involvement in, and commitment to, the course. Third, this arrangement encourages students to develop interpersonal communication skills. However, Thorpe (1995b) notes it is the quality, and not the mere presence, of interaction which influences a student's decision to continue with a course. While interaction can create immediate identification with others in the group and can lessen isolation and anxiety, it can also make the learner feel excluded from the group, as when the learner discovers their background is different from that of the vast majority of the other members of the group. This can trigger powerful feelings that the course is not for them and may make the difference in the decision to continue or to drop out.

Henri and Hotte (1992) note that social interaction, which contributes positively to the learning process, can be brought about by computer-mediated communication (CMC). The authors state that students linked by CMC feel that they have learned from their exchanges with others and that teachers perceive it as an efficient and collaborative medium which promotes a more open learning process. Seaton (1993), in a review of research with three CMC projects, also concludes that CMC is an effective mode of creating an interactive community of learners. However, Hiltz (1993), in a study using software enhancements to CMC called The Virtual Classroom, found that results were superior for well-motivated and well-prepared students who took advantage of the opportunities provided for increased interaction with their instructor and with other students. The author maintains that students who are unable or unwilling to exploit this opportunity may do better in a traditionally delivered course. Kaput and Thompson (1994), identify interactivity as one of the three aspects of electronic technologies that enable a deep change in the experience of doing and learning mathematics. In interactive electronic media, student actions yield a reaction on the part of the machine, which in turn sets the stage for interpretation, reflection and further action on the part of the student. The authors predict a shift in the pedagogical tide towards increased student control of their learning environment as technologies become more powerful.

Moore (1992), writing on the use of teleconferencing in distance education, claims that it is the area of student interaction that is proving to be teleconferencing's main contribution to distance education. It is the dialogue by teleconference between students that is making possible the creation of knowledge by students and the high-level analysis, synthesis and critique of knowledge. However, Bullen (1998) identified three categories of factors which affect (negatively or positively) learner participation in computer conferencing:

1. The attributes of computer conferencing, such as many-to-many communication, place-independent group communication, time-independent group communication, text-based nature of communications, and computer-mediated learning.

2. The design and facilitation of computer conferencing activities, such as whether learners are required to contribute to discussions and whether instructors play a facilitative or directive role.

3. Student situational and dispositional factors. Dispositional factors include the learner's attitudes toward computer conferencing, distance education and the subject matter, as well as computer skills, comfort with the communication medium, the epistemological orientation of the course, and the motivational orientation of the learner. Situational factors include access to computer hardware and software, home situation and available study time.

The author concludes that the factors which affect participation have little to do with the technology of computer conferencing. Student characteristics such as their previous experience with distance education or with independent study, their cognitive maturity, and their experience with participatory and interactive learning environments seem to be the determinants for the successful implementation of computer conferencing characterized by high levels of participation, interaction and critical thinking.

Feedback in Distance Education

Directly related to the interactivity component of distance education is feedback. Blunt and Dent (1999), and Bernard and Naidu (1992) maintain that research has clearly established that learning is greatly enhanced when learners receive meaningful, immediate and continuous feedback. Thorpe (1995a) notes that constructivist and cognitive approaches to learning have worked well in combination with regular feedback in distance education undergraduate courses. L. Ryan (1992) found that students perceive that the provision of generous and constructive feedback on assessed work stimulates interest in the subject. The author suggests that objective and constructive feedback is a powerful tool that can be used to maintain high academic standards in distance education. Kulhavy (1977) (in Bernard & Naidu 1992) found that feedback prior to performing learning activities lowers the effects of the feedback. He concluded that feedback works best after the student has attempted the learning task.

Assignment Submission

Krane (1995) found a significantly higher completion rate in courses containing compulsory assignments and also found that course completion rate fell after the number of assignments exceeded seven or eight. The author asserts that prompt assignment return with constructive feedback is a decisive element for study success and goal achievement in courses at NKS Distance Education. Schuemer (1993) discusses the importance of using telecommunications to reduce the turn around time of submission of assignments or questions by learners. He concludes that a long return time decreases motivation and increases dropout rate. Boondao (1992) found that apart from previous mathematical background, the most powerful variable which influenced achievement was the use of assignments for submission.

Concept Maps

The uses and benefits of concept maps in distance education learning have been researched extensively in recent years. Naidu (1990) defines a concept map as a graphic arrangement of the key concepts in a body of subject matter with connecting lines labeled to show valid and meaningful relationships between the chosen concepts. A typical concept map contains four elements: (a) concepts, (b) propositions, (c) connecting lines, and (d) a hierarchical structure (Bernard & Naidu, 1992). Naidu and Bernard (1992a) state that concept maps have the potential for inducing deeper level processing of content. Naidu (1990) explains that concepts depend on their relationships with other concepts for meaning. A personal grasp of these relationships characterizes meaningful learning. Concept mapping, with its emphasis on linking concepts to other concepts, events and objects, has been shown to be an effective means for enhancing meaningful learning.

Hasemann and Mansfield (1995) state that concept maps provide information about how individual students relate concepts to form organized conceptual frameworks. They also assert that there is no completely correct or ideal concept map since they are idiosyncratic, and depend on the individual's prior learning experience and reflections on those experiences. The authors, in reference to a meta-analysis of the research on the effectiveness of concept maps by Al-Kunifed and Wandersee (1990), state that the meta-analysis showed that concept mapping had generally positive effects on student achievement and attitudes. The meta-analysis also provided evidence that the effectiveness of teacher-prepared or student-prepared concept maps differs very little in improving students' achievement. However, when students prepared their own concept maps, achievement gains were much higher if the students were required to supply the key terms necessary to construct the concept map. In their study of fourth and sixth graders, Hasemann and Mansfield found that although in the short-term concept maps showed changes in student understanding, delayed concept maps showed these changes may have been superficial and not embedded deeply in students' cognitive frameworks. Bernard and Naidu (1992) also note several studies (e.g. Novak, Gowin & Johansen, 1983) which found that concept mappers tend to perform lower on knowledge-type examinations initially, but outperform non-concept mappers later on. Bernard and Naidu explain this 'early deficit' by stating that continued practice with concept mapping is needed to affect achievement outcomes and they caution that the effects of concept mapping may not appear in the short term (Naidu & Bernard, 1992b). This may explain the results found by Hasemann and Mansfield (1995). Their subjects typically constructed a limited number (e.g. three) of concept maps and perhaps this number was only enough to create superficial, as opposed to a deep, understanding of the concepts in the study. Bernard and Naidu (1992) provide evidence for their claim in another study (Naidu & Bernard, 1992b) in which they found a significant increase in academic performance among nurses, enrolled in a distance education course, who completed at least eleven or twelve concept mapping exercises.

Bernard and Naidu (1992), and Naidu and Bernard (1992b) offer explanations for the increase in achievement of persistent concept mappers. These results may be attributable to the effects of the concept-mapping task itself, which requires students to deal with the material in more depth and translate it into personal meaning. However, time on task may be the cause of some of the increased achievement, since students who create concept maps frequently also spend more time with the content. Finally, since concept mapping is fundamentally a graphic procedure, subjects with a preference for visual or graphic learning techniques may have a tendency to persist with the strategy and perform better as a consequence.

Finally, Hereen and Collins (1993), in a study of the use of an electronic, shared, concept mapping tool in support of collaborative learning, state that concept mapping stimulates and supports four classes of cognitive processes: organization, elaboration, exploration and comprehension monitoring processes. The authors note that the collaboratively developed concept maps in their study are not representations of an individual's interpretation of some collection of information but instead serve as a shared knowledge base that functions as a cognitive tool for individuals and for the whole group.

Epistemological Autonomy

Garland (1995) states that helping students achieve epistemological autonomy is a cornerstone in fostering their self-direction and enabling them to exert personal control over their learning, and should be an essential component of effective distance education. In order to achieve epistemological autonomy, theories such as constructivism and situated cognition must be embedded in a curricular structure which supports such views. The components of such a structure include:

1. Taking into account learners' prior experience and knowledge.

2. Providing scaffolding for learners (i.e. providing a framework to connect and make sense of ideas and facts).

3. Embedding a 'teacher as guide' approach in the curriculum structure.

4. Solving realistic problems appropriate to the students' context.

5. Providing technologically rich, situated learning.

6. Providing a high degree of interactivity that includes extensive dialogue which involves the idea that humans in communication are engaged actively in the making and exchange of meaning and not merely in the transmission of messages.

7. Emphasizing critical thinking skills.

Self-Regulation

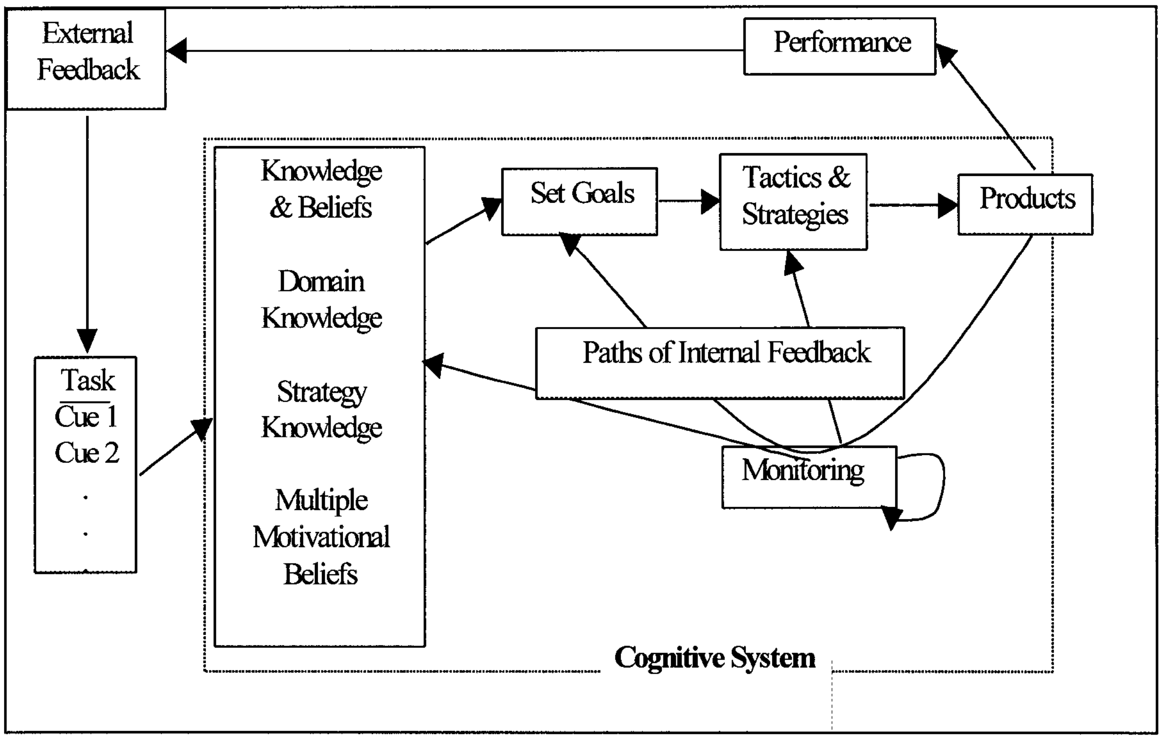
Butler and Winne (1995) note that there is unanimous agreement that the most effective learners are self-regulating. Schunk and Zimmerman (1994) define self-regulation as "the process whereby students activate and sustain cognitions, behaviors and affects, which are systematically oriented toward attainment of their goals" (p. 309). Boekaerts (1997) notes that self-regulated learners differ from those who need large amounts of external regulation in that self-regulators rely on internal resources to govern their own learning process and are aware of what they know and feel about the domain of study, including which general cognitive and motivational strategies are effective in attaining learning goals, how difficult it is to achieve mastery in the domain and whether they have the capacity and motivation to perform. The author notes that although there is a universal preference for self-regulating learners, the prevalent instructional model, with teacher as expert and learner as novice, does not enable students to develop their cognitive or motivational self-regulatory skills. This model assumes that: (a) students can activate or generate successful cognitive strategies in the time allotted by the teacher, (b) transfer will occur spontaneously during practice, and (c) students will gradually become independent of the teacher. However, transfer and the generation of context-sensitive strategies does not occur spontaneously and a significant amount of time is needed for students to become self-regulating learners. The author claims that for self-regulated learning to develop, teachers must create powerful learning environments in which students design their own learning experiments, and this implies that students should be motivated to actively participate in the learning process and construct their own knowledge base on the basis of direct and indirect learning experiences. Boekaerts describes three types of tasks which may help learners become self-regulating:

1. Procedural tasks which encourage students to activate declaratively encoded knowledge and proceduralize it.
2. Cognitive self-regulatory tasks which inspire students to time-share between the learning process and self-regulatory skills.
3. Motivational self-regulatory tasks which pertain to long-term learning goals and invite students to mentally represent these goals as well as their behavioral intentions.

Boekaerts found that students, who completed these tasks and received differential feedback and support from instructors, displayed significantly more deep level processing after six months than students in the control group.

In the academic context, Butler and Winne (1995) describe self-regulation as a style of engaging with tasks in which students exercise powerful skills such as: (a) setting goals for upgrading knowledge, (b) deliberating about strategies to select those that balance progress towards goals with unwanted costs, and (c) monitoring the effects of engagement. The authors synthesize recent research on the relationship between feedback and self-regulation, and offer a model for self-regulated learning (SRL), which analyses the cognitive processes involved in self-regulation, and integrates the findings of relevant, sometimes disparate, literature (see Figure 3).

Figure 3. A model of self-regulated learning.[3]



Note. From "Feedback and Self-Regulated Learning: A Theoretical Synthesis," by D. L. Butler and P. H. Winne, 1995, Review of Educational Research, 65 (3), p. 248. Copyright 1995 by the American Educational Research Association. Reprinted with permission.

Butler and Winne explain that as self-regulating learners engage in academic tasks, they draw on knowledge and beliefs to construct an interpretation of a task's properties and requirements. Based on the interpretation they construct, they set goals.

Goals are then approached by applying tactics and strategies that generate products, both mental (cognitive and affective/emotional) and behavioral. Monitoring these processes of engagement and the progressively updated products they create generates internal feedback (that which students generate for themselves by monitoring their engagement with learning tasks (Bangerts-Drowns, Kulik, Kulik, & Morgan, 1991)). This information provides grounds for reinterpreting elements of the task and one's engagement by setting new goals or adjusting extant ones. Learners may reexamine tactics and strategies and select more productive approaches, adapt available skills, and sometimes even generate new procedures. If external feedback (that which is generated by a source external to the student (Bangerts-Drowns et al., 1991)) is provided, that additional information may confirm, add to, or conflict with the learner's interpretations of the task and the path of learning. As a result of monitoring task engagement, students may alter knowledge and beliefs, which in turn might influence subsequent self-regulation.

Butler and Winne (1995) note that, regardless of the source, feedback is contextualized according to a student's prior knowledge and beliefs before cognitive tactics and strategies are applied. Hence feedback which informs students only about content in a domain is minimally sufficient to affect knowledge construction. The authors propose that feedback should provide information about cognitive activities for learning and about relations between cues and successive states of achievement. They conclude that SRL is inherent in knowledge construction, although it may be carried out suboptimally and contrary to educational objectives. Monitoring is the hub of self-regulated task engagement and the internal feedback it generates is critical in shaping the evolving pattern of a learner's engagement with a task. The authors acknowledge that feedback blends with other information to affect a learner's knowledge and beliefs about the domain and tasks, learning processes and products, and performance.

Summary

The quality of the learner's experience in a distance education context can be enhanced by encouraging and supporting a deep approach to learning through proper course design, with provisions for sufficient interaction, timely, constructive feedback, and opportunities for the construction of a sufficient number of quality concept maps. As well, the use of assessment in general, and projects and assignments in particular, which demand deep-level processing, can promote learning. Helping students become metacognitively active and self-regulatory can also increase learning, as can helping them achieve epistemological autonomy.

RESEARCH PROCEDURES

Chapter Six: Research Procedures

Introduction

This chapter contains three main sections. The first section provides necessary background information and contains both a description of the context in which the web-based version of Mathematics 4225 was developed and implemented and a description of the synchronous component of the course. The second section justifies the selection of a qualitative research methodology, while the third section lists the actual methods of data collection.

Context

On May 1, 1998, the Vista School District of Newfoundland and Labrador publicly released its five-year Strategic Education Plan, 1997-2002 (available: *http://www.k12.nf.ca/vista/aboutus/strategiceducation/index.html*), which was designed to guide the change and growth of the school district into the next millennium. This plan included strategies aimed at ensuring that students' talents were developed, their needs met, and that student understanding of the use of technology was increased. Specifically, by the year 2000, the plan called for the expansion of distance learning opportunities for students, enabling them to pursue individual curriculum opportunities and interests, and for the provision of more individualized programs, designed to meet the intellectual needs of the students. The plan also called for the creation of a district Intranet, that would be used to support the activities of the board, and the development of independent study activities and courses for students using technology.

At the time of the release of this plan, work had already begun on the development of a web-based version of the Advanced Placement (AP) Calculus AB course (available: *http://www.stemnet.nf.ca/~vsddi/*) in the Centre for TeleLearning and Rural Education at Memorial University of Newfoundland. The Advanced Placement Program was endorsed by the Department of Education, as an integral component of the senior high program for high achieving students, in its senior high report entitled *The Senior High School Program: New Directions for the 21st Century*, released in June 1997. The AP Calculus AB course would become one of the first courses offered, as Mathematics 4225, to students via the Vista School District Digital Intranet beginning in the fall of 1998. The intranet would enable students with an aptitude for mathematics to participate in a college level, first calculus course, and possibly gain one university mathematics credit before graduation from secondary school. After one year of piloting, the course would be made available to the other nine school districts in the province.

The development and implementation of the project was guided and monitored by the Vista School District Digital Intranet Advisory Committee whose members included personnel from the Centre for TeleLearning and Rural Education, Memorial University of Newfoundland, the Department of Education, STEM~Net, and the College of the North Atlantic

Student selection was based on the successful completion of Advanced Mathematics 3201 and the recommendation of their high school mathematics teacher. Initial registration for the course included 7 students from a level one to level three high school with an enrolment of 184 students, 6 students from a grade 9 to level three school with an enrolment of 484 students, and 2 students from an all-grade school with a student population of 197. In June of 1999, these students were invited to meet, to interact socially, and to receive some preliminary training from their instructor. At that time, students completed a form outlining their level of comfort with, and expertise in, the use of computers, including their ability to search the Internet and to navigate through web pages, as well as their familiarity with e-mail, chat rooms, and bulletin boards. Those students who had access to computers outside the school were issued a username and password that would allow them access to the Mathematics 4225 website over the summer. The intent was to allow students ample time to become familiar with the website and to better inform their decision to officially enroll in the AP mathematics course in September of 1999.

Asynchronous Interaction (A Description of the Mathematics 4255 Website)

The Mathematics 4225 website first presented a splash page with a "Mathematics 4225" icon (see Figure 4) which allowed access to the login screen (see Figure 5), where the student entered their personal user name and password.

Figure 4. Splash page containing the "Mathematics 4225" icon.

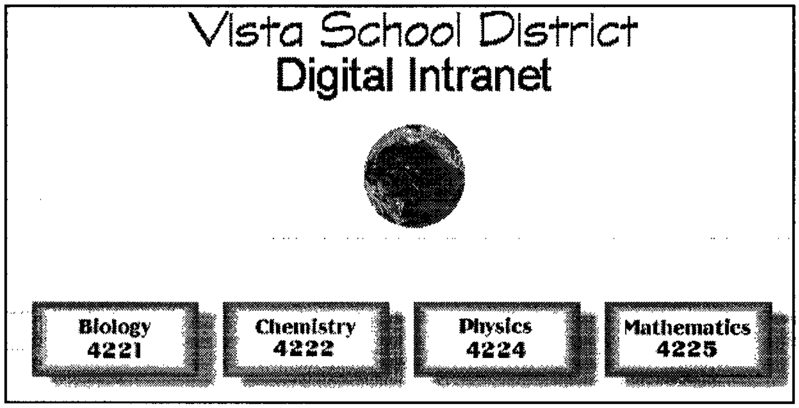
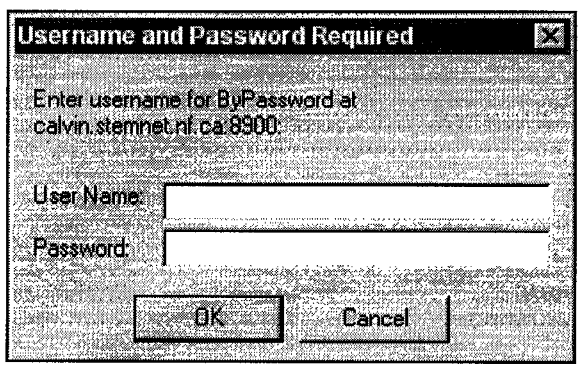


Figure 5. Login screen for Mathematics 4225 website.



The login screen allowed access to the "Welcome to Mathematics 4225" page (see Figure 6) that included of a number of icons which are described in Table 1.

Figure 6. Welcome Page for Mathematics 4225.

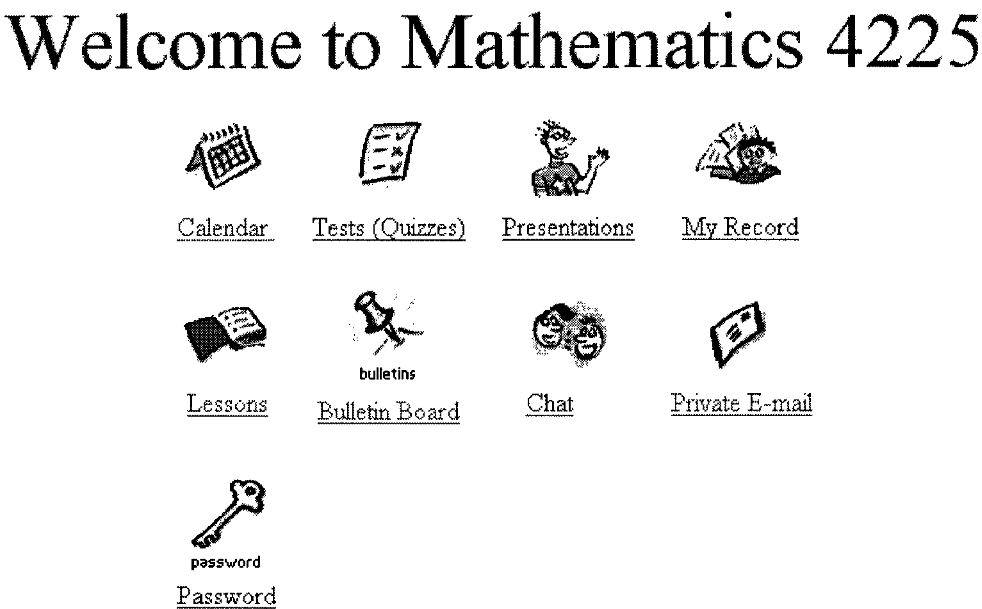
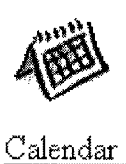


Table 1. Description of Icons on the "Welcome Page"

Icon	*Description*
	The electronic calendar icon gave access to a monthly record of assignment due dates, test dates, special instructions, and online and offline class times. The calendar could be compiled for a selected time period and printed to create a compact hardcopy of important course dates and instructions.
	The electronic bulletin board allowed asynchronous communication between any participants in the course. Participants could enter private or public messages which could be responded to at any time. For example, any changes made to the course calendar would be posted on the bulletin board as well.

Private E-mail

This option worked similar to customary e-mail except that the only possible senders and recipients were those enrolled in the Mathematics 4225 course.

Password

The password icon allowed students to change the password assigned to them by their instructor. This was recommended to all students.

Presentations

This option allowed students to give asynchronous presentations individually or as a group.

Tests (Quizzes)

The tests option gave students access to a database of randomly generated multiple choice, short answer and long answer tests. These tests were automatically graded and the score returned to the student.

Chat

This chat feature is similar to customary chat rooms with the exception that only participants in the course could enter.

My Record

This option allowed students to view their individual scores on the various evaluation instruments used in the course.

Lessons

The "Lessons" icon gave the students access to the lessons developed by the instructor, as well as a variety of other course resources.

Selecting the "**Lessons**" hyperlink on the Welcome page presented students with a frame (see Figure 7) with the choice of three units. Each "**Unit**" hyperlink could be expanded to list the topics covered in that unit. The "**Topics**" hyperlink could be further expanded to list all the sections in the text associated with that topic. Each "**Section**" hyperlink could then be expanded to list the lessons for that section. Each "**Lesson**" hyperlink would bring the student to the appropriate online lesson, which could be read onscreen or printed. Each lesson referred to associated pages from the course text, *Calculus of a Single Variable: Early Transcendental Functions* (5^{th} edition) by Larson, Hostetler and Edwards. At the end of each section is a "**Quiz**" hyperlink that presents the student with a short, multiple-choice quiz which tests the concepts covered in that section. This quiz is automatically scored and the result returned to the student, along with a list of the correct answers for each incorrect answer that the student supplied. At the end of each topic are a more comprehensive multiple-choice test and a long answer test that assess student knowledge of the entire topic. The multiple-choice test is scored as with previous tests of this type and the solutions to the long answer test is provided in a separate, linked, online page. Also at the end of each topic is an "**Assignment**" hyperlink that contains references to assignment questions in the text. These are submitted by fax or interoffice mail, graded by the instructor, and returned to each student.

Figure 7. Frame linked from the "Lessons" hyperlink.

Mathematics 4225

Click ▸ to Expand The List and ▾ to Contract The List

Acknowledgements

▾ **Unit 1 - Functions, Graphs, and Limits**
- ▸ **Prerequisites** - The Cartesian Plane and Functions
- ▾ **Topic 1** - Limits and Their Properties
  - ▸ **Section 1.1** - A Preview of Calculus
  - ▸ **Section 1.2** - An Introduction to Limits
  - ▸ **Section 1.3** - Properties of Limits
  - ▾ **Section 1.4** - Techniques for Evaluating Limits
    - Lesson 1
    - Lesson 2
    - Lesson 3
    - Quiz
  - ▸ **Section 1.5** - Continuity and One-Sided Limits
  - ▸ **Section 1.6** - Infinite Limits
  - **Unit 1: Topic 1 Sample Test**
  - **Assignment 1**

Also included in this same frame are a number of links which provide the student with several resources for the course (see Figure 8). The "**Concept Map**" hyperlink opens a new page containing a concept map for the entire course. The "**Course Outline**" hyperlink links to a new frame containing the percentage of the course covered in each unit, hyperlinks to the lessons for each section, and a time schedule for the completion of each lesson. The "**Outcomes Linked to Lessons**" hyperlink opens a frame containing the course outcomes for each lesson and a hyperlink to the specific lesson which deals with each outcome. The "**Course Evaluation**" hyperlink opens a frame giving the weights assigned to the various evaluation instruments for the course. The "**Quizzes for Self Evaluation**" hyperlink opens a frame containing hyperlinks to all the multiple choice quizzes for each section of each topic. The "**Tests for Self Evaluation**" hyperlink opens a frame containing hyperlinks to a multiple choice and long answer test for each topic. These multiple-choice examinations are graded in a manner similar to the others and the answers to the long answer portion of the test are provided online. The "**Assignments**" hyperlink opens a frame containing the text references to the assignments for each topic and the "**External Links**" hyperlink opens a frame with hyperlinks to various calculus resources found on the Internet, such as tutorials, labs, projects, models and a variety of mathematics applets which demonstrate many of the important concepts in the course. Any student participating in the course who had Internet access could access these resources and features at any time.

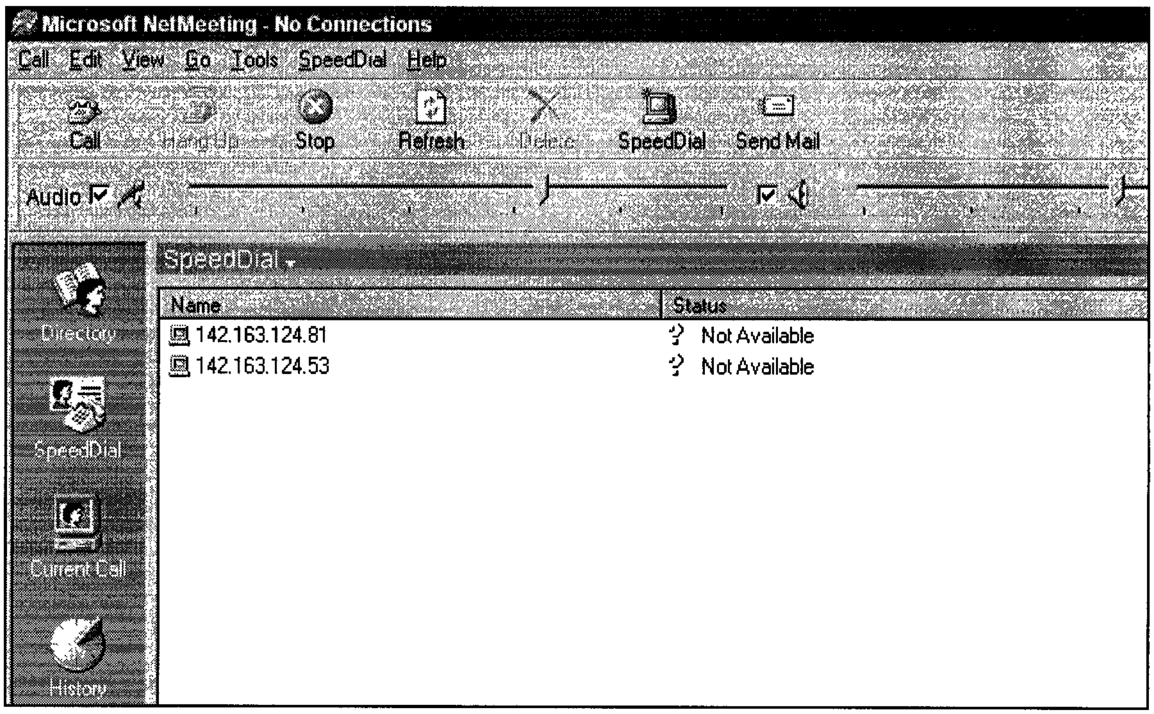
Figure 8. Links to Mathematics 4225 Course Resources.

Concept Map
Course Outline
Outcomes Linked to Lessons
Course Evaluation
Quizzes for Self Evaluation (MC)
Tests for Self Evaluation
Assignments
External Links

Synchronous Interaction (Online Classes)

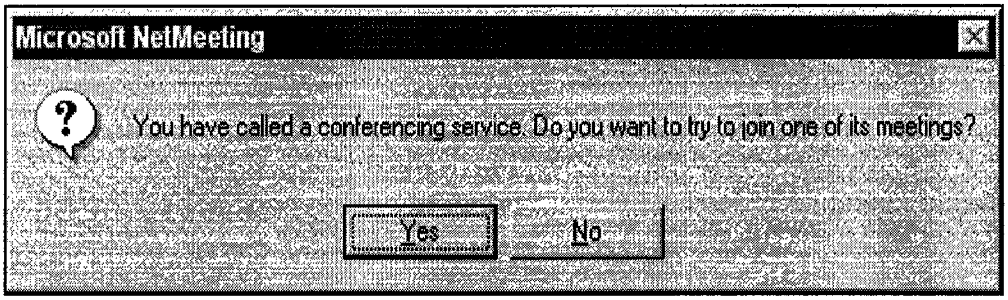
In the 1999-2000 school year, students in Mathematics 4225 were assigned five, fifty-five minute periods every eight days, three of which were online sessions and two of which were offline. The three online classes were scheduled in the morning session when Internet traffic was less and more bandwidth was available for audio, video and whiteboard signals. Students connected to the Internet from the student AP computer using a modem connection provided by a local Internet service provider and then opened Microsoft NetMeeting (see Figure 9) to connect to one of the two servers located in the Stem~Net facilities at Memorial University of Newfoundland. The server used Meeting Point software to route any incoming signals from one participant to all other participants in the NetMeeting.

Figure 9. Opening page of Microsoft NetMeeting containing the IP address of the Meeting Point server and the backup server.



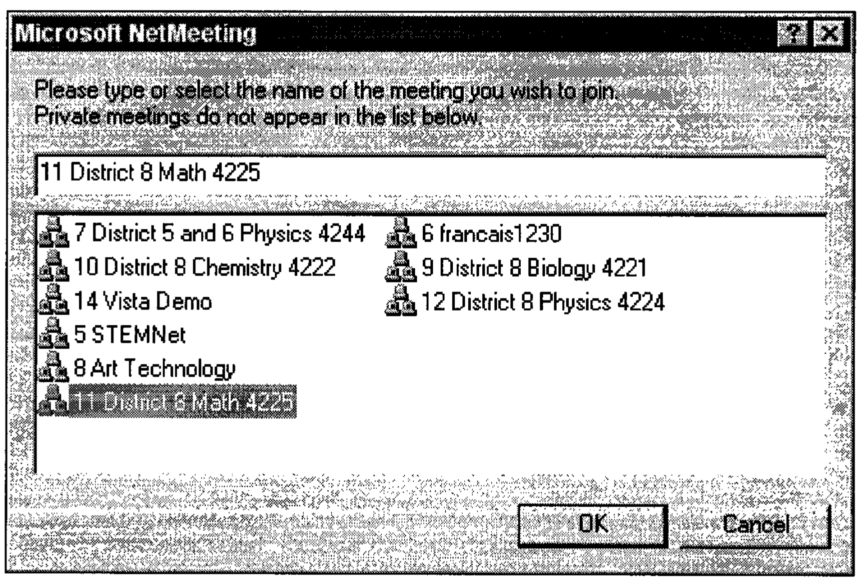
Double clicking on the first IP address presented the participant with a screen asking them if they wished to enter a meeting (see Figure 10). The second IP address connected to the backup server.

Figure 10. Request to join a conference in Microsoft NetMeeting.



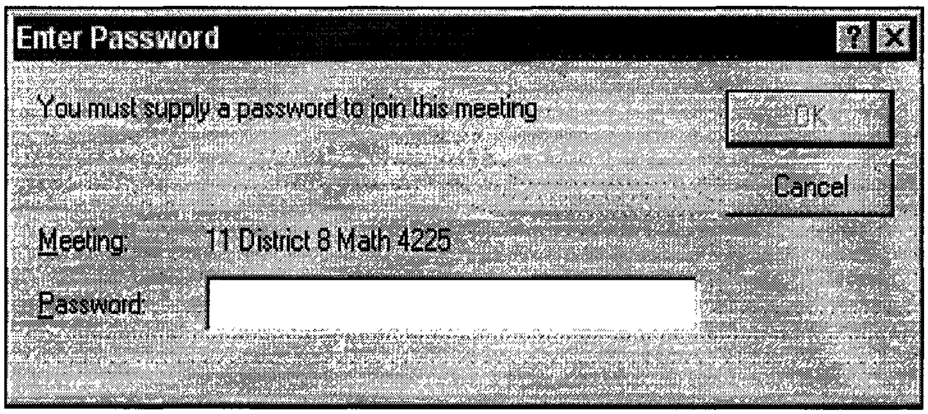
Clicking **"Yes"** opened a window which allowed the student to select the Mathematics 4225 conference (see Figure 11).

Figure 11. Selection of Mathematics 4225 conference.



Choosing "**OK**" presented another screen requesting the appropriate conference password (see Figure 12).

Figure 12. Password request screen.



Having entered the correct password, the student is permitted to join the Mathematics 4225 online class.

Once connected to the online class, participants communicated using a combination of audio, chat, whiteboard and video tools, and through the sharing of computer applications such as the Geometer's Sketchpad or Derive. The student AP computer at each site was equipped with a microphone and one set of speakers which were used to transmit and receive audio signals. The instructor received and transmitted the audio signal through a set of headphones equipped with a boom microphone. The chat feature in NetMeeting allowed participants to communicate using the keyboard. The whiteboard allowed freehand work such as the drawing of symbols and graphs, and the working of exercises. These three features allowed simultaneous student input and control. The video feature allowed participants to view remote sites. The sharing applications feature allowed a participant at one site to open an application on the computer at that site and allowed participants at remote sites to take control of and manipulate the application. For example, a Geometer's Sketchpad file could be opened on the instructor computer and manipulated by students at each of the remote sites even if that site did not have the Geometer's Sketchpad software on the student AP computer. A typical online class would consist of an interactive presentation by the instructor followed by a "question and answer" period where students gained help with specific text exercises or course concepts.

Methodology

Over the last three decades, there has been a shift in educational research methodology from one dominated by quantitative methods to those which have a more qualitative focus. According to Bogdan and Bilken (1982), the reason for this shift was that many educational researchers felt that quantitative research had not proven itself adequate in solving educational problems. These educational researchers began to adapt a variety of qualitative research methods that had proven useful in other fields, including observations and interview methods derived from case study, ethnographic, phenomenological, and critical social theory research. Simultaneously, objects characteristic of quantitative research, such as measurement, operationalized definitions, and variables, were extended for use in qualitative educational research.

Bogdan and Bilken (1982) outline several main characteristics of qualitative research:

1. Qualitative research is concerned with context, and uses natural settings as direct sources of data. This is in contrast with studies in which variables are manipulated and their effects upon other variables observed.
2. Qualitative research is descriptive, making use of words rather than numbers. Written results use quotations from the data for illustration and substantiation of findings.
3. Qualitative research concerns itself with process, rather than focusing solely on outcomes or products.
4. Qualitative research tends to analyze inductively as opposed to deductively. Specifically, qualitative researchers build hypotheses and abstractions from the data gathered, as opposed to collecting data to accept or reject hypotheses generated before the study begins.

Constructing hypotheses and abstractions from gathered data leads to theory that emerges from this gathered data and Glaser and Strauss (1967) refer to this theory as grounded theory. According to the authors, grounded theory fulfills qualities desired of theory because its mode of generation guarantees it is useful for prediction, explanation, interpretation and application. It also serves to guide subsequent research and contributes to theoretical advancement. In comparison, research that merely tests or verifies hypotheses assigns a more passive role to theory.

In this study emphasis was placed on qualitative methodologies. However, the study also adopted a critical approach as aspects of the structure, content, and delivery of the web-based AP Calculus AB course were investigated to determine how the learning of calculus concepts in this context can be improved.

Method

Several qualitative methods were used to gather data which was used to determine how learning could be improved in the context of a web-based, Advanced Placement Calculus AB course which was delivered via a district-wide digital Intranet. Methods of data collection included:

1. Surveying participants to obtain student views on aspects of the AP course, specifically the features of the Mathematics 4225 website, the course text, the online classes, evaluation, collaboration, and social interaction. (See Appendix A)
2. Analyzing the course text to determine its suitability for the course and for the epistemological message(s) contained in the text. This textbook was chosen by the development team because it was the text used in first year calculus at Memorial University of Newfoundland, the university attended by the majority of university bound students in Newfoundland and Labrador. Another text, such as the Consortium Calculus text by Hughes-Hallett, may be a more appropriate course text.
3. Observing participants at the three AP sites to determine their specific difficulties with the course. In this study, the researcher acted as "full participant" (Glesne & Peshkin, 1992, p. 40) on the participant-observation

continuum since the researcher is simultaneously a functioning member of the community under investigation and an investigator.

4. Comparing recent and previous AP Calculus AB examinations published by The College Board. The AP Calculus AB examination typically consists of two parts. Part one consists of multiple-choice questions and part two consists of six, long answer questions with parts to each of these questions. Initially, a scientific calculator was the only computational aid used in the examination. However, in recent years, parts one and two have allowed some use of a graphing calculator and part two, in particular, has become much more application-oriented. Students were provided with entire copies of examinations, published by The College Board, for the years 1985 and 1988, when only scientific calculators were permitted. After writing a sample AP mathematics examination as a course final, and the 2000 Calculus AB examination, these students were a rich source of data for a comparison of recent and earlier examinations in terms of the changes enabled by the use of graphing calculators.

Data collected from these activities was used to determine the student learning experience in an online AP calculus course. It was also be used to identify features of the course that students found beneficial and those which needed to be improved, replaced or removed, and to identify ways in which learning can be improved. On the basis of these findings, the student's experience of learning calculus was described and recommendations were made for the improvement of learning AP calculus in a web-based environment.

RESULTS

Introduction

The results are contained in the four following chapters. Chapter seven categorizes students as completers or noncompleters and provides a description of each student as compiled from the student surveys. Chapter eight presents the results of part one of the student survey. This part of the survey examined the features of the Mathematics 4255 website, course text, online classes, evaluation, collaboration and social interaction. Chapter nine presents the results of part two of the student survey. This part of the survey elicited students' opinions on factors which research literature indicated was important in learning calculus concepts and in learning at a distance. The quantity of results presented in chapters eight and nine necessitated that the discussion of the results for each feature occur immediately after the results themselves were presented, instead of being delayed to a later chapter. Chapter ten contains a comparison of the course text and an alternate text in terms of the features and epistemological messages contained in each. This chapter also compares College Board Calculus AB examinations from 1988 and 1997 in terms of the effects of the increased use of graphing calculators.

Chapter Seven: Introduction and Description of Participants

After the course had ended and the evaluation was concluded, the fifteen participants were asked to complete a survey (see Appendix A, Part 1) which required them to indicate frequency of use or access, usefulness, and importance of the features of the:

(i) Mathematics 4225 Website. The website contained hyperlinks to a calendar, a record of students' marks, lesson outcomes and notes for each corresponding section of the text, a concept map for the course, the course outline, the percentage weights assigned to the course evaluation instruments, a bulletin board, a chat room, private e-mail, and a number of calculus related resource sites which were external to the website itself.

(ii) Course Textbook. This text contained a prerequisites chapter, tables of mathematical rules, colour graphics, technology tips, historical notes, summaries and guidelines, remarks, questions which varied in variety and difficulty, selected answers, and a number of appendices. Accompanying the text was a student study and solution guide.

(iii) Online Classes. This refers to the real-time communication between the instructor and the students at each site when they were simultaneously connected using computer technology to create a virtual classroom.

(iv) Course Evaluation. This refers to the evaluation instruments used in the course and the percentage weight assigned to each.

(v) Collaboration in the Course. This refers to the process in which students interacted with other students as they worked, studied, and learned in the course.

(vi) Social Interaction of Students in the Course. This refers to the interaction of students during activities specifically designed to enable students to make contact, with students at other sites, and with their instructor, in a social setting.

Students were also asked to give information about their decision to enroll in Mathematics 4225, their previous experience with distance education courses, reasons for withdrawal from the course, study habits, career plans, use of technology, and access to computers at home and at school. This data will be used to help future students make an informed decision concerning their participation in the course. In the part two of the survey, students were asked to indicate their degree of agreement with statements about a number of factors which research literature indicated was important in the learning of calculus and in learning at a distance (See Appendix A, Part 2). The data from parts one and two of the survey will be used to determine which aspects of Mathematics 4225 could be transformed to improve student learning.

This section begins with a description of the fifteen students in the course, who were grouped into two categories. Completers refers to the group of six students who wrote the Mathematics 4225 final examination for high school credit. Noncompleters refers to those nine students who withdrew from Mathematics 4225 before writing the final examination. The descriptions were derived directly from students' responses to items in part one of the survey.

Student Descriptions

Noncompleters

Student 1: This student enrolled in Mathematics 4225 to lighten the first year university course load and to have a better understanding of mathematics. The student had not previously enrolled in any other distance education courses and did not have a home computer with Internet access. She withdrew from the course in mid November due to difficulty with the course and dropping marks. She stated "Math 4225 interfered with my other high school work because many hours were spent on math when I had a lot of other work to do. I found it very disappointing to put so much work into something and achieve no results." The student spent 10 to 12 hours per week studying Mathematics 4225 and completed approximately 80% of the homework while enrolled in the course. She reported that her post-secondary career plans were undecided and therefore were not changed because of participation in Mathematics 4225. The student intended to enroll in post secondary mathematics courses.

Student 2: This student enrolled in Mathematics 4225 because of high achievement in high school advanced mathematics courses, because of the challenge Mathematics 4225 would provide, and because it would help in university. She had not previously enrolled in any other distance education courses and did not have a home computer with Internet access. The student withdrew from the course in January, indicating that the course was difficult to do over the Internet and that the teaching was not well done. This student spent 10 to 15 hours per week studying Mathematics 4225 and completed approximately 70% of the homework while enrolled in the course. The student reported that her post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post secondary mathematics courses.

Student 3: This student enrolled in Mathematics 4225 because of interest. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. He withdrew from the course in January because there was "not adequate communication with the teacher." Initially, the student spent 10 to 20 hours per week studying Mathematics 4225 but this gradually decreased as the course progressed. The student did not indicate what percentage of the homework was completed while enrolled in the course. He reported that his post-secondary career plans were undecided and therefore were not changed because of participation in Mathematics 4225. The student was unsure with respect to enrollment in post secondary mathematics courses.

Student 4: This student enrolled in Mathematics 4225 because of the challenge and the opportunity to get ahead in university studies. The student had not previously enrolled in any other distance education courses and had no home computer with Internet access. She withdrew from the course in mid November stating "I had too much work, and with AP math, I was neglecting my other studies. I went back to Math 3201 to improve my mark." The student spent 10 to 12 hours per week studying Mathematics 4225 and completed approximately 80% of the homework while enrolled in the course. She reported that her post-secondary career plans were undecided and therefore were not changed because of participation in Mathematics 4225. The student intended to enroll in post-secondary mathematics courses.

Student 5: This student enrolled in Mathematics 4225 because all high school advanced mathematics courses had been completed by the end of level 2 and she did not want to go a whole year without mathematics, especially in the year before university. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. She withdrew from the course in January, stating: "I was finding it very difficult. I didn't want to spend every free moment working on math and I was annoyed with the online classes." The student spent 8 hours per week studying Mathematics 4225 and completed approximately 40% of the homework while enrolled in the course. She reported that her post-secondary career plans were changed because of participation in Mathematics 4225. This student indicated discouragement with Mathematics 4225 and cited a realization of a hatred of mathematics. This caused her to pursue a career where calculus was not required.

Student 6: This student enrolled in Mathematics 4225 because of the opportunity to get a relatively inexpensive university credit. She had completed all high school advanced mathematics courses by the end of level 2 and stated that doing Mathematics 4225 in level 3 would help her retain previously learned mathematics. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. She withdrew from the course in February, stating she "didn't understand because of trouble with the audio and not having a teacher in the classroom. Math 4225 took a lot of time and I had other courses to pass." The student spent 5 hours per week studying Mathematics 4225 and completed approximately 40% of the homework while enrolled in the course. She reported that her post-secondary career plans were undecided and therefore were not changed because of participation in Mathematics 4225. This student also indicated that participation in post-secondary mathematics courses depended on career choice.

Student 7: This student enrolled in Mathematics 4225 because all high school advanced mathematics courses had been completed by the end of level two and she did not want to go a whole year without mathematics. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. She withdrew from the course in February, stating, "I wasn't interested in the course. I fell behind, didn't enjoy the online classes, and decided I was going to do nursing and didn't need math." This student spent 6 hours per week studying Mathematics 4225 and completed approximately 40% of the homework while enrolled in the course. She reported that her post-secondary career plans had changed because of participation in Mathematics 4225. Initially, she wanted to pursue a science-related career but then decided against it and chose instead to enter a field not requiring calculus. The student intended to enroll in a post-secondary statistics course.

Student 8: This student enrolled in Mathematics 4225 because all her high school advanced mathematics courses had been completed by the end of level two and she did not want to go a whole year without mathematics. She was also encouraged by family and teachers. The student had not previously enrolled in any other distance education courses and had no home computer with Internet access. She withdrew from the course in February, stating: "I wasn't motivated to put in the time and effort into the course, as I was also doing AP Biology." This student spent 5 hours per week studying Mathematics 4225 and completed approximately 20% of the homework while enrolled in the course. She reported science-related post-secondary career plans, which had not changed because of participation in Mathematics 4225. The student intended to enroll in post-secondary mathematics courses.

Student 9: This student enrolled in Mathematics 4225 because all her high school advanced mathematics courses had been completed by the end of level 2 and she wanted to do a mathematics course in the last year of high school. This student also listed university preparation as a reason for enrolment. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. However, her home computer was not equipped with Internet Explorer 4.x or Netscape 4.x, which prevented access to some of the information on the web pages at the mathematics Mathematics 4255 website. This student withdrew from the course in February because of difficulty with the pace of the course and difficulty in understanding the topics. The student spent 6 hours per week studying Mathematics 4225 and completed approximately 85% of the homework while enrolled in the course. She reported that post-secondary career plans had changed because of participation in Mathematics 4225, and stated that a career involving mathematics was no longer an option. However, she intended to enroll in post-secondary mathematics courses.

Completers

Student 10: This student enrolled in Mathematics 4225 to improve her university mathematics skills. The student had not previously enrolled in any other distance education courses but had a home computer with Internet access. This student did write the Mathematics 4225 final examination but did not write the AP Calculus AB final examination stating, "I felt I would not get the grade I wanted." However, she also noted, "I probably would have written the exam if I had known it would be easier than the May 5 exam." This student spent 10 to 15 hours per week studying Mathematics 4225 and completed approximately 75% of the homework in the course. She reported that post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post-secondary mathematics courses.

Student 11: This student enrolled in Mathematics 4225 because calculus was required for university acceptance. She had not previously enrolled in any other distance education courses but had a home computer with Internet access. This student spent 20 hours per week studying Mathematics 4225 and completed 100% of the homework in the course, even though she was also enrolled in AP Chemistry. She reported that post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post secondary mathematics courses.

Student 12: This student enrolled in Mathematics 4225 for the experience, for the opportunity to obtain a relatively inexpensive university credit, and because of encouragement by teachers. He had not previously enrolled in any other distance education courses but had a home computer with Internet access. This student spent 4 hours per week studying Mathematics 4225 and completed approximately 2% of the homework in the course. He was also enrolled in AP Biology. This student reported that his post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post secondary mathematics courses.

Student 13: This student enrolled in Mathematics 4225 so she would not have to do first semester calculus in university if she received credit and would have a background in calculus if she did not receive university credit. She was also encouraged by parents and teachers. This student had not previously enrolled in any other distance education courses and did not have a home computer with Internet access. She spent 5 hours per week studying Mathematics 4225 and completed approximately 4% of the homework in the course. This student reported that her post-secondary career plans had changed, but not because of participation in Mathematics 4225. The student did not intend to enroll in post-secondary mathematics courses.

Student 14: This student enrolled in Mathematics 4225 because all his high school advanced mathematics courses had been completed by the end of level 2 and he did not want to go a whole year without doing a mathematics course. This student had previously enrolled in other distance education courses, having completed high school advanced mathematics and science courses using the facilities of the Telemedicine Centre. The student had no home computer with Internet access. This student did write the Mathematics 4225 final examination but did not write the AP Calculus AB final examination because of a fear of not doing well. He spent 5 hours per week studying Mathematics 4225 and completed approximately 70% of the homework in the course. This student reported that post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post-secondary mathematics courses.

Student 15: This student enrolled in Mathematics 4225 to get an idea of what a first calculus course would be like in university. He had also completed all high school advanced mathematics courses by the end of level 2 and felt it was necessary to take a mathematics course in level 3. This student had previously enrolled in other distance education courses, having completed high school advanced mathematics and science courses using the facilities of the Telemedicine Centre. The student had no home computer with Internet access. This student did write the Mathematics 4225 final examination but did not write the AP Calculus AB final examination because of a fear of not passing it. He spent 2 hours per week studying Mathematics 4225 and completed approximately 10% of the homework in the course. This student reported that post-secondary career plans were not changed because of participation in Mathematics 4225. The student intended to enroll in post-secondary mathematics courses.

Chapter Eight: Results of Part One of the Survey

Results compare six completers (those who wrote the Mathematics 4225 final examination), and nine noncompleters (those who did not write the Mathematics 4225 final examination, and in most cases, withdrew before or near the midpoint of the course). Average results presented in each table range on a continuum from 1 to 5 and can be interpreted using Table 2.

Table 2
Interpretation of Results from Part One of the Survey

Evaluation Criteria	Result				
	5	4	3	2	1
Frequency of Access/Use	> 6 times per week	5-6 times per week	3-4 times per week	1-2 times per week	Less than once per week
Usefulness	Extremely useful	Very useful	Fairly useful	Somewhat useful	Not at all useful
Importance	Extremely important	Very important	Fairly important	Somewhat important	Not at all important

Access to Internet-Ready Computers

Table 3
Access to Internet-Ready Computers: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Access to Internet-Ready Computer During Offline Classes	Frequency of Use/Access	3.0	0.89	2.3	1.41
	Usefulness	2.8	1.83	2.1	1.05
	Importance	2.7	1.86	2.2	1.09
Access to Mathematics 4255 Website Using Home Computer	Frequency of Use/Access	2.5	1.29	3.0	1.58
	Usefulness	2.6	1.14	2.7	1.63
	Importance	2.8	1.64	2.8	1.83

Completers, on average, used a computer during offline classes more frequently than noncompleters, although noncompleters' frequency of access ranged from less than once per week to more than six times per week. Although both groups indicated that it was only somewhat useful and important to have access to a computer for offline classes, usefulness and importance were rated higher by completers. Access to an Internet-ready computer was not perceived as beneficial by one completer, who stated, "The computer wasn't much help. It always came down to the fact that we could only rely on ourselves to help us complete the course."

Students with home Internet-ready computers could have accessed the Mathematics 4255 website at any time and used the chat, e-mail or the bulletin board features to communicate. With the addition of a microphone and speakers, students could have also achieved synchronous audio using NetMeeting and Meeting Point software. A whiteboard would have also allowed students to communicate in written form. Noncompleters who had Internet access at home reported greater access to the Mathematics 4255 website, although both groups indicated that it was only fairly useful and important to have access to an Internet-ready computer at home. Two completers did not have Internet access at home. Four noncompleters did not have Internet access at home and one did not have an Internet browser that allowed complete access to the web pages at the website.

Internet access at home was beneficial to, or perceived as beneficial by, some students. A noncompleter stated: "If I had more access to a computer, I probably would have explored more of the sites and used them more." Another stated, "If I did not have Internet access at home, I would not have continued in the course as long as I did." One completer indicated a home computer would have allowed him to spend more time at his homework.

Features of the Mathematics 4255 website

Introduction

This section presents students' assessment of the features of the Mathematics 4225 website, as obtained from the results of part one of the survey.

Online Calendar

Table 4
Dates for Classes: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Dates for Online Classes	Frequency of Use/Access	2.7	1.37	3.6	0.88
	Usefulness	3.2	1.33	3.9	1.05
	Importance	2.8	1.47	3.7	1.00
Dates for Offline Classes	Frequency of Use/Access	2.0	0.89	3.0	0.82
	Usefulness	2.3	0.81	2.6	0.79
	Importance	2.0	0.89	2.6	0.53
Dates for Extra Online Classes	Frequency of Use/Access	1.2	0.41	2.5	1.20
	Usefulness	2.5	1.52	2.8	1.09
	Importance	2.5	1.64	3.1	1.05

The dates for all classes were posted in the online calendar. Noncompleters indicated more frequent access to the online calendar for class dates and considered them more useful and important. One completer went by slots in the school schedule for online classes and not the dates posted in the online calendar. She also used the school schedule for offline class dates as scheduling difficulties caused that site to reschedule the offline classes. This removed an opportunity for students at that site to interact with students from other sites and with the instructor, as both attended other classes during those times.

One noncompleter indicated it was nice to know the dates of online classes so she could be prepared with questions. Another noncompleter stated that "The calendar was not very useful in the beginning because the teacher did not write when our classes were on it. In the end it was much better." All extra class dates were also announced during online classes and sent to students via the bulletin board and e-mail.

Table 5
Dates for Evaluation: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Examination Dates	Frequency of Use/Access	3.2	1.83	3.3	1.32
	Usefulness	4.2	0.75	3.7	1.12
	Importance	4.7	0.52	3.9	0.93
Assignment Due Dates	Frequency of Use/Access	2.8	1.48	3.4	1.13
	Usefulness	3.8	0.75	3.7	1.22
	Importance	4.3	0.82	3.9	1.09

The dates of examinations and assignment due dates were also posted in the online calendar in September, but these dates were somewhat flexible. Both completers and noncompleters reported equal access to the online calendar for examinations dates, but noncompleters reported more frequent access to the online calendar for assignment due dates. Both groups thought these features were very useful and important. One noncompleter indicated it was useful to know the dates of examinations well in advance.

Table 6

Special Notices: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Special Notices	Frequency of Use/Access	2.3	1.51	3.1	0.93
	Usefulness	2.5	1.38	3.6	0.88
	Importance	2.7	1.63	3.6	0.92

Noncompleters reported more frequent access to the online calendar for special notices and thought that this feature more useful and important. One noncompleter indicated it was very important to know if examination or assignment dates had been rescheduled. Special notices were also sent to each student via private e-mail, were posted on the bulletin board and were announced during online classes. Students may have chosen any of these options to access these notices.

Table 7

Calendar Compile Option: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Calendar Compile Option	Frequency of Use/Access	1.4	0.55	2.2	0.97
	Usefulness	2.0	1.00	3.2	1.48
	Importance	1.6	0.55	3.1	1.05

The calendar compile option allowed students to produce a hard copy of the contents of the calendar for any desired time period. The entries in the calendar were usually totally completed a month in advance, so students using the compile feature needed only to access the feature early in the month. However, any adjustments made to the calendar during the month would make any previous hardcopy inaccurate, so students were advised to check the calendar on a weekly basis. Noncompleters reported more frequent access to this option, and thought it more useful and important. One noncompleter noted that this was an easy way to look ahead at upcoming examinations and assignments. A completer indicated that she never used this feature.

My Record

Table 8
My Record: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
My Record	Frequency of Use/Access	3.5	1.38	2.4	1.13
	Usefulness	4.3	0.82	4.0	1.00
	Importance	3.7	1.51	3.9	1.05

Completers reported more frequent access to the *My Record* feature, although both groups thought the feature very useful and important. This feature allowed students to view all their marks online and, for each piece of evaluation, compare their mark with the highest and lowest mark, and the mean and median mark. The feature also generated a histogram of the results for each piece of evaluation. One noncompleter stated that it was "nice to be able to see all my marks and to be able to compare them with the class statistics." A completer indicated it was good to be able to see how you were doing at different points in the course.

Lessons

Table 9
Lessons: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Curriculum Outcomes	Frequency of Use/Access	2.3	1.75	2.7	1.32
	Usefulness	3.0	1.90	3.4	1.33
	Importance	2.8	1.83	3.6	1.24
Text Reference Pages	Frequency of Use/Access	2.8	2.04	3.4	1.24
	Usefulness	3.0	2.19	3.6	1.24
	Importance	3.0	2.19	3.4	1.13
Advance Organizers	Frequency of Use/Access	2.0	1.0	2.3	0.76
	Usefulness	2.8	1.10	3.1	1.21
	Importance	2.6	1.52	2.7	0.95
Online Notes	Frequency of Use/Access	2.2	1.47	3.2	1.30
	Usefulness	2.7	1.97	3.6	1.59
	Importance	2.5	1.97	3.8	1.48
Links to Illustrations Within the Lesson Notes	Frequency of Use/Access	1.0	0.00	2.1	1.35
	Usefulness	1.5	0.55	2.9	1.68
	Importance	1.3	0.52	2.6	1.81

Links to Other Resources Within the Lesson Notes	Frequency of Use/Access	1.2	0.41	1.3	0.76
	Usefulness	1.3	0.52	2.2	0.98
	Importance	1.3	0.52	2.0	1.10
Summary at the End of the Lesson Notes	Frequency of Use/Access	2.7	1.21	2.7	1.11
	Usefulness	2.5	1.38	3.0	1.20
	Importance	2.3	1.21	2.9	1.13
Homework Assignment at the End of the Lesson Notes	Frequency of Use/Access	3.0	1.67	4.0	0.87
	Usefulness	4.2	1.17	4.2	0.67
	Importance	3.8	1.47	4.1	0.78
Reading Assignment at the End of the Lesson Notes	Frequency of Use/Access	3.3	1.86	3.0	1.32
	Usefulness	3.7	1.75	3.9	0.83
	Importance	3.7	1.75	3.8	0.71

The curriculum outcomes were derived from the AP Calculus AB topical outline which was published by The College Board. Noncompleters reported greater access to the curriculum outcomes for each lesson in the online notes, although access in each group ranged from infrequent to very frequent. Both groups thought the feature fairly useful and important with noncompleters assigning higher importance and usefulness to the feature. One completer indicated the feature was not important for him because he could already see what was important through the lessons. The curriculum outcomes could also be obtained using the *Outcomes Linked to Lessons* hyperlink.

The corresponding page numbers of the textbook were listed at the end of each curriculum outcome. Noncompleters reported more frequent access to the text reference pages for the lesson outcomes, and thought the feature more useful and important. Individual completers indicated either very frequent or very infrequent access to the feature and that the feature was either extremely useful and important or not useful nor important at all. Students could also obtain this information from other parts of the Mathematics 4255 website using the *Course Outline* or the *Outcomes Linked to Lessons* hyperlinks. As well, the reading assignments at the end of each lesson provided students with the appropriate text pages for the following lesson.

The beginning of the online lesson notes contained a simple advance organizer in the form of a statement of the form "In this lesson you will … " which gave a brief overall view of what would be covered in the lesson. Noncompleters reported greater access to the advance organizer. Both groups thought it fairly useful and important.

One of the important goals of learning calculus is to read mathematical texts. Hence the course developers refrained from placing a set of notes online that could replace those in the text. The online notes consisted mainly of the curriculum outcomes for each lesson, comments about examples in the text, and homework and reading assignments. Noncompleters indicated greater frequency of access to the online notes, and thought them much more useful and important. Completers thought this feature either extremely useful and important or not at all useful nor important.

Within a small fraction of the lesson notes were links to illustrations of the concepts being explained in the lesson. For example, in the lesson notes for section 6.2, there was a hyperlink to an animation that illustrated the solid of revolution formed when the region bounded by $y = x\sin x$ and the x-axis was rotated around the x- axis. Noncompleters indicated greater frequency of access to the hyperlinks to illustrations within the online lesson notes. They also thought the feature more useful and important than completers, who thought the feature neither useful nor important.

Also within a small fraction of the lesson notes were links to other resources. For example, in the lesson notes for section 6.2, there was a hyperlink to an external website that contained an animation which illustrated the approximation of the solid of revolution formed when n representative discs are rotated around the x-axis. Both groups reported equal access to the hyperlinks to other resources within the online lesson notes. Noncompleters thought the feature more useful and important than completers, who thought the feature neither useful nor important.

Near the end of the lesson notes was a brief summary of the lesson contents. Both groups reported equal access to the summary at the end of the online lesson notes. Noncompleters thought the feature more useful and important.

Also near the end of the lesson notes was a list of odd-numbered questions selected from the exercises in each section of the text. Noncompleters reported more frequent access to these homework questions and both groups thought this feature very useful and important. One noncompleter requested a solution guide for these questions. However, a student study and solution guide was available which contained the worked solutions to the odd-numbered questions, and the importance and availability of this guide was stressed by the instructor in both the June and September orientation sessions. The answer for each odd-numbered question was also located at the back of the text.

At the end of the notes for each lesson was the reading assignment for the next lesson, which contained the pages of the text that dealt with the concepts of the next lesson. Both groups reported nearly equal access to the reading assignment and both groups thought the feature very useful and important.

Online Self-Evaluation

Table 10
Online Self-Evaluation: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Online Section Multiple Choice Quiz	Frequency of Use/Access	3.0	1.58	2.4	1.13
	Usefulness	3.7	1.37	3.3	0.71
	Importance	3.8	1.47	3.2	0.83
Online Solutions to Section Multiple Choice Quiz	Frequency of Use/Access	3.0	1.58	2.7	1.33
	Usefulness	3.8	1.47	2.9	1.27
	Importance	3.7	1.51	3.2	1.30
Online Topic Multiple Choice Quiz	Frequency of Use/Access	3.0	1.87	2.1	0.83
	Usefulness	3.8	1.30	2.8	1.39
	Importance	3.8	1.30	2.9	1.46

Online Solutions to Topic Multiple Choice Quiz	Frequency of Use/Access	3.2	2.05	2.3	1.39
	Usefulness	3.8	1.30	2.5	1.51
	Importance	4.0	1.41	2.9	1.64
Online Topic Long Answer Test	Frequency of Use/Access	3.0	1.55	1.4	0.79
	Usefulness	3.8	1.47	2.3	1.25
	Importance	4.0	1.55	2.3	1.25
Online Solutions to Topic Long Answer Test	Frequency of Use/Access	3.0	1.55	1.4	0.79
	Usefulness	3.8	1.47	2.3	1.25
	Importance	4.0	1.55	2.3	1.25

At the end of each section of the online notes was a hyperlink to a multiple-choice quiz which tested the concepts covered in that section. Each question had four distracters and the correct distracter was selected by clicking the appropriate radio button. When all questions had been attempted, the student submitted the quiz and it was automatically corrected and a mark immediately reported to the student. The report also contained the correct answer for any questions the student answered incorrectly. Completers reported greater access to the multiple-choice quizzes at the end of each section and both groups thought the feature useful and important. Completers indicated that the feature helped them study for examinations and was an indicator of how well they understood the section. A noncompleter indicated the quizzes were very good for extra practice. Another indicated that she did not have time to do the quizzes, as she was behind in most of her work.

Completers reported greater access to the solutions to the multiple-choice quizzes at the end of each section. They also thought the feature more useful and important. This result may be expected given that completers indicated higher access to the multiple-choice quizzes. One completer indicated that some of the solutions were incorrect and very misleading. A noncompleter made a similar statement.

At the end of each topic was a hyperlink to a multiple-choice quiz which tested the concepts covered in that topic. The quizzes were similar in format to the section quizzes and were corrected and reported in the same manner. However, these quizzes were generally longer and tested more material. Completers reported greater access to the multiple-choice quizzes at the end of each topic. They also thought the feature more useful and important.

Completers reported greater access to the solutions to the multiple-choice quizzes at the end of each topic. They also thought the feature more useful and important. This result may be expected given that completers indicated higher access to the multiple-choice quizzes. One completer again indicated that some of the solutions were incorrect.

At the end of each topic was a hyperlink to a long answer test on the topic. These tests were copies of the long answer sections of tests given previously in the course by the instructor. Completers reported greater access to the online long answer tests at the end of each topic. They also thought the feature much more useful and important. One completer indicated that the long answer tests were a great indication of how well he knew the material. A noncompleter indicated that she did not have time to complete the online tests and quizzes. Another indicated that she never used this feature. A third noncompleter indicated that she did not use this feature as much as the multiple choice quizzes.

At the end of each long answer test was a hyperlink to the solutions for that test. Completers reported greater access to the solutions to the online long answer tests at the end of each topic. They also thought the feature much more useful and important. This result may be expected given that completers indicated higher access to the long answer examinations.

Concept Map

Table 11
Concept Map: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Concept Map	Frequency of Use/Access	2.2	1.33	1.6	0.98
	Usefulness	2.8	1.33	1.8	1.04
	Importance	2.8	1.33	1.9	1.36

From the *Concept Map* hyperlink (see Figure 8) students could link to one concept map covering the entire course. Completers reported greater access to this map. This group thought the feature more useful and important than noncompleters, who thought the feature only somewhat useful and important. Two noncompleters indicated they did not know this feature existed. A completer stated that the concept map "put everything at my fingertips and made it easy to relate the things I learned."

Course Outline

Table 12
Course Outline: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Course Outline	Frequency of Use/Access	1.8	1.33	2.7	1.60
	Usefulness	2.7	1.03	3.0	1.41
	Importance	2.5	1.04	3.1	1.55

The *Course Outline* hyperlink (see Figure 8) linked to a outline containing the percentage weight given to each of the three units, hyperlinks to the lessons of each section, and the suggested amount of class time which should be spent on each lesson. At the end of the outline was an allotment of time for the online classes, tests and the midterm. Noncompleters reported greater access to the course outline. They considered it more useful and important than completers. One completer indicated it was nice knowing what material had to be covered in the course. Two noncompleters indicated that they never used this feature. The sections of the text covered in the course were identified by the instructor during the online classes and could also be ascertained from the reading assignments.

Outcomes Linked to Lessons

Table 13
Outcome Linked to Lessons: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Outcomes Linked to Lessons	Frequency of Use/Access	2.5	1.00	2.6	1.51
	Usefulness	2.8	0.50	3.3	1.58
	Importance	2.8	0.50	3.3	1.58

The *Outcomes Linked to Lessons* hyperlink (see Figure 8) contained a list of the curriculum outcomes for each section and a hyperlink to the corresponding lesson associated with each outcome. It was a convenient way for students to quickly review the outcomes of the course and, if necessary, link to the lessons which reviewed those outcomes. Both groups reported equal access to the outcomes linked to lessons. Noncompleters thought the feature more useful and important. One completer indicated he did not use this feature and another indicated she did not know it existed. A noncompleter indicated she used the feature infrequently. Students could also link to the lesson notes, which also contained the associated outcomes, using the *Unit 1*, *Unit 2*, or *Unit 3* hyperlinks (see Figure 7).

Course Evaluation

Table 14
Course Evaluation: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Course Evaluation	Frequency of Use/Access	2.6	1.52	2.5	1.51
	Usefulness	3.0	0.82	3.3	1.28
	Importance	3.3	0.96	3.3	1.16

The instructor, with the approval of the School District Board Office, predetermined the percentages assigned to each evaluation category. Test were weighted at 30%, assignments at 15%, one midterm examination at 25%, a final examination at 25% and one project at 5%. Both completers and noncompleters groups reported equal access to the course evaluation. Both groups thought the feature equally useful and important. A completer indicated that she did not know the feature existed.

External Links

Table 15
External Links: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Ask Mr. Calculus	Frequency of Use/Access	1.0	0.00	1.5	0.71
	Usefulness	2.3	1.53	2.0	0.00
	Importance	2.3	1.53	2.0	0.00
Ucalc	Frequency of Use/Access	1.0	0.00	1.0	0.00
	Usefulness	2.5	0.71	1.0	0.00
	Importance	2.5	0.71	1.0	0.00
Calculus Toolkit	Frequency of Use/Access	1.0	0.00	1.0	0.00
	Usefulness	2.5	0.71	1.0	0.00
	Importance	2.5	0.71	1.0	0.00
Real World Applications of Differential Equations	Frequency of Use/Access	2.0	1.00	1.0	0.00
	Usefulness	2.0	0.82	1.0	0.00
	Importance	2.0	0.82	1.0	0.00
Online Integrator	Frequency of Use/Access	1.5	0.71	1.0	0.00
	Usefulness	2.3	0.58	1.0	0.00
	Importance	2.7	0.58	1.0	0.00

Mathematical Models	Frequency of Use/Access	2.0	1.41	1.0	0.00
	Usefulness	2.7	0.58	1.0	0.00
	Importance	2.7	0.58	1.0	0.00
Calculus Projects	Frequency of Use/Access	1.0	0.00	—	—
	Usefulness	2.5	0.71	—	—
	Importance	2.5	0.71	—	—
Calculus Labs	Frequency of Use/Access	1.0	0.00	—	—
	Usefulness	2.5	0.71	—	—
	Importance	2.5	0.71	—	—
Academic Assistance	Frequency of Use/Access	1.0	0.00	—	—
	Usefulness	2.5	0.71	—	—
	Importance	2.5	0.71	—	—
Calculus Tutorials	Frequency of Use/Access	2.0	0.00	—	—
	Usefulness	2.5	0.71	—	—
	Importance	2.5	0.71	—	—
Mathematical Applets	Frequency of Use/Access	1.3	0.58	3.0	0.00
	Usefulness	2.3	0.50	4.0	0.00
	Importance	2.3	0.50	3.0	0.00

Note. Noncompleters indicated unawareness of four of the external links.

Students could access several external websites that provided academic assistance in the form of tutorials and labs, and applications of calculus in the form of models and projects. One website contained a number of java applets, which illustrated important calculus concepts but often took minutes to download. With the exception of the link to the mathematical applets, completers reported greater access to the eleven external links and thought them more useful and important. While only one noncompleter reported access to the applets, he accessed them more frequently and indicated they were much more useful and important. Two-thirds of students in the course never used the external links or did not know they existed, although they were made aware of their existence at both orientation sessions. Students who used the external links thought them fairly useful. One completer indicated that the external links should receive more emphasis.

Communication

Table 16
Communication: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Bulletin Board	Frequency of Use/Access	3.3	1.51	3.1	1.54
	Usefulness	3.3	1.51	3.3	1.12
	Importance	3.5	1.51	3.2	1.20
Chat	Frequency of Use/Access	1.0	0.00	1.3	0.58
	Usefulness	2.3	1.15	1.3	0.58
	Importance	2.3	1.15	1.3	0.58
Private E-mail	Frequency of Use/Access	3.8	1.60	2.2	1.39
	Usefulness	4.0	1.26	2.2	1.20
	Importance	4.2	1.33	2.3	1.32

Both groups reported equal access to the electronic bulletin board on the welcome page (see Figure 6) and both groups thought it fairly useful and important. Messages on the bulletin board were always repeated through private e-mail and were posted in the online calendar. This gave students a choice as to how they received information. One noncompleter indicated she obtained her messages through e-mail instead of through the bulletin board.

Chat

The chat room (see Figure 6) at the Mathematics 4225 website was similar to other Internet chat rooms except that only participants in the course could enter it. Both groups accessed the chat room very infrequently. Completers thought it a more useful and important feature. One completer indicated that this feature should have been used to confirm students' interpretations of questions or to have a question explained by someone other than the instructor.

Private E-mail

This feature (see Figure 6) was similar to regular Internet e-mail except that it could only be accessed by participants in the course. The feature allowed messages to be transferred privately, so sender and recipient only could view them, or publicly, so messages could be viewed by all participants. Completers reported more frequent access to the private e-mail feature and thought it much more useful and important. One completer indicated she only used the e-mail to contact the instructor for solutions and workings for the homework questions. A noncompleter indicated that she used the e-mail only to contact the instructor. Another indicated that she used the feature only to receive messages and not to send them.

Password

Table 17
Password: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Password	Frequency of Use/Access	2.5	1.97	2.4	1.81
	Usefulness	3.2	2.04	3.1	1.77
	Importance	3.2	2.04	3.1	1.77

The instructor initially assigned students a password which was required to access the Mathematics 4255 website. Students were then directed to use the password feature to obtain a secure password. Individual passwords allowed the student-tracking feature of WebCT to be used, which permitted the instructor to track the date of the student's first and last access to the mathematics website, the number of times the student accessed the site, and the number of articles posted and read. A password also enabled students' personal information, such as marks, to remain confidential. Both groups reported equal access to the password option and thought it equally useful and important.

Overall opinion of the Mathematics 4255 website

Most students thought the website was well done overall, being well laid out, easy to access, and very easy to use. Students used the website mainly to access features such as the lesson notes, calendar, private e-mail, bulletin board, online tests and student records. One completer stated: “I mainly used [the website] for checking the calendar, e-mail and bulletin board as well as lessons and online tests. Mainly, I stuck to the textbook and only used the website for keeping up to date on events and getting the questions I had to do.” However, students were unaware of other features available on the website that might have benefited them. One noncompleter stated, “There were a lot of tools that may have been a benefit to me, had I known they existed. I was in the course three months but knew very little about the website.” Students indicated that the lack of availability of an Internet ready computer at home, or their computer skills, hindered them from exploring the website, and that the other features of the website were not needed or were of little importance. The majority of students did not know that any of the eleven external links existed and few used the concept map, course evaluation, course outline, links to illustrations and other resources within the lesson notes, or chat. One completer indicated that the external links should receive more emphasis. Another indicated a desire to use the chat feature to determine if other students “see questions like you see it and perhaps sometimes they can explain a question better than the teacher can.” A noncompleter stated, “I would have been a lot more comfortable using a book in a normal classroom setting because I am not a computer person so I just used the website when I knew I had to.”

Discussion of Results (Features of the Mathematics 4225 Website)

Introduction

This section presents possible explanations for unexpected survey results and for disparities between completers and noncompleters as they relate to the features of the Mathematics 4255 website. The lower importance assigned to Internet-ready computers by both groups was surprising given the essential role of computer technology in a web-based learning environment. Also unexpected was the low use of the external links and the widespread unawareness of the existence of these links. As well, the effectiveness of the concept map as reported by students was far below its demonstrated potential for improving student learning. The value assigned to some of the website features varied greatly from student to student and this variation was not confined to a single group but existed in both groups. The results of this section of the survey also point to a performance orientation to learning. Subsequent survey results indicate this orientation exists to a greater degree in noncompleters. As well, noncompleters did not value the self-evaluation to the degree reported by completers. In contrast, completers assigned less value to the online notes.

Internet-Ready Computers

Given the special emphasis on computer technology in Mathematics 4225, it would be expected that access to, and use of, Internet ready computers would be more important than the results in Table 3 indicate. At least one computer with an Internet connection was present in each school, as each site had a high-end computer specifically dedicated to the AP courses in the school. However, this AP computer was used for up to three other AP courses so access to the computer outside of the online mathematics classes was limited to times when it was not in use by another AP class or student. Access to other Internet-ready computers was also limited, as computer labs at all three sites were frequently fully occupied by other classes. The student AP computer at one site was not connected to the school network. Hence AP students at this site did not have access to the network hardware and software. The software used for online classes also dictated that connection to the Internet using the AP computer be made through a modem, and heavy Internet traffic during the school day usually meant that access to the Mathematics 4255 website was much slower than access using the satellite dish that served the computer lab at each school. The same access problems occurred for students with home Internet-ready computers. As well, students' home computers did not have the special hardware and software needed for full synchronous communication. These limitations made Internet capable computers less useful and important to students than would be expected in a web-based course offering both synchronous and asynchronous communication.

Alternate Access to Website Features

Table 18
Alternate Access to Website Features

Feature(s)	Methods of Access
Dates for Classes	Calendar, E-mail, Bulletin Board
Dates for Examinations, Assignment Due Dates, Special Notices	Calendar, E-mail, Bulletin Board, Online Classes
Curriculum Outcomes, Text Reference Pages, Online Notes	Course Outcomes hyperlink, Outcomes Linked to Lessons hyperlink, Lessons hyperlink
Self Evaluation	Quizzes for Self Evaluation hyperlink, Tests for Self Evaluation hyperlink, Topic hyperlink

Students indicated that some components of the online calendar were only fairly or somewhat useful and important. However, for many of these components, students had multiple methods of obtaining the same information (see Table 18), some of which did not require access to the online calendar. The slots for the online and offline classes were identified in each student's course schedule. Dates for extra classes, dates for examinations, assignment due dates, and other special announcements were also communicated during the online classes, and sent to students via e-mail and the bulletin board. Students who used these alternate means to access the special announcements and class and evaluation dates would not have needed to access the same information through the online calendar, and so may have attributed less importance to the components of the calendar containing this information. As well, if students had obtained elsewhere the information contained in the online calendar, they would not have had as great a need for the calendar compile option.

Some or all of the information in other links at the Mathematics 4255 website, such as the *Course Outline*, *Outcomes Linked to Lessons*, and *Course Evaluation* hyperlinks, may also have been obtained elsewhere. The *Course Outline* hyperlink contained the percentage weights given to the three units, hyperlinks to each lesson and the suggested amount of class time which should be spent on each lesson. Students were given the percentage weights early on in the course during an online class, and these weights could also have been obtained using the *Course Evaluation* hyperlink. The lessons could have been linked from the *Lessons* hyperlink, and from the *Outcomes Linked to Lessons* hyperlink. The online calendar also contained lessons timelines. These timelines were notices which the instructor placed in the calendar to inform students that they should have completed a certain group of lessons by a particular date. Students who viewed these notices could have scheduled their time without referring to the suggested class time in the *Course Outline* link. The *Outcomes Linked to Lessons* hyperlink contained the curriculum outcomes for each section and a link to the lesson associated with each outcome. The outcomes were also repeated in the lesson notes, which could be reached from the *Lessons* hyperlink or the *Course Outline* hyperlinks. Students who used these alternative means to access the information contained in these links may have then attributed less importance to the *Course Outline*, *Outcomes Linked to Lessons*, and *Course Evaluation* hyperlinks.

Online Notes

Completers consistently indicated that the components of the lessons on the Mathematics 4255 website were less useful and important (see Table 9). In particular, completers reported much less access to the online notes and would therefore have gained much less exposure to the other components of the lessons which were also part of the online notes. Secondary school mathematics students have been accustomed to having a complete set of notes other than those in the text and the class may have had the same expectation for Mathematics 4225. However, the online notes augmented, instead of replaced, those in the textbook, and some students may have found them less useful and important as a result. Completers in particular may have been able to learn the concept using the notes from the online classes and by reading the text itself. As a result their need for online notes may not have been as great.

Homework and Reading Assignments

In a reversal of the trend in most of Table 9, both groups indicated that the homework and reading assignments at the end of the each lesson were very useful and important. Some students indicated that these components were the main reason they accessed the lessons. One noncompleter stated, "I didn't have much access to computers with [the] Internet, but when I did I made use of the lessons and homework questions. I found them useful." A completer noted, "Mainly, I stuck to the textbook and only used the website for keeping up to date on events and getting the questions I had to do."

Students may have assigned higher priority to these components as they realized it was necessary to complete the homework questions and read the textbook to fully understand the material.

Communication

Table 19
Communication: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	On-Site Participants		Off-Site Participants	
		Average	Standard Deviation	Average	Standard Deviation
Bulletin Board	Frequency of Use/Access	2.0	0.93	4.3	0.83
	Usefulness	2.4	0.73	4.1	0.93
	Importance	2.3	0.70	4.3	0.83
Private E-mail	Frequency of Use/Access	1.7	0.70	3.9	1.45
	Usefulness	2.1	0.64	3.8	1.48
	Importance	2.0	0.53	4.0	1.50

The usefulness and importance of the bulletin board and e-mail may have been misrepresented by the average results given in Table 16. With the exception of the students who were onsite with the instructor, completers and noncompleters reported that at least one of these two features was very or extremely useful and important (see Table 19). Those attributing low usefulness and importance to one of the features also reported low frequency of access to that feature, indicating they used only one of the features. The onsite students had frequent face-to-face contact with the instructor and would not have needed these features to any great degree. Hence the lower usefulness and importance attributed to these features by the onsite students masks that attributed to the features by students not onsite with the instructor.

Performance Versus Mastery Learning Orientation

The *My Record* feature allowed students to view their marks and compare them with those of their classmates. Both groups indicated that this feature was very useful and important. A completer stated, "You can see how you're doing at different times." A noncompleter noted, "It was nice to be able to see all my marks and compare them with the class statistics." All evaluation was returned to students, and since they could have obtained the weights of each category of evaluation from the Mathematics 4255 website, they could have easily calculated their own marks. This indicates that students may have valued the comparison aspect of the feature. The importance attributed to this aspect may indicate that many of the students in the course adopted a performance, as opposed to a mastery, orientation to learning. Learners with a performance orientation believe ability is evidenced by doing better than others and self worth is determined by one's ability to perform. Learning itself is viewed only as a way to achieve and enhance self worth. Associated with a mastery orientation is a focus on the intrinsic value of learning, where individuals are focused on mastering and understanding content and are willing to engage in the process of learning (Ames, 1992). A performance orientation to learning often does not lead to the level of understanding of concepts necessary to be successful in a first calculus course.

External Links

During the September 1999 orientation session, the instructor met with the students in Mathematics 4225 in the computer lab at one of the sites so students could become familiar with some of the features of the Mathematics 4255 website. The instructor briefly explained what was contained in each of the eleven external links and the class viewed some of the calculus java applets obtained using the Mathematics Applets hyperlink. However, ten students either never used the external links or reported that they did not know they existed. In not using these external links, students missed the opportunity for tutorials, labs, modeling, and the application of calculus concepts in real world situations. The tutorials and labs could have helped students more fully understand calculus concepts that were only partially explored during online classes because of time restrictions resulting from technical problems. Modeling and applications could have increased the connection between calculus and other disciplines and made the course more relevant and realistic.

Self Evaluation

Noncompleters consistently indicated that the components of the online self-evaluation were less useful and important (see Table 10). One noncompleter noted that for the multiple- choice examinations, "some of the answers were incorrect and misleading." For some noncompleters, the confusion created by these incorrect answers may have led them to avoid further use of the self-evaluation. Some noncompleters also indicated that they did not have time to use the self-evaluation and indicated it was not important to them. As well, some noncompleters withdrew from the course very early in the year and may not have had sufficient experience with the online evaluation to realize any benefit from it.

The disparity between completers and noncompleters in the importance and usefulness of the self-evaluation was most apparent with the long answer self-tests. The lower priority attributed to the long answer tests by noncompleters may arise from the fact that feedback from the long answer tests was not instantaneous since the test was not corrected automatically nor the answers displayed immediately, as occurred with the multiple choice quizzes. The long answer tests were much more time consuming as students had to write out much longer solutions for these questions than for the multiple choice questions. They then had to link to the solutions to the long answer tests and had to review and correct their own solutions. The time required to complete and correct the long answer tests may have been a deterrent to noncompleters, some of whom indicated that they did not have enough time to complete the much shorter multiple-choice tests.

Concept Map

This feature may have been less effective because of the low number of concept maps created and because the map was constructed by the instructor rather than the students. Naidu and Bernard (1992a) state that meaningful learning is characterized by a personal grasp of the relationships between concepts. However, an instructor-constructed map meant that students did not have to personally develop the relationships between the various calculus concepts presented in the course in order to construct the map. Bernard and Naidu (1992) also state that consistent and prolonged practice with concept maps is needed if a deep understanding of concepts is to be achieved. The one instructor-constructed concept map in the course was not enough to positively affect student learning and may explain the low priority given to the feature.

Features of the AP Mathematics Text Book

Introduction

The textbook selected for Mathematics 4225 was the fifth edition of *Calculus of a Single Variable: Early Transcendental Functions* by Larson, Hostetler and Edwards. It was chosen because it was the text used in first year calculus at Memorial University of Newfoundland, the university attended by the majority of Newfoundland and Labrador high school graduates, and because it was deemed acceptable by the online course developers, as it matched the 1999 topical outline for AP Calculus AB, published by The College Board. This section presents students' assessment of the different features of the course text. Average results presented in each table range on a continuum from 1 to 5 and can be interpreted using Table 20.

Table 20

Interpretation of Results from Part One of the Survey

Evaluation Criteria	Result				
	5	4	3	2	1
Frequency of Access/Use	> 6 times per week	5-6 times per week	3-4 times per week	1-2 times per week	Less than once per week
Usefulness	Extremely useful	Very useful	Fairly useful	Somewhat useful	Not at all useful
Importance	Extremely important	Very important	Fairly important	Somewhat important	Not at all important

Rules and Formulae

Table 21
Rules and Formulae: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Basic Differentiation Rules	Frequency of Use/Access	4.5	0.84	3.7	1.41
	Usefulness	4.8	0.41	4.4	0.88
	Importance	4.7	0.82	4.3	1.00
Basic Integration Rules	Frequency of Use/Access	4.5	0.84	4.5	0.71
	Usefulness	4.7	0.52	5.0	0.00
	Importance	4.0	1.67	5.0	0.00
Geometry Formulas	Frequency of Use/Access	3.3	1.63	3.1	1.46
	Usefulness	3.8	1.17	4.4	0.97
	Importance	4.0	1.26	4.0	1.29
Trigonometry Rules	Frequency of Use/Access	3.2	1.83	3.6	1.24
	Usefulness	3.5	1.22	4.2	0.97
	Importance	3.3	1.51	4.2	0.97
Algebra Rules	Frequency of Use/Access	2.8	1.60	2.9	1.62
	Usefulness	3.2	1.47	3.9	1.13
	Importance	3.2	1.72	3.8	1.04

The basic differentiation rules were located inside the front cover of the text and in two chapters. Completers reported greater access to the basic differentiation rules although both groups thought them very useful and important. One noncompleter referred to these rules frequently and another indicated they were extremely useful.

The basic integration rules were also located inside the front cover of the text and in two chapters. Both groups reported very frequent access to the basic integration rules and both groups thought them extremely useful and important. However, only two noncompleters reported use of the integration tables as the remaining noncompleters had withdrawn from the course by the time integration began.

The formulas of geometry were located on the page following the inside of the front cover and included the area and volume formulas for common geometric figures plus the law of cosines and the Pythagorean Theorem. Both groups reported equal access to the rules of geometry. Noncompleters found them slightly more useful, although one noncompleter indicated they were useful, but usually unnecessary. Both groups indicated they were very important.

The rules of trigonometry were located inside the back cover of the text and included the definitions of the trigonometric function in terms of the sides of a right triangle plus the trigonometric identities and formulas from level three Advanced Mathematics 3201. Noncompleters reported greater access to the rules of trigonometry. This group also thought them more useful and important. These rules were frequently needed to simplify trigonometric expressions before and after using the basic differentiation or integration rules.

The rules of algebra were located on the page before the inside of the back cover of the text and included factoring rules, The Fundamental Theorem of Algebra, the Binomial and Rational Roots Theorems, factoring rules and the laws of exponents. Both groups reported equal access to the rules of algebra. Noncompleters thought them slightly more useful and important.

Graphics, Tips, Notes, Remarks, & Summaries and Guidelines

Table 21
Graphics, Tips, Notes, Remarks, & Summaries and Guidelines: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Colour Computer Graphics	Usefulness	3.2	1.60	3.1	1.45
	Importance	3.2	1.60	3.0	1.41
Technology Tips	Usefulness	2.5	1.38	2.1	1.46
	Importance	2.3	1.21	2.0	1.41
Historical Notes	Usefulness	1.7	0.82	1.6	0.74
	Importance	1.3	0.52	1.6	0.74
Summaries and Guidelines	Frequency of Use/Access	3.5	0.84	3.3	1.12
	Usefulness	3.8	1.17	3.9	0.93
	Importance	3.8	1.17	3.9	1.17
Remarks	Usefulness	3.0	1.26	2.9	1.36
	Importance	2.8	1.33	3.2	1.39

Colour graphics were used in the text to help students establish the connection between the graphical, numerical and algebraic representations of mathematical concepts. Both groups indicated that the colour graphics were fairly useful and important. One noncompleter indicated she did not notice this feature and another stated that the graphics confused her.

Dispersed throughout the notes in the text were technology tips which were intended to encourage students to use graphing utilities or symbolic algebra systems for exploration, problem solving, visualization, and verification of results. The tips also identified errors that might have resulted from the use of technology. Both groups indicated that the technology tips were somewhat useful and important. One noncompleter indicated she knew they existed, but did not use them. Two other noncompleters indicated that they only read some of the tips. One completer noted the tips were not updated for current advanced calculators.

The text included some notes in the margins that related some of the history of calculus and described some famous mathematicians. Both groups indicated that these historical notes were neither useful nor important. One noncompleter indicated she did not notice the notes existed.

Many of the sections contained summaries that identified the core ideas and procedures of the section. Both groups reported equal access to the summaries and guidelines in the text and indicated they were very useful and important.

The text also contained special instructional notes to students in the form of remarks. They appeared after definitions, theorems, or examples and were designed to give additional insight, help avoid common errors, or describe generalizations. Both groups indicated that the remarks in the text were fairly useful and important. One noncompleter indicated she did not read them and another did not use them. One completer noted that the remarks "were good because they showed little mistakes that can be made and how the problem should be done right."

Exercises and Problems

Table 22
Exercises and Problems: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Graphing Calculator Exercises	Usefulness	3.3	1.63	2.9	0.93
	Importance	3.2	1.83	2.9	0.93
Challenging Homework Problems	Usefulness	3.5	1.22	3.1	1.36
	Importance	3.2	1.47	3.5	1.31
Variety of Homework Problems	Usefulness	4.3	1.03	3.8	1.09
	Importance	4.3	1.03	3.7	1.12
Many Basic Computation Problems	Usefulness	3.3	1.51	4.0	0.71
	Importance	3.3	1.51	4.0	0.71

Facility with graphing calculators is a very important skill in AP Calculus AB and is a requirement for the AP Calculus AB examination. The exercises contained some questions that were identified as requiring a graphing calculator. However, questions on

Mathematics 4225 evaluation instruments rarely required the use of a graphing calculator. Completers indicated that the exercises using the graphing calculator were slightly more useful and important. In both groups, survey responses ranged from 1 to 5. One completer noted they were "VERY USEFUL" and drew a happy face after her comment. One noncompleter indicated she rarely did any questions requiring a graphing calculator.

The text included challenging exercises that could be assigned as special projects to individuals or groups. Completers indicated that challenging homework problems were slightly more useful but slightly less important. One noncompleter indicated that she omitted many of these problems. Another indicated the solution manual for these questions would be very useful.

The exercise sets contained a range of problems that provided opportunities for writing, projects, multi-part applications, manipulation of real data and for solving qualitative and quantitative problems. The exercises also included true or false questions that typically required students to produce a counter example for those statements which were false. Both groups indicated that a variety of homework problems were useful and important.

The beginning of each exercise set contained several questions aimed at skill development. Noncompleters indicated that many basic computation problems were more useful and important. One noncompleter indicated a preference for the solution guide for these questions. One completer indicated that these questions "helped you to understand the basics before doing the more difficult problems."

Appendices

Table 23
Appendices: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Appendix A: Proofs of Selected Theorems	Frequency of Use/Access	1.3	0.58	2.5	2.12
	Usefulness	3.0	2.0	3.0	1.83
	Importance	3.0	2.0	3.0	1.83
Appendix B: Basic Differentiation Rules for Elementary Functions	Frequency of Use/Access	1.3	0.58	3.0	0.00
	Usefulness	3.4	1.14	4.0	0.00
	Importance	3.4	1.14	4.0	0.00
Appendix C: Integration Tables	Frequency of Use/Access	1.8	0.84	4.0	0.00
	Usefulness	3.8	0.98	4.0	0.00
	Importance	3.8	0.98	4.0	0.00
Appendix D: Real Numbers and the Real Number Line	Frequency of Use/Access	1.5	0.58	3.0	0.00
	Usefulness	3.0	0.71	4.0	0.00
	Importance	3.0	0.71	4.0	0.00

The text sometimes referred students to the proofs of selected theorems in Appendix A. Only four students accessed Appendix A, as the remaining students were unaware of its existence. One noncompleter indicated it was extremely useful and extremely important, the other indicated that Appendix A was somewhat useful and important. One completer indicated it was fairly useful and important, the other indicated the appendix was neither useful nor important. One noncompleter and two completers indicated that the appendix would have been very useful, had they known it existed. Another completer indicated she did not have time to browse the book, she only had enough time to finish the homework questions.

Only four students accessed Appendix B of the text, as the remaining students were unaware of its existence or referenced another part of the text for the same rules. One noncompleter accessed Appendix B of the text and indicated it was very useful and very important. Three completers indicated it was fairly useful and important.

Only one noncompleter accessed Appendix C of the text, as the remaining noncompleters had withdrawn from the course before the appendix was needed. He indicated it was very important and useful. Four completers accessed Appendix C and indicated it was useful and important. The two completers who did not know Appendix C existed, indicated that the appendix would have been extremely useful and important. As with the differentiation rules, integration rules could have been referenced in other parts of the text, although the tables in the appendix were more comprehensive.

Only one noncompleter accessed Appendix D of the text. He indicated it was very important and useful. Three completers accessed Appendix D. They indicated it was fairly useful and important. The remaining eleven students were unaware of its existence.

Answers and Solutions

Table 24
Answers and Solutions: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Answers to Odd-Numbered Exercises	Frequency of Use/Access	3.7	1.51	4.2	0.97
	Usefulness	4.5	0.84	4.6	0.52
	Importance	4.5	0.84	4.3	1.00
Study and Solution Guide	Frequency of Use/Access	3.3	1.63	3.0	1.15
	Usefulness	3.8	1.33	4.0	0.58
	Importance	3.8	1.33	3.9	0.69

Noncompleters indicated slightly higher access to the answers for the odd-numbered exercises located at the back of the text. Both groups indicated they were very useful and important. Several students indicated they would have preferred the answers to all the questions in the solution manual or that the solutions to the assigned even-numbered questions be posted online since these questions tended to be difficult. Providing the solution to all exercises was problematic as many of the assignment questions were selected from the even-numbered exercises in the text. The solutions for the homework questions that were assigned from the even numbers were available from the instructor upon request, and at least one student did request the solutions to many of the even-numbered questions that were assigned.

The questions set for homework were almost exclusively the odd-numbered questions and the student study and solution guide provided students with the complete solutions for the odd-numbered exercises in the text. It was indicated at both the June and September orientation sessions that a solution manual was available and should be purchased, as it allowed students to check their entire solution instead of just the final answer. Both groups reported nearly identical access to the student study and solution guide and both groups indicated that it was very useful and important. One noncompleter indicated it needed to be larger and in more detail. Several students indicated they did not have it. One completer reported that the guide was beneficial in understanding a question when you were not sure how to do it. Another completer expressed a preference for all the solutions.

Prerequisites Chapter

Table 25
Prerequisites Chapter: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Prerequisites Chapter	Frequency of Use/Access	3.2	1.33	3.3	1.32
	Usefulness	4.0	0.89	4.0	0.87
	Importance	4.0	0.89	4.0	0.87

Chapter one of the text was a review of many of the concepts learned in Advanced Mathematics 3201. Very few new concepts were presented in this chapter. Both groups reported equal access to this prerequisites chapter and indicated that it was very useful and important. One noncompleter reported he would have been lost without it. Another reported that it was "the only chapter I actually understood." A completer indicated that the prerequisites chapter was "really good to remind you of what you did" and did a better job at explaining concepts than the high school, level three, Advanced Mathematics 3201 textbook.

Overall Opinion of the Textbook

Students reported that the text was very good, with helpful reference tables, good examples, and elaborate explanation at the beginning of each section. One noncompleter noted that the many practice problems were a great help in studying for examinations. A completer noted that the text "explained everything well, and didn't assume you remembered everything from [the] math you did previously." She added, "The proofs (that I knew were there) were well written and easily understood." However, three noncompleters indicated that they were unaware of some important features of the text. Three completers and one noncompleter expressed a preference for more examples with explanation. Another completer noted that the text should be Canadian, with metric instead of standard units. Many of the students requested a solution guide for all practice problems.

Discussion of Results (Features of the AP Mathematics Textbook)

Introduction

This section provides possible explanations for the high rating received by some features of the course text and the low rating received by others. The rules of differentiation and integration, and the prerequisites chapter were rated highly by both groups, whereas the appendices of the text were seldom used. The summaries and guidelines in the text were viewed as more important and useful than the graphics, tips, notes and remarks. Noncompleters afforded a higher rating to the rules of algebra, geometry and trigonometry, and to certain types of exercises and problems.

Rules and Formulae

Students indicated that the rules of differentiation and integration were either very or extremely important (see Table 21). They also indicated a high frequency of access to these rules. The importance attributed to these rules may indicate an emphasis on questions requiring symbolic manipulation and reference to these rules. Had emphasis been placed on the interpretation of the result of symbolic manipulation instead of on symbolic manipulation itself, students would not have needed to reference these rules as frequently as much of the symbolic manipulation could have been performed automatically by the graphing calculator, software such as Derive, or dedicated symbolic manipulation software such as the online integrator (see Table 15). The emphasis placed by students on these rules as opposed to the use of technology to find derivatives and antiderivatives indicates that skill with symbolic manipulation was deemed to be an important part of the course. Significant portions of The College Board Calculus AB examination required students to directly apply the basic rules of differentiation and integration without the aid of technology. Therefore facility in symbolic manipulation was a necessity for those students wishing to earn university credit.

Noncompleters consistently rated the algebraic, geometric, and trigonometric rules and formulas as more useful and important. However, the vast majority of the rules of geometry, trigonometry, and algebra presented in the text are part of the provincial high school advanced mathematics curriculum and should be familiar to all students in Mathematics 4225, as Advanced Mathematics 3201 was a prerequisite for enrolment in the calculus course. The relative importance attributed to these rules by noncompleters may indicate that these students have not retained material covered at the high school level as well as completers, due to a surface, as opposed to a deep, understanding of these rules.

Graphics, Tips, Notes, Remarks, & Summaries and Guidelines

With the exception of the summaries and guidelines, the features of the text contained in Table 21 were rated at best as being only fairly useful and important. However, at least one of these features may be very important in the learning of mathematical concepts. The colour computer graphics feature was a very important aspect of the text as it enabled the connection between the graphical, numerical and algebraic representations of concepts. This connection is necessary if the important yet difficult process of encapsulation is to occur. This feature may have been rated as less useful and important because students were not encouraged to make the connection between the different representations of concepts.

Students reported a low rating for the technology tips and historical notes. The rating received by the technology tips may have been a result of students' capability with graphing calculators. Students in the course were well acquainted with graphing calculators and may have already been cognizant of many of the tips presented in the text. However, some of the tips were still very informative and important, especially for those students who were not completely familiar with the graphing calculator. For instance, it was entirely possible never to experience a situation in high school mathematics where the graph given by the calculator was incomplete. However, students were informed of this possibility early in the text and given an example where the graphing calculator drew an incomplete graph. The instructor did not refer to the historical notes nor were they necessary for any of the homework or evaluation in the course. As a result students did not view them as useful or important.

Students rated the summaries and guidelines in the text relatively higher. This feature often contained the steps that students could follow in solving word problems or performing procedures such as finding the intervals on which a function is increasing or decreasing. The relative importance and usefulness attributed to this feature of the text may have resulted from a need for students to refer frequently to the summaries and guidelines to complete template questions as opposed to questions requiring students to grapple with the ideas of calculus.

Exercises and Problems

Noncompleters reported that completing challenging homework problems and graphing calculator exercises was less useful and important than completing many basic computation problems and being exposed to a variety of homework problems (see Table 22). There was no insistence by the instructor that homework be completed, students were not required to submit their homework to the teacher for examination, nor was homework a component of the course evaluation. The technical problems experienced during the online classes may have made it especially difficult for noncompleters to understand the basic calculus concepts and they instead relied on the basic computation problems and a variety of homework questions to develop an understanding of these concepts.

Appendices

Most students did not access any of the appendices of the text, although three completers and one noncompleter indicated that at least two of the appendices would have been very or extremely useful and important, had they known they existed. Students may have been unaware of appendices because the instructor seldom, if ever, referenced them. Proofs do not appear on the AP Calculus AB examination and so received little emphasis in Mathematics 4225. The information contained in Appendices B and C was available elsewhere in the text, and the instructor referenced these locations much more frequently. The information in Appendix D was also available elsewhere in the text and should have already been part of each student's mathematical knowledge.

Prerequisites Chapter

Students indicated that the prerequisites chapter was very useful and important, although the vast majority of the content of this chapter had been covered in previous high school mathematics courses, mostly in Advanced Mathematics 3201. An excellent mark in Advanced Mathematics 3201 was prerequisite for enrolment in Mathematics 4225 so students should have had an excellent understanding of the material in the prerequisites chapter and should not have assigned such importance to the chapter. The importance assigned to the prerequisites chapter may indicate that students had a surface level understanding of the concepts in Advanced Mathematics 3201 and required the chapter for a review of those concepts. While reviewing the concepts in the prerequisites chapter, students were also adjusting to a new way of interacting and learning and the familiarity of the prerequisite material may have helped ease the transition into this new, web-based learning environment.

Features of the Online Classes

Introduction

The sites were connected synchronously for three, fifty-minute periods every eight days using Microsoft NetMeeting and Meeting Point. The Meeting Point software allowed the server connecting all four sites to send an incoming signal from one site to all other sites. This allowed audio, video, whiteboard, and chat signals to reach each site simultaneously. The instructor and the students at each site used a dedicated computer located in the distance education room in each school. This high-end computer was equipped with one microphone, one set of speakers, and one whiteboard and pen. If necessary, sites were provided with additional monitors so the number of students viewing a single monitor was kept below seven.

Length and Number of Online Classes

Students in Mathematics 4225 had five, fifty-minute classes every eight days, three of which were online. The remaining two classes were offline. Noncompleters reported a need for four to five, one-hour classes per week. This was higher than completers, whose modal response was three, one-hour classes per week. However, one completer preferred four to five, one-hour classes per week and two others preferred one class at the end of the week for one to two hours. One of these completers also indicated that the classes should have consisted of the teacher and only one site, as the audio quality was superior in this situation.

Communication

Table 26
Communication: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Audio	Usefulness	4.5	0.84	4.4	0.53
	Importance	4.5	0.84	4.6	0.53
Whiteboard	Usefulness	4.7	0.52	4.3	0.50
	Importance	4.8	0.41	4.3	0.50
Chat	Usefulness	2.5	1.22	2.6	1.72
	Importance	3.0	1.55	2.7	1.89
Video	Usefulness	1.5	0.58	3.0	0.00
	Importance	1.3	0.50	3.5	0.71

The four survey items in Table 26 refer to the functions of Microsoft NetMeeting that were available for use during the online classes. Both groups indicated that the audio was very useful and important. One completer noted that the audio was really important but added, “too bad it didn’t work half the time.” A noncompleter made a similar comment. Another noncompleter indicated the audio was “really terrible” and that “a lot of time was wasted ensuring that all schools could hear the teacher or telling students to turn off their microphones to reduce the feedback.” This problem was also noted by another noncompleter who stated, “We spent a lot of time trying to fix audio problems.” A fourth noncompleter stated that “If the audio had worked, the classes would have been very useful, but we missed a lot of online classes because of technical problems." A fifth noncompleter noted that the audio "was useful when it worked, which wasn't often." Still another noncompleter noted that, "while I was in the course, we had lots of audio problems and it made it difficult to hear." In all, nine students indicated some kind of consistent problem with the audio.

The whiteboard feature in Mathematics 4225 allowed participants to create written solutions to questions, as it permitted writing and the freehand drawing of symbols. The whiteboard also allowed students to view previous pages, to quickly add graphics, and to save entire lessons as a whiteboard file. Both groups indicated that the whiteboard was very useful and important. One noncompleter indicated it was "great" and "was probably all that worked." Another noncompleter noted that sometimes it would lock up and the instructor would be writing but nothing would appear on student monitors.

The chat allowed students in the course to communicate only using the keyboard, and therefore did not allow the use of mathematical symbols. Messages could be sent privately or publicly. Both groups indicated the chat was somewhat useful and fairly important. Two noncompleters noted that it was great when the audio was not working. A completer made a similar comment.

Video was not available in the course because of bandwidth restrictions. The two noncompleters who completed this survey item indicated that video would have been fairly useful and important. Completers indicated it was neither useful nor important.

Saving Whiteboard Files

Table 27
Saving Whiteboard Files: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Ability to Save Whiteboard Files	Usefulness	3.8	1.84	4.2	0.67
	Importance	3.8	1.84	4.3	0.71

Both groups indicated that the ability to save whiteboard files was very useful and very important. One completer indicated, "It was good being able to go to a lesson we may have missed." Another thought it was a great feature because "you don't have to keep your mind on writing the notes. Rather you can watch and observe the instructor and get the notes after class." Three noncompleters made similar comments. Another noted this feature was very helpful and drew a happy face after his comment.

Student Expectations versus Student Perception of Instructor Activity

Table 28
Student Expectations vs. Student Perceptions of Instructor Activity During Online Classes

Group	Online Class Activity				
	Explain Big Picture	Explain Major Concepts	Show How Concept is Applied	Use Concept in Problem Solving	Other
Completers					
Expectation	0	5	3	5	2
Perception	0	5	2	5	1
Noncompleters					
Expectation	0	7	7	8	0
Perception	0	7	7	2	2

The activities of the instructor during online classes matched the expectations of students. However, noncompleters wanted the instructor to spend more time using the concept in problem solving situations. A completer requested that the instructor use worksheets to identify areas where students need remediation and then use the online classes to give remediation in that area. One completer and two noncompleters indicated that the instructor used a significant amount of online class time to diagnose and correct technical problems, especially those related to the audio.

Overall Opinion of Classes

Table 29
Importance and Use of Classes: Mean and Standard Deviation of Survey Responses

Survey Item	Evaluation Criteria	Completers		Noncompleters	
		Average	Standard Deviation	Average	Standard Deviation
Online Classes	Importance	2.8	1.47	3.9	0.78
Extra Online Classes	Importance	2.3	1.37	2.9	1.45
Offline Classes	Importance	3.3	1.51	3.3	0.87

Noncompleters indicated that online classes were more important although responses from completers ranged from extremely important to not important at all. Noncompleters also indicated that extra online classes were more important. Both groups attributed equal importance to offline classes. One noncompleter noted that online classes were a very important aspect of AP mathematics courses, as it was one of the few times interaction occurred with the teacher and with students from other sites. She also expressed a need for more training with Microsoft NetMeeting, the software used to connect the sites for online classes. Ten students stated that the technology used for online classes needed improvement, especially the audio, and that the usefulness of these classes suffered as a result. A completer indicated that she believed "the online classes would have been really good if the Internet was dependable enough for it and if everything worked like it should." One completer found it difficult to interact with the teacher and ask questions during online classes. She stated that "turning the mike on and off [to reduce the feedback] was a hassle and it was harder to stop him online in the middle of an example." A noncompleter indicated that the time lost due to the audio problems never gave her the chance "to ask all my questions or time to fully understand what the instructor was teaching." Eleven students stated that technical problems were the main reason why they missed online classes. Four completers and two completers indicated a preference for an on-site teacher as opposed to online classes and most students wanted more face-to-face classes.

Discussion of Results (Features of the Online Classes)

Introduction

This section offers possible explanations for the variation in the results reported by the two groups and for the difference in value assigned to certain features of the online classes. The audio and whiteboard components of communication received a higher rating than the chat and video components. Noncompleters attributed greater importance and usefulness to the online classes and reported a discrepancy between the expected and actual activity of the instructor during online classes.

Communication

Students assigned a much higher rating to two of the four functions of NetMeeting (see Table 26). The audio and whiteboard feature were very or extremely useful and important to both groups. The functioning of the audio feature in NetMeeting was similar to the audio system used in other high school distance education courses delivered by the Telemedicine Centre, although the technical difficulties made the NetMeeting audio system more difficult to use. However, the whiteboard feature of NetMeeting offered several advantages over the system used in current high school non-AP courses. The whiteboard in the Telemedicine model does not allow viewing of previous pages, as one page must be erased before the next can be started. This requirement prevents students from saving the whiteboard contents for later viewing. As well, this model does not allow graphics to be quickly added to a whiteboard page. Any diagrams must be drawn and stored on the distance education system at all sites. A command to open the diagrams must be sent from the teacher site to each site over telephone lines which may at times have heavy traffic and a slow transfer rate. The whiteboard function of NetMeeting allows students to save entire lessons on their computer and it allows graphics to be quickly drawn or inserted. These graphics are transferred almost instantly to the computer at each site. However, it is the ability to share applications among sites that most distinguishes the NetMeeting system from the Telemedicine model. Sharing applications allows remote sites to view and manipulate files from computer applications, even if the application itself does not reside on the computer at the remote site. Applications such as Geometer's Sketchpad and Derive could be opened on the instructor's computer and control of the computer screen could be transferred to any site. This permits the collective exploration of mathematical concepts. Unfortunately, bandwidth restricted the sharing of applications.

The chat and video functions received much lower ratings. Students could have accessed the whiteboard feature only with the AP student computer, as the feature required special input software and hardware. However, they could have easily used the chat feature for communication at any time, as it required only a keyboard. However, the chat feature did not allow the use of mathematical symbols and the absence of this important attribute almost certainly detracted from the functionality of the feature. The low use of the chat feature may also have been caused by a lack of access to computers. The AP student computer was sometimes in use by other AP classes when an offline mathematics class was scheduled. For half the year, one site had practically no access to the school computer lab during their offline classes, as it was filled to capacity by another class. As well, a significant number of students could not use the chat feature to communicate outside of school hours, as they did not have Internet access at home. The low use of the chat feature may have also been due to students' unwillingness to contact students at other sites. Students seemed reluctant to contact students outside their immediate site, and since face-to-face communication would have been more efficient within a site, the chat feature may not have been a useful tool for many students.

Bandwidth restrictions prevented the use of real time video as the use of this feature further deteriorated the audio and whiteboard signals, although it was possible to have a still picture of the instructor or one other site. However, this feature may be important in helping students from remote sites become familiar with each other and in fostering a sense of belonging to a virtual classroom.

Student Expectation versus Student Perception of Instructor Activity

Students generally indicated that the instructor activity during online classes matched their expectations. However, noncompleters reported a discrepancy between the expected and perceived use of concepts in problem solving during online classes. These students expected the instructor to use the concepts learned in problem solving situations to a much greater extent. During online classes, the instructor placed emphasis on first explaining major concepts and showing how they could be applied, relegating problem solving to the last part of the class. Often, a significant amount of class time was lost diagnosing and correcting audio problems, and as a result, little or no time remained for problem solving. This time may have been adequate for completers, but not for noncompleters.

Importance and Use of Classes

Although both groups thought offline classes equally important, noncompleters assigned greater importance to both types of online classes. The online classes permitted more interaction with the instructor and other students and enabled the construction of knowledge through dialogue. They also allowed students to test their knowledge, receive immediate feedback, and compare their knowledge with their classmates. It may also have made them feel less isolated. Noncompleters may have had a greater need for the synchronous interaction provided by the online classes, especially since varying offline class schedules and a lack of access to an Internet-ready computer in school or at home made synchronous interaction during these offline classes somewhat difficult.

Features of Evaluation

Introduction

This section presents the results of the student surveys with regard to the percentage weights assigned to the examinations, assignments and projects, and the frequency, length and difficulty of these evaluation instruments.

Evaluation Weights

In Mathematics 4225, unit tests were assigned twice the value of assignments (tests-30%, assignments-15%) and the midterm and final examinations were given equal weight (25% each). The project was worth 5%. Noncompleters generally preferred that unit tests be given a weight only slightly higher than that actually assigned to assignments. They also preferred that the final examination be assigned more weight than the midterm examination, and that the weight of the project be doubled. Completer preferred that assignments be weighted slightly higher and that the midterm examination be weighted slightly lower than in the actual course evaluation. The weights that completers preferred assigned to the unit tests, final examination, and project were similar to the course evaluation.

Frequency, Length and Difficulty of Evaluation

Students generally agreed that six unit examinations was an appropriate number, and most noncompleters indicated the length and difficulty of these examinations was appropriate. However, three noncompleters indicated that examinations needed to decrease in length and difficulty. Completers generally indicated that unit examinations were appropriate in length and difficulty, although two completers indicated that they needed to decrease in length. One completer indicated that the difficulty of the examinations needed to be more consistent.

Students generally agreed that the number, length and difficulty of assignments was appropriate. However, two noncompleters indicated that the assignments were too long in some cases and another indicated that the assignments were too difficult. One completer indicated that the assignments were too long.

Overall Opinion of Evaluation

Overall, students reported that the evaluation was acceptable except for the unit tests, which some students found inconsistent in length and often too difficult. One completer indicated that a greater number of assignments of shorter length should have been given. Another completer indicated that the evaluation prepared him well for the AP Calculus AB final examination.

Discussion of Results (Features of Evaluation)

The examinations were inconsistent in structure and length. Early examinations consisted mainly of multiple-choice questions, which students could do in the time allotted, whereas examinations at the end of the course consisted entirely of long answer questions that some students were only able to complete with extra time. The difficulty of the examination questions was similar to that of the online evaluation. Multiple-choice questions were selected from a teacher's guide designed to accompany the textbook, and were very similar in nature to the online multiple-choice examinations. Long answer questions were similar to those in the online long answer examinations. Occasionally, long answer questions were taken from previous College Board AP Calculus AB examinations.

Examinations were usually written during the offline classes. The different offline class schedule at one of the sites and the rescheduling of examinations in a particular school due to school activities made it difficult to ensure that the common examination was still a valid and reliable measure of students' knowledge. However, there was no indication that any examination in the course was compromised.

With the exception of the assignment on the prerequisites chapter, all assignment questions were selected from the even-numbered exercises in the text. However, these assignments were inconsistent in length. Usually, each chapter or part chapter had its own assignment and the amount of material covered by each assignment could vary greatly. Consequently, the number of questions on assignments varied from six to thirty-two. One assignment was so long that only one student fully completed it. The assignments were also inconsistent in difficulty. The prerequisites assignment posed little difficulty for most students. However, the assignment on applications of the derivative was a source of frustration for many students.

Students' suggestions were not limited to the length and difficulty of tests and assignments, as one student suggested an increase in the number of projects. Students may take more ownership of the course if they have input into the types of evaluation used in the course and the percentage weight assigned to each type of evaluation.

Features of Collaboration

Students reported that on-site, student-student collaboration was frequent and very important. However, student-student collaboration between sites was practically nonexistent and was not at all important to students. Nine students indicated that they participated in a study group and those students who did not indicated that they would have benefited from such a group. Four noncompleters and four completers indicated that they received help from someone other than the instructor. Overall, students reported benefits from collaboration with fellow students. One noncompleter indicated that she needed another student with which to discuss homework. She also indicated that working with another student helped to keep her on schedule in terms of course work. Another indicated that communicating with students on a regular basis "helps us to build our knowledge better." A completer indicated that without student-student collaboration, finishing the course would have been impossible. However, three noncompleters and one completer indicated a need for increased student-teacher collaboration. Another completer noted, "The ability to collaborate is there although it is hindered by technology."

Features of Social Interaction

Students reported that the meeting in June 1999 was not worthwhile, and many did not attend this orientation session, possibly because they had not made the decision to enroll in Mathematics 4225. However, they indicated that the orientation session in September 1999 was important because it allowed them to interact with the teacher and their fellow classmates and to get some exposure to the Mathematics 4255 website. One noncompleter indicated that the orientation needed to be more explanatory as there were a lot of aspects of the course of which she was unaware. Students preferred an increase in the number of social events during the year. Noncompleters requested an average of three to four such events and completers requested three events. Overall, socialization was important to students and they indicated that socialization between sites needed to be increased.

Discussion of Results (Features of Collaboration and Social Interaction)

The lack of emphasis placed on collaboration between remote sites is an area of concern, especially for the welfare of a single student who may be a future participant in the course. Without collaboration between sites, this student would not realize many of the benefits experienced by students who have others with which to discuss their work and aid in their understanding of concepts. This type of collaboration may also be crucial in overcoming feelings of isolation that a lone student may quickly develop if the student views themselves as a disjointed subset of the whole. The course placed little emphasis on the development of collaboration between remote sites. Students were not required to collaborate with other students at remote sites, and in some cases were unaware that this was even permitted. During an informal discussion with the instructor, one completer noted that she did meet students from remote sites during her personal social activities. When the instructor suggested that she contact some of them for help with assignments or homework, her response was, “We can do that?”

Seven of the eight students who received help from someone other than the instructor were not onsite with the instructor. Another noncompleter indicated that he did not receive extra help but that teachers at his site should have been more helpful. Students who were onsite with the instructor received extra help from the instructor after the online class periods and during the offline classes when no students from the remote sites requested help. This indicates that the request by some students for more student-teacher collaboration is justified, especially considering the difficulties experienced during online classes.

During the orientation in September 1999, all AP students met at one location and spent one hour in the mathematics computer lab with their instructor exploring the Mathematics 4255 website. Students spent the afternoon socializing and becoming acquainted with their classmates and instructor. Student responses indicate that the amount of time exploring the textbook and website should be increased. For this reason alone, their request for other opportunities for socialization appears to be appropriate.

Chapter Nine: Results of Part Two of the Survey

Introduction

In part two of the survey, the six completers and nine noncompleters were asked to indicate their degree of agreement with statements about a number of factors which research literature indicated was important in the learning of calculus and in learning at a distance. The results are presented as the numerical averages of the responses of completers and noncompleters and can be interpreted on a continuum using Table 30. Responses that differ greatly from the average response to each statement are noted.

Table 30
Interpretation of Results from Part Two of the Survey

Result	Interpretation
1	Strongly Disagree
2	Disagree
3	Agree
4	Strongly Agree

Table 31
Mathematical Modeling: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Lack of Mathematical Modeling	2.0	0.89	2.0	0.00
Preferred Self-Developed Models	2.0	0.00	2.1	0.69

The homework and assignment questions, as well as a limited number of test questions, required students to work with mathematical models. In all cases, the model was provided to the students. Both completers and noncompleters disagreed with the statement that there was a lack of mathematical modeling and two completers strongly disagreed with the statement.

All completers and most noncompleters disagreed with the statement that they would prefer to develop mathematical models for themselves. However, two noncompleters indicated they would prefer to develop their own models.

Table 32
Class Size: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Class Size Too Large	1.5	0.55	1.2	0.44
Class Size Too Small	2.2	0.98	1.6	1.01

Both completers and noncompleters were satisfied with the class size, with the exception of one completer and one noncompleter, both of whom thought the class size was too small.

Table 33
Amount of Material: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Too Much Material	2.5	0.55	2.5	0.55

The course material was determined by the 1999-2000 topical outline for AP Calculus AB. Overall, the class was undecided as to whether there was too much material in the course. However, two completers and one noncompleter, who were onsite with the instructor, disagreed with the statement. Another noncompleter agreed with the statement.

Table 34
Word Problems: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Too Many Word Problems	2.3	0.82	2.5	0.53
Too Few Word Problems	2.0	0.63	1.5	0.53
Too Many Template Word Problems	1.8	0.75	2.0	1.20
Word Problems Relevant and Realistic	3.3	0.52	2.9	0.35

Completers thought the number of word problems was sufficient, with the exception of one student who believed there were too many word problems and one who thought there were too few. Noncompleters were undecided as to whether there were too many word problems but disagreed with the statement that there were too few word problems. Four noncompleters strongly disagreed with this statement.

The majority of the questions in the exercise sets of the sections of the text specifically dedicated to word problems had a solution nearly identical to the sample problems presented in the notes for the section. Generally both completers and noncompleters disagreed with the statement that there were too many of these template word problems. Four noncompleters and two completers strongly disagreed with the statement. However, one completer and three noncompleters indicated that there were too many problems of this type. One completer indicated a need for an example in the text similar to each homework problem.

Both completers and noncompleters generally agreed with the statement that word problems were relevant and realistic. Two completers strongly agreed with this statement. However, one noncompleter disagreed with this statement.

Table 35
Answers: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Not Required to Justify Answers	1.7	0.52	1.7	0.49
Emphasis on Getting the Correct Answer	2.0	0.63	2.6	0.73
Emphasis on Determining the Reasonableness of the Answer	3.2	0.75	2.4	0.52
Emphasis on Getting the Right Answer the First Time	1.8	0.41	2.2	0.67

Both completers and noncompleters disagreed with the statement that they were not required to justify their answers. Two completers and two noncompleters strongly disagreed with the statement.

Practically all completers disagreed with the statement that there was an emphasis on getting the correct answer as opposed to understanding the meaning of the answer, and one completer strongly disagreed with the statement. However, one completer agreed with the statement. Six noncompleters indicated an emphasis on getting the correct answer. However, two noncompleters disagreed with this statement and one noncompleter strongly disagreed with the statement.

Most completers agreed with the statement that there was an emphasis on determining the reasonableness of the answer. Two completers strongly agreed with this statement. One completer and five noncompleters disagreed with the statement.

Completers and most noncompleters disagreed with the statement that there was an emphasis on getting the right answer the first time, and one completer and one noncompleter strongly disagreed. However, three noncompleters agreed with the statement.

Table 36
Memorization: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Calculus a Collection of Rules to be Memorized	2.5	0.55	2.3	0.71
Expected to Memorize a Lot of Rules	3.0	0.63	3.0	0.71

The class was undecided as to whether they viewed calculus as a collection of rules and procedures that must be memorized in order to succeed in the course. One noncompleter strongly disagreed with the statement. However, three completers and four noncompleters did view the course in this manner.

On average both completers and noncompleters agreed with the statement that they expected to have to memorize lots of rules and procedures when they enrolled in the course. However, one completer and two noncompleters disagreed with this statement.

Table 37
Symbolic Manipulation: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Large Amount of Symbolic Manipulation	3.0	1.10	2.5	0.53
Could Do Well With Symbolic Manipulation	2.6	0.49	1.9	0.60

An emphasis on symbolic manipulation is a common weakness identified in many traditional calculus courses. Completers agreed with the statement that there was a large amount of symbolic manipulation in the course, and three completers strongly agreed with this statement. Overall, noncompleters were undecided. However, four noncompleters agreed with the statement.

Completers were undecided as to whether they could do well in the course by being good at symbolic manipulation instead of having an understanding of the material. Most noncompleters disagreed with this statement, indicating that an understanding of the material was required. Two noncompleters strongly disagreed with the statement. However, one noncompleter agreed with the statement.

Table 38
Student-Teacher Interaction: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Teacher and Students Should Work Together More	3.0	0.63	3.4	0.53
Too Little Interaction with Teacher	2.5	0.84	3.4	0.73
Too Many Interactions with Teacher	1.8	0.41	1.3	0.50
More Face-to-Face Meetings with Instructor	3.0	0.71	3.4	0.53

Most completers and all noncompleters agreed that students and the teacher should work together more. One completer and four noncompleters strongly agreed with this statement. One completer disagreed with the statement.

In addition to the synchronous interaction that occurred during the online classes, students could have interacted asynchronously with the instructor using e-mail, the bulletin board, the calendar, and the fax and telephone. Completers were undecided as to whether there was too little interaction with the teacher, but two completers indicated that there was too little interaction. Noncompleters agreed with the statement. One completer and five noncompleters strongly agreed with the statement. One completer thought the interaction was sufficient.

During the previous year, the instructor made several visits to each site and interacted with the students in small groups, answering specific questions or reviewing concepts. However in 1999-2000, the instructor made only two visits to each of the remote sites. Both completers and noncompleters indicated that they preferred more face-to-face meetings with the instructor and one completer and four noncompleters strongly agreed with this statement. However, one completer disagreed with the statement. Near the end of part two of the survey, students were again asked if they would have liked more face-to-face meetings with the instructor and what they would liked to have done in these meetings. Eight noncompleters and three completers felt the need for more face-to-face meetings with the instructor, indicating that they would have liked to discuss problems with homework and their understanding of concepts. The four students indicating that more face-to-face meetings were not needed were onsite with the instructor or had previous distance education experience.

Table 39
Writing: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Too Little Writing	1.8	0.41	2.1	0.35
Too Much Writing	2.7	0.82	2.0	0.00

The evaluation instruments or homework questions seldom required students to write more than a brief statement of their answers to any question, or to write an explanation of the reasoning behind their solutions. Completers disagreed with the statement that there was too little writing in the course but were generally undecided in their decision about too much writing. However, one completer strongly agreed that there was too much writing. Most noncompleters indicated the level of writing was acceptable. However, one noncompleter indicated that there was not enough writing in the course.

Table 40
Topics Linked to Careers: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Topics Applied to Career	2.2	0.98	3.0	0.71
Wide Range of Application	2.3	0.82	2.8	0.67

Practically all completers disagreed with the statement that they would have preferred more emphasis on how topics could be applied to their chosen career and one completer strongly disagreed with the statement. However, one completer strongly agreed with this statement. Noncompleters generally agreed with the statement, and two noncompleters strongly agreed. However, two noncompleters disagreed with the statement.

Completers were undecided as to whether they preferred exposure to a wide range of applications, but one completer strongly disagreed with the statement. On average noncompleters agreed with the statement and one noncompleter strongly agreed. Three noncompleters disagreed with the statement.

Table 41
Feedback: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Enough Good Feedback on Homework	2.2	0.75	1.7	0.50
Prompt Feedback on Homework	2.2	0.75	1.9	0.64
Enough Good Feedback on Assignments	2.7	0.52	2.3	0.46
Prompt Feedback on Assignments	2.8	0.41	3.0	0.00
Enough Good Feedback on Tests	2.5	0.84	2.3	0.46
Prompt Feedback on Tests	2.8	0.41	3.0	0.00

Students received feedback on homework mainly during online classes but the technical problems experienced during these classes meant that many questions remained unanswered. However, only one completer consistently requested feedback on homework questions outside of the online class periods. Most completers and all noncompleters disagreed with the statement that they received enough good feedback from the instructor on their homework, and one completer and three noncompleters strongly disagreed. Three of these students were on-site with the instructor and could have obtained feedback on their homework during offline, as well as online classes. Two completers, who were not onsite with the instructor, agreed with the statement.

Completers and noncompleters generally disagreed with the statement that they received prompt feedback from the instructor on their homework. Two noncompleters and one completer strongly disagreed. However, two completers and one noncompleter agreed with the statement. The noncompleter who agreed with the statement was onsite with the instructor and received feedback on homework during offline classes. One completer who agreed with the statement had previous distance education experience and the other frequently asked that the solutions to homework questions be done during online classes and consistently requested that the instructor fax the full solutions to the homework questions that could not be answered online.

Assignments were faxed or mailed to the instructor, corrected, and returned to students by fax or mail. However, a complete copy of the solutions was not faxed to each site, as happens with the secondary school distance education courses in the province that are delivered using the Telemedicine model. Four completers and two noncompleters indicated they received enough good feedback on assignments. However, six noncompleters and two completers indicated the opposite.

Five completers and eight noncompleters agreed that they received prompt feedback on assignments. Only one completer indicated the opposite.

Examinations were received and returned in the same manner as assignments. Again, complete solutions for the entire examination were not provided to students. Four completers and two noncompleters agreed that they received enough good feedback on tests. However, one completer disagreed and another strongly disagreed. Six noncompleters indicated they did not receive appropriate feedback.

With the exception of one completer and one noncompleter who did not respond to the item, all students indicated that they received prompt feedback on tests.

Table 42
Pace: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Not Enough Time to Understand all the Topics	2.2	0.75	3.0	0.00
Pace too fast	2.0	0.63	2.9	0.69
Pace too Slow	1.8	0.41	1.7	0.49
Often 2 or 3 Sections Behind	3.2	0.75	3.7	0.50

The class time allotted to each of the three units was based on the percentage weights given to each unit of the course (Unit 1 - 15%, Unit 2 - 45%, Unit 3 - 40%). The difficulties experienced during online classes meant that concepts were often not dealt with in detail to ensure the course material was covered in the time allotted. Most completers disagreed with the statement that there was not enough time to understand all the topics, and one completer strongly disagreed. Two completers agreed with the statement. Noncompleters agreed with the statement.

Mathematics 4225 did not extend over the full school year, as is customary in most two credit, high school courses, as the AP Calculus AB examination was written before mid-May. The course progressed at a pace which allowed the course material to be finished by the end of April to allow students a full week to prepare to write the Mathematics 4225 final examination, and nearly another week to review this final and prepare for The College Board examination. Completers indicated that the pace was acceptable. However, one completer indicated the pace was too fast. Noncompleters also indicated the pace was too fast. However, two noncompleters indicated the pace was acceptable.

Most completers and all noncompleters agreed that they were often 2 or 3 sections behind the instructor. Two completers and six noncompleters strongly agreed with this statement. However, one completer disagreed.

Table 43
Standards and Expectations: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Standards and Expectations of the Course too High	2.2	0.75	2.3	0.50
Standards and Expectations of the Course too Low	1.8	0.41	1.9	0.35
Standards and Expectations Higher than Tests	2.2	0.41	2.0	0.00
Standards and Expectations Lower than Tests	1.8	0.41	2.7	0.82
Standards and Expectations Higher than Assignments	2.2	0.75	2.1	0.38
Standards and Expectations Lower than Assignments	1.8	0.41	2.6	0.53
Standards and Expectations Higher than Examination on May 5	2.0	0.63		
Standards and Expectations Lower than Examination on May 5	2.3	0.51		
Standards and Expectations Higher than Examination on May 11	2.0	0.00		
Standards and Expectations Lower than Examination on May 11	1.7	0.58		

Overall, both completers and noncompleters indicated that the standards and expectations of the course were reasonable. However, two completers and three noncompleters indicated they were too high.

Completers indicated that the standards and expectations of the course were on a par with those of the examinations. However, one completer thought standards and expectation of the course were higher than those for the tests. Noncompleters indicated that they felt the standards and expectations of the course were not higher than those of the examinations but were undecided as to whether or not the course standards and expectations were lower than those of the examinations.

Completers indicated that the standards and expectations of the course were on a par with those of the assignments. However, two completers indicated the course standards and expectations were higher than those of the assignments. Noncompleters indicated that the standards and expectations of the course were not higher than those of the assignments, but were undecided as to whether or not these standards were lower than those of the assignments.

Completers indicated that the course standards and expectations were on a par with those of the Mathematics 4225 final examination. One completer indicated that course standards and expectations were higher, and two completers indicated that course standards and expectations were lower. Noncompleters did not respond to this item as none of these students wrote the Mathematics 4225 final examination.

Completers indicated that the course standards and expectations were on a par with those of the AP Calculus AB examination. Noncompleters and two completers did not respond to this item, as they did not write the AP final examination.

Table 44
Proof: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Too Many Proofs	1.8	0.41	2.0	0.00
Too Few Proofs	2.3	0.82	2.3	0.46

Completers disagreed with the statement that there were too many proofs in the course. However, they were undecided as to whether or not there were too few proofs in the course. Noncompleters indicated the number of proofs was acceptable. However, two noncompleters and three completers indicated there were too few proofs.

Table 45
Numerical Methods: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Too Little Emphasis on Numerical Methods	2.2	0.41	2.5	0.58

Nearly all completers disagreed with the statement that there was too little emphasis on numerical methods. However, one completer thought there was too little emphasis. Noncompleters were undecided. Five noncompleters did not respond to the item.

Table 46
Class Presentations: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
More Opportunity for Class Presentations	1.5	0.55	1.6	0.53

Teacher guides published by The College Board indicate that independent or group presentations are effective in helping students learn early calculus concepts. However, both groups disagreed with the statement that there should be more opportunity for class presentations and three students from both groups strongly disagreed with the statement.

Table 47
Realism: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Course Not Realistic	2.0	1.10	2.4	0.53

Some of the homework questions involved applications of the concepts in the course but only the course project specifically required students to use what they learned in a truly novel, real-life situation. Most completers disagreed with the statement that the course was not realistic (i.e. not connected to things in the real world). However, one completer strongly agreed with the statement. Noncompleters were undecided, although three noncompleters agreed with the statement.

Table 48

Technology: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Graphing Calculator Helped Understanding	3.5	0.55	3.0	0.71
Geometer's Sketchpad Helped Understanding	1.2	0.41	1.3	0.49
Derive Helped Understanding	1.3	0.82	1.3	0.46
Used Technology for Symbolic manipulation and Focused on Understanding and Application	2.3	0.52	2.5	0.58
Initially Apprehensive Because I Knew I Would Have to Use a Computer	1.7	0.52	2.4	1.01

It was expected that students would use at least three pieces of technology to help them understand the concepts in the course. Mathematics 4225 required that students possess, and be familiar with, a graphing calculator. Students were also expected to use a dynamic geometry software package called Geometer's Sketchpad and a computer algebra system tool called Derive. However, students were given little training in the use of either of the software packages.

All completers and most noncompleters agreed that the graphing calculator aided their understanding. Three completers and two noncompleters strongly agreed with the statement. However, two noncompleters indicated that the graphing calculator did not aid in their understanding.

The developers of the course created several files using Geometer's Sketchpad to help illustrate some of the concepts of the course. These files were sent to each site on a disk and it was recommended that a copy of the software be installed on the AP computer at each site. The lesson notes directed students to these files at the appropriate time. These files could also have been illustrated to the sites using the software on the instructor's computer, even if the remote sites did not have the software on their own computer. However, the lack of bandwidth prevented the sharing of applications during online classes. Both completers and noncompleters disagreed with the statement that Geometer's Sketchpad helped their understanding. Five students from each group strongly disagreed with the statement.

It was also recommended that students obtain a copy of Derive, which is used in some first year calculus classes at Memorial University of Newfoundland. A free demonstration version of Derive could have been obtained online at the Texas Instruments website. This piece of software could have performed much of the symbolic manipulation required in the solution to homework and assignment problems, and allowed students to more easily explore different mathematical approaches in solving a problem. This application could also have been shared from the instructor computer, had sufficient bandwidth been available. Most completers and all noncompleters disagreed with the statement that Derive helped their understanding. Five completers and six noncompleters strongly disagreed with the statement. However, one completer indicated that Derive did help in his understanding of the concepts.

Facility with symbolic manipulation was a requirement of AP Calculus as parts of the AP Calculus AB examination required the straightforward applications of basic calculus rules but prohibited the use of any technology that could perform the symbolic manipulation. Completers generally disagreed with the statement that the course used technology for symbolic manipulation so emphasis could be placed on applications and understanding. However, two completers indicated that technology was used for this purpose. Noncompleters were undecided. Five noncompleters omitted the item.

Students were asked to self-assess their degree of computer competency on their Mathematics 4225 registration form (see Appendix B). Many were familiar with standard word processing and spreadsheet software and some indicated they had constructed their own web pages. However, no student indicated they were familiar with any of the communication software used in the course and only two had previously used a whiteboard. Completers disagreed with the statement that they were initially apprehensive about doing the course because of computer use and two completers strongly disagreed with the statement. On average, noncompleters were undecided, although four noncompleters agreed with the statement and one strongly agreed.

Table 49
Link to Other Disciplines: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Calculus Linked to Other Disciplines	3.0	0.00	2.4	0.74

The instructor did set homework questions that made some connection to other disciplines, such as physics and biology. However, only the project allowed students to choose from a group of problems connected to a wide range of disciplines. This project was scheduled for the time between the writing of the AP Calculus AB examination in mid-May and the closure of school in mid-June, so only completers would have submitted it. Completers agreed with the statement that calculus was linked to other disciplines. Noncompleters were undecided, although one noncompleter strongly disagreed with this statement.

Table 50
Learning: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Web-Based Course Suited the Way I Learn	2.0	0.89	1.4	0.73
This Course Helped Me Become a Better Learner	3.0	0.63	2.0	0.76
I Needed to be an Independent Learner to be Successful	2.5	1.05	3.1	1.05
I needed to Take Responsibility for My Own Learning	3.3	0.52	3.7	0.50

None of the students in Mathematics 4225 had previously participated in a web-based course. Students from two of the sites had only experienced learning in the traditional classroom. The two students from the other site had completed other distance education courses, but none of these were web-based. Most completers and nearly all noncompleters disagreed with the statement that the web-based format suited the way they learned. Two completers and six noncompleters strongly disagreed with this statement. However, two completers and one noncompleter indicated that this format did suit their learning style.

Completers agreed that participating in Mathematics 4225 made them better learners and one completer strongly agreed with this statement. Noncompleters generally disagreed and two noncompleters strongly disagreed with this statement. However, two noncompleters indicated it made them better learners.

Completers were undecided as to whether or not it was necessary to be an independent learner to be successful in the course. One completer strongly agreed with the statement and one strongly disagreed. Noncompleters generally agreed with the statement. Four noncompleters strongly agreed with the statement and one noncompleter strongly disagreed.

Both completers and noncompleters agreed that they needed to take responsibility for their own learning in order to be successful. Two completers and six noncompleters strongly agreed with this statement.

Table 51
Isolation: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Felt Isolated	2.2	0.98	2.6	0.88

Initially, one site had seven students, another had six students, and the third site had two students. After noncompleters withdrew from the course, each site had two students.

Most completers disagreed with the statement that they felt isolated in the course, although one completer strongly agreed with this statement. Noncompleters were generally undecided. One noncompleter strongly agreed and another strongly disagreed with the statement.

Table 52
Teacher Support: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Had Enough Support From the Teacher	2.7	1.03	2.3	0.50

On average completers agreed with the statement that they had enough support from the teacher. However, one completer disagreed with the statement and another strongly disagreed. Noncompleters generally disagreed with the statement. However, three noncompleters agreed with the statement.

Table 53
Connection to Previous Knowledge: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Material Connected to Previous Knowledge and Experience	2.3	0.52	1.9	0.60

Nearly all completers and noncompleters disagreed with the statement that the material was connected to their previous knowledge and experience. Two noncompleters strongly disagreed with this statement. However, two completers and one noncompleter agreed with the statement.

Table 54
Discussion of Material: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Material Discussed Until Understood by Everyone	2.8	0.41	1.6	0.73

The instructor often anticipated technical problems during the online class and gave priority to covering new material instead of answering questions, although some time was usually available at the end of class for student questions. Students were also reminded that they could fax their questions to the instructor if they could not be answered during the online classes. However, using the fax to ask and answer questions meant that only the site asking the question received the solution, so any discussion of the question was limited to the instructor and the students at that site. Nearly all completers agreed that the material was discussed until everyone understood it. However, one completer disagreed with this statement. Most noncompleters disagreed with the statement and five noncompleters strongly disagreed with the statement. However, one noncompleter did agree with the statement.

Table 55
Previous Distance Education Experience: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Previous DE Experience Helped Determine What Would Be Expected of Me	2.8	1.21		

The two completers who responded to this item had completed other high school distance education courses using the Telemedicine model and indicated that this experience helped them determine what would be expected of them in Mathematics 4225. All other participants had no previous exposure to distance education. These two students remained in the course even though their academic achievement was below that of some noncompleters. They were also less apprehensive about the technical difficulties experienced during the online classes.

Table 56
Deep Versus Surface Learning: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Did the Work Because of Interest	3.2	0.41	2.2	0.44
Did the Work Because It Had to Be Done	3.2	0.75	2.8	0.67
Tried to Determine What the Work Meant	3.2	0.41	2.7	0.50
Tried to Determine How the Work Fit into the Overall Scheme of Things	2.7	0.52	2.3	0.50
Could Relate the Things Learned to Previous Knowledge and Experience	3.0	0.63	2.0	0.71
Relied on Memorization	2.7	0.82	2.9	0.33
Worried About Time	2.8	0.75	3.3	0.50

Completers agreed that they did the work in the course because they were interested in it. One completer strongly agreed with the statement. Noncompleters generally disagreed, although two noncompleters agreed with the statement.

Completers and practically all noncompleters agreed they did the work because it had to be done. Two completers strongly agreed with the statement. However, one noncompleter strongly disagreed with the statement.

All completers and most noncompleters agreed that they tried to determine what the work meant as they completed it. One completer strongly agreed with the statement. However, three noncompleters disagreed.

Completers generally agreed that they tried to determine how the work they did fit into the overall scheme of things. However, two completers disagreed. On average noncompleters disagreed with the statement, although three noncompleters agreed with the statement.

Practically all completers agreed they could relate the things they learned to their previous knowledge and experience. One completer strongly agreed with the statement and one completer disagreed. Noncompleters disagreed and two noncompleters strongly disagreed with the statement.

Completers were undecided as to whether they relied on memorization in Mathematics 4225, although one completer strongly agreed with this statement. Noncompleters agreed that they did rely on memorization.

Most completers and all noncompleters agreed that they worried about the time required to do the work in the course. One completer and three noncompleters strongly agreed with this statement. However, two completers disagreed with the statement.

Table 57
AP Calculus AB Examination Formats: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Preferred Older AP Calculus AB Examination Format	2.0	1.0		

All students were given copies of The College Board AP Calculus AB examination from 1985 and 1987 as a general guide to the format and expectations of the May 2000 examination. The Mathematics 4225 final written by completers was a diagnostic test identical in format, and similar in expectations, to the May 2000 examination. The two earlier examinations were different from the diagnostic examination in that they did not allow the use of a graphing calculator for any part of the examination, and had less application-type problems. Completers disagreed with the statement that they preferred the older AP Calculus AB examination format, and two completers strongly disagreed with the statement.

Table 58
Thinking Ability: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Course Improved My Ability to Think	3.2	0.41	2.3	0.49

Completers agreed that the course improved their ability to think, and one completer strongly agreed with the statement. Noncompleters generally disagreed, although two noncompleters agreed with the statement.

Table 59
Application of Concepts: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Difficult to Apply the Things I Learned to Applications	2.0	0.00	3.4	0.53

Completers disagreed with the statement that it was difficult to apply the things they learned. Noncompleters agreed with the statement, and four noncompleters strongly agreed with the statement.

Table 60
Mathematical Language: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Mathematical Language Was Confusing	2.2	0.41	2.7	0.71

Most completers disagreed with the statement that the mathematical language was confusing. However, one completer found it confusing. Noncompleters were generally undecided, although one noncompleter strongly agreed with the statement.

Table 61
On-Site Competition: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
On-Site Competition Helped Me Work Harder	2.8	0.98	2.4	0.88

Completers agreed that competition with an on-site classmate helped them work harder, and two completers strongly agreed with the statement. Noncompleters were undecided, although one noncompleter strongly agreed with the statement and another strongly disagreed.

Table 62
Relevance and Interest of Project Topics: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Project Topics Relevant and Interesting	3.2	0.41		

The one project in the course was weighted at 5%. Completers were presented with six topics from which to choose and could work on one topic alone or as part of a group. Completers agreed that the project topics were relevant and interesting, and one completer strongly agreed with this statement.

Table 63
Participation on Other Web-Based Courses: Mean and Standard Deviation of Survey Responses

Survey Item	Completers		Noncompleters	
	Average	Standard Deviation	Average	Standard Deviation
Would Participate in Another Web-Based Course	2.5	0.84	1.8	0.83

Four of the six completers agreed that they would participate in another web-based course like Mathematics 4225. However, one completer disagreed with the statement and another strongly disagreed. Noncompleters disagreed with the statement and four noncompleters strongly disagreed.

Near the end of part two of the survey, students were asked to indicate topics on which they would have preferred to spend more time and aspects of the course which should have received more emphasis. They were also asked to indicate ways in which their learning could be improved. Results are presented in Table 64, Table 65, and Table 66.

Table 64
Topics That Should Have Received More Time

Topic	Number of Students	
	Completers	Noncompleters
Limits	0	1
Related Rates	1	1
Applications of Differentiation	0	1
Volumes of Revolution	2	0
Integration by Parts	1	0
Precalculus Topics	0	2
All Topics	0	3

Table 65
Aspects of the Course That Should Have Received More Emphasis

Aspect	Number of Students	
	Completers	Noncompleters
Manipulation and Procedure	6	1
Problem Solving	4	5
Theory and Proof	2	0
All Three Aspects	0	1

Table 66
Means to Improve Learning

Student Suggestion	Number of Students	
	Completers	Noncompleters
Improved Technology	1	3
Better Audio for Online Classes	1	4
More Reliable Computer Equipment	0	1
Access to Internet at Home	1	0
More Interaction	0	2
More People in the Class	0	1
More Complete Orientation Session in September	0	1
More Face-to Face Meetings with the Instructor	1	3
More Online Classes	0	1
Offer the Course in the Traditional Manner Instead of Online	2	4
Complete Calculus Readiness 3105 Before Enrolling in Mathematics 4225	0	2
Have a Good Knowledge of Computers	0	1
Have a Good Knowledge of the Mathematics 4255 website	0	1
A Better Solution Guide	1	1
Better Explanation of Initial Concepts	0	1
A Teacher On-Site	0	2
A Tutor at Each Site	2	0
More Projects	1	0

Shorter and Easier Tests	0	1
Shorter Assignments	1	0
Less and More Focused Homework	0	1
More Emphasis on Getting Homework Done	0	2
Noncompulsory Online Classes	1	0
Increased use of the Chat Feature	1	0
Increased use of the Links	1	0
Improved Teaching	0	1

Students indicated that their learning could be enhanced if the technology was improved to allow reliable, consistent interaction with teacher and students, if a more complete orientation occurred at the beginning of the course, if there were more face-to-face meetings with the instructor, if there were more online classes, a more complete solution guide, more consistent evaluation, a tutor or teacher for each site, and more projects. They indicated that the experience of participating in this course could have been very valuable with these improvements. However, the problems made the experience very frustrating.

Overall Opinion of the Mathematics 4225

Noncompleters, in general, were not satisfied with Mathematics 4225. One noncompleter stated, "Overall, I did not enjoy this course. The technology was not very good and I did not like the teaching methods. The classes were too small and the students were not given a chance to interact." Some noncompleters thought that offering

Mathematics 4225 was worthwhile, but not as an online course. One commented, "The math course itself would probably [have] been okay, but the way it was taught over the Internet was not one of its strong points. There were many technical problems [and] sometimes the site wouldn't open. The audio was really bad. It was difficult to hear the teacher. The feedback was terrible. This made it very frustrating. This was the main reason I dropped the course."

Completers generally indicated that the experience was worthwhile. A completer commented, "I think that the Math 4225 course is great because it prepares us for post-secondary school. It helps us learn earlier, to push ourselves to do work, and how to be more independent in our studying. I felt that the e-mail, whiteboard, links, lessons records, bulletin board, calendar and chat are all extremely important" Another completer added, "Overall I thought it was challenging but I feel that it will benefit me when I begin my university studies in the fall. I found it an excellent way for students who are planning on attending post-secondary schools [to] get a feel of what the different university courses will be like." Another completer stated, "It was a good experience in that we got some university learning and a new learning experience. I felt the course suited me well." Even a noncompleter wrote, "If I had continued in the course, I think that the course would have been really great. It would have brought us to a different level of our understanding about math."

Discussion of Results of Part Two of the Survey

Introduction

The items in part two of the survey focused on features which research literature deems important in learning calculus and in learning at a distance. The discussion which follows offers an explanation for some of the results of the survey and, in some cases, presents possible consequences if the weaknesses of the course are not addressed.

Mathematical Modeling

Although most students indicated they were comfortable with having mathematical models provided, they should be encouraged, not only to formulate models from given data, but to gather the actual data itself, as an important part of the scientific process involves the accurate and precise measurement of some characteristic of a real-life phenomenon. Readily available technology such as Texas Instrument's (TI) Calculator Based Laboratory (CBL) allows the graphing calculator to be used by the student for data collection, and TI Interactive software allows the retrieval of data from Internet documents to the graphing calculator for analysis. The recent emphasis placed on the use of graphing technology in mathematics, and the availability of graphing calculators in provincial high schools now permits more meaningful investigation of mathematical concepts through mathematical modelling.

Answers

Seven students indicated that there was an emphasis on getting the correct answer as opposed to understanding the meaning of the answer. This result, and the indicated lack of emphasis on determining the reasonableness of answers, may indicate an emphasis in the course on product instead of on process and the meaning of the process outcome. This emphasis may allow students to be successful as Collectors (Frid, 1992), who have the ability to successfully manipulate symbols and apply rules but may attain only a surface level understanding of concepts. In addition, this emphasis does not send the appropriate message to students about the nature of calculus knowledge nor will calculus courses with such an emphasis produce graduates who are suited for science, engineering, and academia.

Memorization

Seven students also reported that they viewed calculus as a collection of rules and procedures that had to be memorized. One student noted that, "All the course is is memorizing rules." Another, during a discussion with the instructor comparing the merits of the course text with that of another text which was much more application oriented, stated in reference to the alternate text, "That's the way it should be." However, success on the AP Calculus AB examination requires that students memorize many of the rules of differentiation and integration, as portions of the multiple choice section of the examination expect students to use these rules, without the aid of technology, merely to obtain the correct distractor. Until the AP Calculus AB examination places less emphasis on the simple application of rules and procedures, memorization will continue to be an important aspect of the course.

Both groups expected to have to memorize a good deal of material in the course. This expectation may be a result of the way students have typically learned mathematics in the past. Some high school mathematics students have been successful in previous mathematics courses by memorizing procedures and examples and using these procedures to solve template problems on evaluation. Hence they achieve a surface level understanding of mathematical prerequisites that is not sufficient to ensure success in a first calculus course.

Symbolic Manipulation

Tables 37 and 48 may indicate that, for completers at least, emphasis was placed on computational, as opposed to interpretative, skills. Homework questions, and to a lesser extent evaluation instruments, required a considerable amount of symbolic manipulation. For sites which did not have a copy of Derive, many questions which involved implicit functions could not be attempted using graphing calculator technology, and had to be approached using paper and pencil techniques. For functions explicitly written in terms of the independent variable, graphing calculators could produce only the numerical results of differentiation and integration and not the symbolic result. While pure computational skill is required on the AP Calculus AB examination, the prevalence of questions requiring symbolic manipulation without technology may send improper epistemological messages to students.

Student-Teacher Interaction

The responses in Table 38 indicate overall dissatisfaction with the level of interaction with the instructor. This dissatisfaction with synchronous interaction is justified given the frequency of technical difficulties during online classes and the resulting loss of communication between students and instructor. It may also have been a result of an expectation on the part of the student that the instructor would be visiting the sites regularly, as happened in the previous year. These visitations did not occur, as the cost was prohibitive. However, many of the avenues for asynchronous interaction were seldom or never utilized, especially by noncompleters. Students in Mathematics 4225, like those students in distance education courses delivered using the Telemedicine model, could contact the instructor by e-mail, telephone, or fax. In addition, they could also have used the bulletin board feature. However, many students did not make full use of the avenues available for interaction with the instructor. The private e-mail feature was used only by a very limited number of students, and consistently by only one student. This student was also the only one who contacted the instructor by telephone. In addition, students in Mathematics 4225 did not contact the instructor by fax, other than to submit assignments and tests. According to Hiltz (1993), students who are not motivated to take advantage of these opportunities for interaction may be more suited to courses delivered in the traditional manner.

Writing

Although students were generally satisfied with the amount of writing in the course, their response may have been based on a comparison between the quantity of writing required in high school mathematics course and that of AP calculus. Secondary school mathematics courses generally have very limited writing requirements, and in comparison, the amount of writing required in Mathematics 4225 was greater. However, writing requirements in Mathematics 4225 are still far below that desired for a revitalized calculus course.

Topics Linked to Careers

Students were generally undecided as to whether a wide range of calculus applications should be presented or if applications should be career specific. This result may stem from student indecision about career plans while enrolled in the course. However, the indecision in preference for varied versus career-oriented applications parallels that of mathematics educators and scientists, some of whom desire eclectic applications of calculus concepts and some of whom prefer a more discipline-oriented approach.

Feedback

While students were generally satisfied with the promptness of feedback on evaluation, both groups indicated the quality of the feedback from the instructor on evaluation instruments was unsatisfactory. The instructor did cover some homework questions as requested by students during online classes, but students received no regular evaluative feedback on homework. Accordingly, students missed an opportunity to gain information about teacher expectations and course objectives, and the instructor lost information on student and class performance. The instructor did evaluate and locate student errors on assignments and examinations, and sometimes made suggestions for alternate approaches to a problem, but students did not receive the complete solutions for assignments or tests. The lack of high quality feedback on evaluation instruments and homework assignments was detrimental to student understanding of the course material as it decreased student motivation to complete homework questions and did not provide the direction necessary to help students overcome their misconceptions of the mathematical concepts, as it diminished opportunities for the instructor to create the cognitive conflict which Tall and Vinner (1981) believe is often necessary for the transition from concept image to concept definition. The provision of high quality feedback is especially important in courses like a web-based AP calculus that require students to be independent learners, but provide reduced direct contact with the teacher.

Pace

Students indicated that they were consistently working two to three sections behind the instructor. This may indicate that students did not use their study time effectively or that other demands meant that insufficient time remained to keep on schedule in the course. The results may also indicate that the time allotted for the course in the school schedule is insufficient given the greater demands of the course. As well, some students viewed the course as an extra and gave priority to other courses needed for graduation and entrance into post-secondary institutions and did not spend sufficient time on Mathematics 4225 to ensure they kept pace with the instructor. As a result, students were generally unprepared for online classes, not having read the appropriate pages of the text nor formulated appropriate questions which could lead to the further assimilation and synthesis of material. Consequently, much online class time was used for the introduction, as opposed to the further refinement of understanding, of calculus concepts.

Proof

The course evaluation did contain some proofs, but none were unseen. As well, the AP Calculus AB examination typically does not contain any proofs and hence they were not emphasized in the course, especially given the time constraints imposed by the technical difficulties experienced during online classes. While students generally find proofs challenging, they are part of the work of mathematicians. Hence, students in this course did not often experience an important aspect of mathematics. Five of the fifteen participants indicated a need for more proofs in the course and their response may support Wilson and Albers (1988) claim that proof is needed to attract strong mathematical students.

Numerical Methods

The homework and evaluation questions placed emphasis on the integration and differentiation of numerous functions specifically selected to fit the basic differentiation and integration rules. Students were only occasionally required to differentiate or integrate numerically. The lack of emphasis on numerical methods in Mathematics 4225 parallels that on the AP Calculus AB examination which seldom require numerical methods. Examination questions dealing with integrals that do not fit the basic integration rules require only that the student set up but not evaluate the integral. The lack of emphasis on numerical methods in the course is not indicative of the usefulness or applicability of these methods in solving real-world problems. Questions requiring numerical methods permit solutions using computer programming and may be ideally suited to Dubinsky's ACE Teaching Cycle (1999). Correctly structured activities centred on numerical methods may help students achieve a deeper understanding of mathematical concepts.

Class Presentations

The reluctance of both groups to participate in class presentations may indicate that students were not comfortable presenting to their classmates and this feeling may have been a result of the lack of interaction between sites. The varying offline schedules, plus the demand for computers by other classes at different sites, made it difficult for students at one site to interact with those at another site. Many students could not establish communication after school hours as they lacked Internet-ready computers or other necessary hardware and software. Telephone calls between students could be expensive and, as students were somewhat isolated geographically, face-to-face interaction was limited. These barriers may have been viewed as insurmountable by students, especially for those who might have formed a group with classmates at another site. Hence students did not experience the benefits of collaborative learning to the same extent as was possible by interacting with students at remote sites as well as with their on-site classmates. Their reluctance to participate in presentations may also indicate that they were not comfortable with their level of comprehension of the course material and suggests a surface as opposed to a deep understanding of calculus concepts.

Technology

Both groups indicated that the two pieces of mathematical software recommended for the course did not aid in their understanding of the course material. The Mathematics 4225 course description recommended that each site purchase a copy of Geometer's Sketchpad. Derive was available online as a free demonstration version that was suitable for the course. Both programs were important as they could enhance student understanding of fundamental calculus ideas. However, students did not receive training for either pieces of software and used them infrequently or not at all. As well, bandwidth restrictions prevented the instructor from sharing these applications with the sites that did not have them. In not using these technologies, students lost the opportunity for a deeper understanding of calculus concepts through greater focus on modeling and interpretation of results, more realistic applications, and the relegation of symbolic manipulation to computers.

Links to other Disciplines

While completers indicated that calculus was connected to other disciplines, noncompleters were undecided. The difference in results between the two groups may be due to the timing of the introduction of applications of the major calculus concepts. Applications of differentiation did not occur until the last section of the third chapter and the last half of the fourth chapter. Applications of integration occurred in the second part of the course and the project was not due until the last month of the school year. In all cases, noncompleters had withdrawn from the course well before experiencing the full range of applications of calculus, and so the link to other disciplines was not well established.

Learning

Both groups indicated that it was necessary to be an independent learner and to be responsible for their own learning in order to be successful in the course. However, both groups indicated that the web-based version of the course did not suit the way they learned and noncompleters indicated that participating in the course did not help them become better learners. Their responses indicate that although both groups recognize the need for independence and responsibility in learning in the web-based environment, noncompleters in particular did not achieve the autonomous self-learning which Kasworm and Yao (1992) insist should be part of the learning process, possible because they withdrew from the course before it had any significant positive impact on their learning.

Isolation

While students generally did not feel isolated, each site had two or more students and so had at least one other classmate with which to interact and experience the benefits of collaborative learning. It may be difficult for a single student at one site to overcome feelings of isolation given the current impediments to synchronous interaction with instructor, and with students at other sites. The audio problems experienced during online classes, the difficulty accessing the AP student computers, and the possible lack of a properly equipped computer at home could reduce both the synchronous and asynchronous interaction of a lone student with other course participants and jeopardize that student's chance for success in the course.

Teacher Support

All three noncompleters who indicated they received enough support from the teacher were located at the same site as the instructor and could meet face-to-face with the instructor for fifteen minutes after each online class and for seventy-five minutes during those offline classes when no offsite students requested help. This would indicate that the typical noncompleter in Mathematics 4225 did not receive enough support from the teacher. Completers may have been more independent learners and generally did not require supports such as feedback and solutions for evaluation and homework, on-site visitations, and a complete set of notes to the same extent as noncompleters.

Connection to Previous Knowledge

Most students indicated that the course material was not connected to their previous knowledge and experience. To foster a deep approach to learning, Biggs (1994) indicates that new ideas and concepts must be related to the student's existing knowledge and conceptual framework. If the interconnections between new and previous knowledge are not formed, reification of concepts may be especially difficult as students may not expand nor reconstruct existing schemata.

Discussion of Material

Noncompleters indicated that the discourse was not sufficient to bring about understanding of concepts. This result may be accurate with respect to the dialogue that occurred during online classes. Technical difficulties during these classes often meant that insufficient time was available for an in depth discussion of concepts and hence only a succinct coverage of material occurred to allow students some time for homework questions. However, noncompleters in particular did not avail of the opportunities for asynchronous communication. This group seldom used e-mail, fax or telephone to initiate contact with the instructor or with other students. This meant that for students who were not onsite with the instructor, the opportunities for discussion of material was very limited.

Deep versus Surface Learning

The reported results in Table 56 as to why students completed work, their determination of the meaning of their work, its overall connection to the course, and its relation to their previous knowledge and experience indicate that completers generally had a deeper approach to learning than noncompleters. Noncompleters may have adopted a surface approach to learning because their motivation was not intrinsic, they were passive as opposed to active in the learning process, they had limited discussions about concepts with instructor and classmates, and the material they were learning was often not related to their previous knowledge and experience. These students had no input in the selection of the material covered or in planning learning activities, online classes were often dominated by the instructor with little student input or discourse among participants, and new calculus concepts were often not introduced in detail in order to keep on schedule in the course.

Thinking Ability

Noncompleters indicated that participation in Mathematics 4225 did not improve their ability to think, nor did it help them become better learners. However, as most noncompleters withdrew from the course early on, it is doubtful that noncompleters were enrolled in the course long enough for the experience to have any positive impact on their ability to think or learn, especially if they viewed the course as an attractive but unessential part of their school experience.

Applications of Concepts

Noncompleters also indicated that they had difficulty applying the things they learned. This may again indicate that noncompleters had only a surface level understanding of the concepts which prevented them using their knowledge in realistic situations and expanding and interconnecting their existing schemata. This difficulty may also be a result of a noncompleters' lack of opportunity to apply concepts. The applications of derivatives were not studied until well after the students had learned much of the theory and rules of derivatives. By the time the applications of derivatives appear, many noncompleters had withdrawn from the course and hence had little opportunity to apply the concepts learned earlier.

On-Site Competition

Noncompleters were undecided as to the benefit of on-site competition. For some students, on-site competition may act as a motivator to work harder, but only if the student attributes success to effort instead of ability. Ability is viewed as an internal, stable trait, and students who believe their ability is the cause of their success and failure would not attribute success or failure to effort and hence would not believe the extra effort stemming from on-site competition was a factor in achievement. The result may also indicate that some students may prefer to work as part of a community of learners rather than in competition with one another.

Topics to Receive More Time

Students reported that a variety of topics should have received more time. Several completers indicated that more time should be spent on volumes of revolution. Noncompleters required more time on precalculus topics. However, the vast majority of the precalculus topics were part of the syllabus of Advanced Mathematics 3201, which was a prerequisite for Mathematics 4225. This may indicate that noncompleters may have had only a surface level understanding of these prerequisite topics, even after they had been reviewed in Mathematics 4225. Noncompleters also indicated they needed more time on all the topics in Mathematics 4225. This requirement may be justified given the relatively much greater amount of material covered in the course as compared with other high school, two credit courses and the early examination date for students seeking university credit. The amount and difficulty of the material in Mathematics 4225 greatly exceeds that in other high school advanced mathematics courses. As well, the course was scheduled to be finished by the end of April to allow students time for review before they wrote the AP Calculus AB examination in mid-May. However, in two of the three sites, students were only allotted time equivalent to that of a two credit high school course and were required to spend much more time studying outside of school hours that they were accustomed to.

Aspects to Receive More Emphasis

All completers indicated that manipulation and procedure should have received more emphasis in the course, a direct contradiction to the emphasis on conceptual understanding desired by calculus reformers. Their response may indicate that completers perceive that these aspects are an important component of homework and evaluation and are essential for success in the course.

Nine of the fifteen students in the course also indicated that problem solving should have received more emphasis. This result may stem from students' need to have the practicality of the material they have learned demonstrated through the application of calculus concepts to other disciplines.

Means to Improve Learning

Noncompleters indicated their learning could be improved if technology was improved, particularly the audio component of online classes. The audio problems were a constant source of frustration for both the instructor and students and made online classes less than effective. They were a contributing factor to the withdrawal of a significant portion of the noncompleters. Noncompleters also expressed a need for more interaction through more face-to-face meetings with the instructor, more online classes, and an on-site teacher. They also indicated there should be less but more focused homework with an emphasis on the completion of that homework.

While completers identified a number of means to improve their learning, no one area received special emphasis, indicating they were more satisfied overall with the course.

Chapter Ten: Comparison of Texts and Examinations

The first part of this chapter compares the current Mathematics 4225 course text (*Calculus of a Single Variable: Early Transcendental Functions*, Fifth Edition, by Larson, Hostetler and Edwards) with an alternate text (*Calculus*, Second Edition, by Hughes-Hallett, Gleason, et al.) that has recently received support as a more suitable text for a first calculus course. The chapter begins with a short description of the alternate text, a comparison of the features of both texts, and a comparison of the epistemological messages that each text conveys to students. The second part of the chapter compares The College Board AP Calculus AB examinations from 1988 and 1997 with respect to the effects of the increase in emphasis on graphing technology on recent examinations.

Current and Potential AP Calculus AB Texts

Description of the Alternate Text

The 1998 second edition of *Calculus* (also known as *Consortium Calculus*), produced by a consortium based at Harvard University, has received support as a suitable text for a first calculus course. The focus of the text is depth of understanding of a small number of key calculus concepts as opposed to breadth of coverage.

The text has two guiding principles. The first is a de-emphasis on problems that can be solved using the examples presented in each section as templates. Students must grapple with and learn the ideas and concepts of calculus to become successful problem solvers. Many of the application problems are open-ended with more than one correct approach and have solutions that require common sense knowledge. These problems also encourage students to develop estimating and approximating skills. The second guiding principle is the Rule of Four. Where possible, topics are presented verbally as well as geometrically, numerically, and analytically, to provide alternate avenues through which students can understand the material.

The text develops mathematical thinking by first helping students acquire a clear intuitive picture of the central ideas. The student is then led to reason with these intuitive ideas and to explain their reasoning in plain English. Finally, the student has a choice to subsequently focus on theory or to focus on modeling. Those students favoring a theoretical approach may choose the *Focus on Theory* sections, which exposes them to the formulation of axioms, definitions, and theorems, and the construction of proofs. These sections also guide students to construct definitions and proofs independently. Alternatively, students favoring a more practical approach may choose the *Focus on Modeling* sections, which explore selected calculus applications in depth.

The text also allows for the development of mathematical skills and the use of technology. The *Focus on Practice* sections contain a battery of problems that allow students to develop mechanical skills in differentiation and integration. The text contains many questions that can be approached graphically but is not supported by any specific graphing calculator, graphing software, or computer algebra system. Students are expected to use their own judgment in determining the applicability and suitability of technology.

Features of the Texts

Introduction

The following section compares the features of the Consortium Calculus text with those of the Mathematics 4225 text (Larson Text). Tables 68 to 73 contain the major features of each text. A (✓) indicates that the text possesses the feature, an (✗) indicates that it does not. Any differences between common features are noted. The latter part of the section identifies other differences between texts.

Rules and Formulae

Table 67
Rules and Formulae: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Basic Differentiation Rules	✓	✓
Basic Integration Rules	✓	✓
Geometry Formulas	✓	✓
Trigonometric Rules	✓	✓
Algebra Rules	✓	✓

Although both texts contain the features of Table 67, there are some significant differences in the comprehensiveness of most sets of rules and formulae. The basic rules of differentiation and integration, the trigonometric rules, and the geometry formulas are much more exhaustive in the Larson text than in the Consortium text. The basic rules of differentiation appear in three separate locations in the Larson text, and the most comprehensive listing in chapter five contains 24 rules. Only 12 of these rules appear in one location in the Consortium text. The basic rules of integration appear in three separate locations in the Larson text, and the most comprehensive listing in Appendix C contains 91 rules. Only 31 of these rules appeared in one location in the Consortium text. Forty-six rules of trigonometry appear on the inside of the back cover of the Larson text as opposed to only 5 such rules inside the front cover of the Consortium text. Twenty-six geometry formulas appear inside the front cover of the Larson text as opposed to 6 inside the front cover of the Consortium text. The algebra formulas of both texts are similar in number and type.

Graphics, Tips, Notes, Remarks, & Summaries and Guidelines

Table 68
Graphics, Tips, Notes, Remarks, & Summaries and Guidelines: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Colour Graphics	✓	✓
Technology Tips	✕	✓
Historical Notes	✓	✓
Summaries and Guidelines	✓	✓
Remarks	✕	✓

The graphics of the Larson text contains a greater number and combinations of colours, and the book is interspersed with full colour photographs. The Consortium text uses black, gray or blue, and contains no full colour photographs. This text contains no technology tips as separate entities, but does contain one reference to the misunderstandings that can result from incorrect window boundaries when using the graphing calculator. It also does not have historical notes as separate entities. Famous mathematicians are briefly mentioned only in reference to the concept being discussed as it relates to some problem that the mathematician attempted to solve. Summaries in the Consortium text are located at the end of each chapter and are very brief, naming only the most important concepts in the chapter as headings with a limited number of subheadings. Summaries in the Larson text are dispersed throughout the chapters and sometimes require an entire section of the chapter. Guidelines, sometimes referred to as Problem Solving Strategies in the Larson text and Practical Tips in the Consortium text, are generally similar, although not as frequent in the Consortium text. Remarks again are not separate entities in the Consortium text but are incorporated into the written discussion of the material.

Exercises and Problems

Table 69
Exercises and Problems: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Graphing Calculator Exercises	✓	✓
Challenging Homework Problems	✓	✓
Variety of Homework Problems	✓	✓
Many Basic Computation Problems	×	✓

Both texts share most of the features of Table 69, but with some differences. The Larson text identifies specific questions requiring technology, particularly those involving the graphing calculator, whereas the Consortium text requires students to decide when and how technology will be useful. While both texts have varied and challenging homework questions, challenging questions are much more prevalent in the Consortium text, as it typically presents fewer skill development questions and proceeds more quickly to more demanding problems. The only part of the text containing a large quantity of skill development questions are the two *Focus on Practice* sections that were aimed at the development of differentiation and integration skills. The Larson text often presents a larger quantity of basic computation problems aimed at skill development in each section before providing challenging questions. The Consortium text does not contain any true or false questions. Those in the Larson text require a high level of understanding of the material and direct the student to find counter examples for those statements they believe to be false.

Appendices

Table 70
Appendices: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Appendix A: Proofs	×	✓
Appendix B: Basic Differentiation Rules	×	✓
Appendix C: Integration Tables	×	✓
Appendix D: Real Numbers and the Real Number Line	×	✓

The appendices of the Consortium text are quite different from those of the Larson text. The proofs of selected theorems found in Appendix A of the Larson text are contained in the *Focus on Theory* sections of the Consortium text. The other appendices of the Consortium text contain material on roots and accuracy, compound interest, polar coordinates, complex numbers, Newton's method and parametric equations.

Answers and Solutions

Table 71
Answers and Solutions: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Answers to Odd Numbered Questions	✓	✓
Student Solution and Study Guide	✓	✓

Both texts provide answers to the odd numbered exercises and have an accompanying student solution and study guide. However, the student guide for the Consortium text contains the solutions to only half the odd-numbered questions.

Prerequisites Chapter

Table 72
Prerequisites Chapter: A Comparison of Texts

Feature	Calculus Consortium Text	Calculus of a Single Variable
Prerequisites Chapter	✓	✓

A large part of the initial chapter of both texts is devoted to a review of functions. However, the Consortium text forgoes the discussion of instantaneous and average rate of change included in the Larson text and instead includes concavity, increasing and decreasing functions, asymptotes, translations, amplitude, period and phase shift of trigonometric functions, and continuity, and contains a *Focus on Theory* section which introduces the student to several theorems and their proofs. The Larson text provides a discussion of these topics in other chapters.

A further comparison of the texts reveals that in the Consortium text:

(a) modeling and applications have been increased.

(b) projects are more numerous and have been introduced much earlier.

(c) problem solving has received more emphasis and problems are more realistic.

(d) the number of problems which can be solved following template examples has been reduced.

(e) the number of basic computation problems has decreased. However, the text supplies some template problems in the *Focus on Practice* sections.

(f) students are required to justify their answers more frequently.

(g) the ability to succeed though the memorization of procedures and rules has been reduced significantly.

(h) the motivation for studying calculus concepts is more readily apparent.

(i) there has been a significant decrease in the amount of symbolic manipulation and more emphasis has been placed on the meaning of the result of symbolic manipulation.

(j) greater emphasis has been placed on numerical methods.

(k) links to other disciplines have been established through modeling and projects.

(l) skill in estimation and approximation is essential.

(m) error approximation has received more emphasis.

(n) a greater number of questions at the end of each section of the text require deeper level mental processing.

(o) deep learning is fostered through motivation, activity, and a structured knowledge base.

(p) a minimalist approach to colour graphics allows emphasis on subject matter itself as opposed to the presentation of the subject matter.

(q) the emphasis on proof has increased.

Epistemological Messages Contained in the Texts

Introduction

Epistemology deals with the origin and nature of knowledge. The epistemological messages conveyed by mathematics textbooks can have a strong influence on the way students view and learn mathematics. An analysis of these epistemological messages can provide some understanding of the difficulties encountered by students as they attempt to learn calculus for the first time. This section compares some of the epistemological messages conveyed by the Larson and Consortium texts, in an attempt to identify inappropriate epistemological messages in each text and hence determine if one text is more epistemologically suitable for Mathematics 4225.

A Comparison of the Epistemological Messages Contained in the Texts

Message One: **Students are expected to learn mathematics for the sake of learning mathematics.**

This message is conveyed more strongly by the Larson text. In the chapter on the applications of the derivative, this text provides no introduction to give the student an indication of how the derivative will be applied. Instead the text jumps immediately to section one on finding the extrema of an interval. The student is asked to accept on faith that the usefulness of derivatives will become explicit somewhere later in the chapter and will justify the effort in learning everything else in the meantime. In its corresponding chapter, the Consortium text provides a short introductory paragraph explaining that the derivative will be used to analyze the behavior of functions and in optimization problems. This makes explicit how the derivative can be useful and creates an appropriate mind set for the student. The text also makes a somewhat nebulous link between the upcoming material and that covered in previous chapters.

In its introduction to continuity, the Larson text provides no motivation for studying the concept. Only after two definitions, three theorems, seven examples, and a good deal of explanation, is there any mention of the Intermediate Value Theorem. Even then, it is three paragraphs later before it is noted that a function that is not continuous does not necessarily possess the intermediate value property, and another paragraph before a use for this theorem is made explicit. Of the four application problems near the end of the exercises, the three most realistic applications involve functions that are *discontinuous* and the one problem using continuous functions seems to be contrived. In its initial brief introduction to continuity, the Consortium text immediately makes it clear that continuity will only be discussed from the graphical and numerical points of view and that the concept will be investigated in more depth in chapter two. Nevertheless, the text gives a general reason why continuity is important by line four of the first paragraph and at least mentions the Intermediate Value Theorem by line seven. The second paragraph goes on to explain why continuity on an interval is needed to guarantee the existence of a zero for a function whose value changes sign on that interval. Hence students are provided with a reason why continuity might be worthy of study. However, none of the exercise sets in either of the chapters in this text have real-life, relevant questions involving continuous functions.

Message Two: **The development of mechanical skills is more essential than the application of concepts**.

This message is conveyed more strongly by the Larson text. The majority of the exercise sets in this text begin with a significant number of questions aimed at skill development before providing any questions which require a deep understanding of material or grappling with calculus concepts. The first section of chapter two of this text deals with the derivative and the tangent line problem. The section provides five examples that require students to find the derivative of a function using the definition of derivative (the limiting process). In each of these examples, the student must also find the slope of the corresponding tangent line at one or more points or is reminded by the text that the derivative can be used to find the slope of the tangent line. The text then illustrates, using graphs and the alternative form of the derivative, three conditions which destroy the differentiability of a function.

The exercise set for this section contains 65 questions. The first two questions ask the student to estimate the slope of the tangent line given the graph with the tangent drawn through a point on the graph. The third question requires that students evaluate a function at particular points in the domain given the graph with the coordinates of these points on the graph. All three questions, plus the first part of question four, do not require any knowledge of the derivative. Only the second part of question four requires the student to make the connection between the slope of the line tangent to the graph of a function at a point and the derivative of the function evaluated at that point. Questions 5 to 26 are very much like the first five examples and could be described as plug-and-chug exercises. Questions 27 through 32 require the student to make the connection between the geometric interpretation of the derivative of a function (the slopes of the tangents) and the graph of the derivative itself. These questions do not require the student to formulate any new knowledge about derivatives since the connection is explicitly stated in the section and is illustrated through numerous examples. The only difference here is that the questions are posed in a somewhat different manner. The same can be said of questions 33 to 36. The first two questions combine techniques used in questions one and two and in questions 17 through 24. The last two are essentially the same as the last part of question three. Questions 37 through 42 ask students to determine if a function is differentiable at a point in its domain using the alternative form of the derivative, but this requires nothing more than evaluating a limit. These questions again amount to little more than plug-and-chug exercises. Questions 43 through 52 only require that the student compare each graph with the three earlier graphs that illustrate how differentiability is destroyed. Questions 53 through 58 are essentially the same as questions 37 through 42 except that the former questions do not require the use of the alternative form of the derivative. The student is left to determine how to best approach the questions. Question 59 leads students to investigate the relationship between odd and even functions and the value of the derivative when the abscissas are additive inverses. Question 60 applies the results of question 59 to a specific even and a specific odd function. Only the true or false questions from 61 through 64 ask students to consider the finer points of some of the concepts to which they have been exposed. Finally, question 65 provides a counter example for question 62, effectively negating the intent of the latter question, and requires little more than the evaluation of a limit using the squeeze theorem. Practically all the questions here require at most a superficial understanding of the concepts presented in the section but do require a facility in evaluating limits. Neither question in this exercise set utilizes the derivative, the tangent to a curve, or the differentiability of a function in any real-life application. Hence the message sent to students by this text is that facility in symbolic manipulation and the ability to complete exercises following template examples in the text is more essential than the formulation of new knowledge, a deep understanding of concepts and the ability to apply calculus concepts in real-life situations.

The Consortium text investigates the derivative at a point, and the differentiability of a function, in two separate sections. It first deals with average rate of change and uses one volume example to illustrate how the average rate of change can vary on different time intervals. It then presents the physical interpretation of the derivative (as an instantaneous rate of change) and then the geometric interpretation (as the slope of the curve at a point and the slope of the tangent to the curve at a point). Of the eight examples presented, only the last uses the actual pen and pencil calculation of a limit to find the derivative of a function. The other examples require the use of a graph to determine the sign of the derivative of a function at a point on the graph or to estimate the value of a function or its derivative at a point on the graph.

The exercises at the end of this section contain thirty-three questions. Twelve of these questions could be answered via the mechanical computation of the derivative using the limiting process, although students are not directed to use this process. These questions do not appear until after twelve other questions that help students to differentiate average and instantaneous rate of change (questions 2, 4, 10 and 11), distinguish between the value of a function and the value of its derivative (questions 3, 7-9), estimate the value and sign of derivative at a point on a curve (questions 1 and 12), and derive the limit which is the definition of the derivative (questions 5 and 6). These exercises may be crucial in the development of a deep understanding of the meaning of the derivative and they are appropriately placed before drill questions 13 through 24. Questions 25 through 30 again require students to estimate the value of the derivative of a function at a point, albeit with nastier functions. Question 31 presents a real-life application on rate of change of population growth. Question 32 helps develop the constancy of the value of the derivative at a point for a family of functions which differ only by a constant, and in doing so sets a basis for a subsequent understanding of why the derivative of a constant function is zero and also why antidifferentiation produces a family of functions. Finally, question 32 illustrates the error that can result from using the table method, and the difference quotient with progressively smaller values of the denominator, to approximate the derivative of a function at a point. The message sent to students by this text is that while mechanical skill in finding the derivative may be necessary, it does not overshadow the need for a deep understanding of derivatives characterized by a knowledge of the difference between average and instantaneous rate of change, an awareness of the connection between the different interpretations and representations of the derivative, an ability to select the appropriate method in determining the derivative, accuracy in estimation of the derivative, and the capability to apply the derivative in real-life problems.

The section on differentiability appears later in Consortium text in the *Focus on Theory* section of chapter two. The section includes a discussion of local linearity, a cognitive root which Tall (1992) believes is more appropriate than the limit concept as a basis for studying calculus. However, it appears late in the chapter after much of the initial work with derivatives has been completed. It is doubtful that local linearity can replace the notion of limit as a basis for studying calculus when the student has been previously exposed to substantial work with limits.

The section first gives the student a graphical method (zooming) for determining if a function is differentiable at a point and, through illustration and explanation, an intuitive understanding of the three ways in which a function can fail to be differentiable.

The limiting process is introduced in the examples but the emphasis is not on the calculation of this limit, but on how the existence or nonexistence of the limit is consistent with the graphical determination of the differentiability of the function at a point.

The exercise set at the end of the section contains only 15 questions. Question 1 has two graphs that can be compared with the graphs at the beginning of the section to determine points at which the given graph is not differentiable. Question 2 illustrates the error that could occur from using graphical techniques to decide if a function is differentiable at a point. Questions 3 through 5 ask the student to test functions for differentiability on an interval, but allow the choice of the graphical or the limiting technique. Questions 6 through 8 apply the concept of differentiability in electromagnetism. Questions 9 and 10 deal with the relationship between continuity and differentiability and questions 11 through 15 examine local linearization. Very few of these questions require the actual calculation of a limit to determine differentiability. Instead, students are often encouraged to investigate the function graphically. The text de-emphasizes the manual calculation of limits to determine differentiability and instead leads students to investigations using a graphical approach. It also makes evident the need for an awareness of the danger of a complete reliance on linear approximation. Finally, the text illustrates the practicality of the differentiability by applying it in real-life situations. The message here is that the manual skill of calculating a limit to determine differentiability is less important than establishing the connection between differentiability and continuity and the real-life application of the concept.

The Consortium text also conveys the importance of the application of concepts through ubiquitous projects and modeling. Every chapter in the text contains a section where students are asked to apply the concepts learned to a real-life situation, and a wider selection of projects is contained in the Appendix H. In addition, three chapters have sections devoted to mathematical modeling where students explore applications of calculus. The Larson text contains no dedicated project or modeling sections.

Message Three: **The theory of mathematics is important.**

This message is conveyed more strongly by the Consortium text. The theory of mathematics is evident in both texts, but many of the important proofs in the Larson text are relegated to Appendix A. According to the survey, students were either unaware of the existence of this appendix or ignored it, and hence missed exposure to a significant amount of the basic theory of calculus. In the Consortium text, instead of being hidden near the back of the text, sections labeled *Focus on Theory* are incorporated into four of the chapters covered in the course. This compares with two *Focus on Practice* sections and three *Focus on Modeling* sections. The number and inclusion of the *Focus on Theory* sections throughout the text sends a message to students that the importance of theory equals that of skills acquisition and modeling.

Message Four: **In a first calculus course, the development of formal reasoning is more important than the development of intuitive reasoning.**

This message is conveyed more strongly by the Larson text. A significant number of proofs are presented in both texts, and the exercise sets of both texts contain questions requiring proofs, but the method of presentation is consistently more formal in the Larson text. This may leave the student with the impression that only the traditional, formal presentation of proof is acceptable at this level. The Consortium text also contains proofs presented in the formal manner, but presents a significant number of proofs using a less formal, written explanation. This allows for the development of intuitive, as well as formal, reasoning.

Message Five: **The history of mathematics is of little importance**.

This statement is conveyed more strongly by the Consortium text, although neither text stresses the history of mathematics to any great degree. A review of the index of both texts finds reference in the Larson text to 41 mathematicians or scientists who were important in the development of calculus, but only 18 in the Consortium text. As well, any historical reference to these scientists in the Consortium text is incorporated into the wording of the text and any description of the person does not extend past that related to the concept being discussed. The Larson text places its historical notes in the margins of the page, and sometimes adds personal details about the person and a reference to articles that further provide further information about the person's life. However, neither text requires that the student learn anything about any mathematician, nor does either text require any research to extend the student's historical grounding in calculus. Hence the message sent to students is that while the history of mathematics may be an interesting sideline for some, it is not worthwhile knowledge nor does it merit extended study.

Message Six: **The textual presentation matters as much as the subject matter being presented.**

This message is conveyed more strongly by the Larson text. This text uses numerous colours and combinations of colours in its graphics and includes a full page, multicoloured picture at the beginning of each chapter. Presumably, these graphics are included to attract students' attention but only succeeded in confusing some students. The purpose of the full page coloured picture at the beginning of each chapter is not readily apparent, nor is any explicit connection made later between the pictures and material covered in the chapter. The purpose of the colourful presentation seems aimed more at attracting the students' attention rather than augmenting their understanding of the concepts. The Consortium text takes a minimalist approach to the use of colours, using only two shades of blue in diagrams and no full page, multicoloured pictures. The aim of this text seems to be to direct students' attention towards the subject matter of the text studied and away from the textual presentation as such (Holmberg 1995).

Message Seven: **The importance of the verbal presentation of concepts equals that of the numerical, geometrical and analytic presentations.**

This message is conveyed more strongly by the Consortium text. One of the guiding principles in the development of the text was that multiple representations encourage students to reflect on the meaning of the material. Furthermore, the addition of a verbal method of presenting an explanation is important since it allows further insight into student thought processes. To this end, the text often presents multiple explanations of concepts, including a common English verbal explanation, and the exercise sets include a plethora of questions that ask students for descriptions, verbal or written justification for their reasoning, or an interpretation of results. In its preface, the Larson text also claims a renewed emphasis on communicating mathematics, and even identifies essay type questions in its exercise sets. However, questions that require verbal or written communication are far less frequent than in the Consortium text.

The 1988 and 1997 AP Calculus AB Examinations

Introduction

The second part of this chapter compares The College Board AP Calculus AB examinations from 1988 and 1997 to determine the effect of permitting the use of graphing calculators in recent examinations. Every few years, The College Board publishes a complete copy of the Calculus AB examination for that year along with the solution and scoring guide. The 1997 examination was chosen because it was the most recent examination published that allowed the use of graphing calculators. The 1988 examination was chosen because it was representative of earlier examinations that did not permit the use of graphing calculators. This part of the chapter first presents the format of each examination. It then compares the two examinations in terms of the number of multiple-choice items that can be answered by matching an item to a memorized rule and then manipulating symbols ("plug and chug" items). This is followed by a comparison of the free response section of each examination to determine the effects of the use of graphing calculators on more recent examinations.

Examination Formats

The 1988 AP Calculus AB Examination

The 1988 examination consisted of two, equally weighted sections. Section 1 contained 45 multiple choice items to be completed in 90 minutes and was marked using a correction factor of the number of items answered correctly minus one-quarter the number of items answered incorrectly. Section 2 contained six free response problems to be completed in 90 minutes. Equal weight was assigned to both sections of the examination and to each item within each section. No calculators or reference materials were permitted for either section of the examination.

The 1997 AP Calculus AB Examination

The 1997 examination consisted of two, equally weighted sections. Section 1 contained two parts. Part A contained 25 multiple choice items to be completed in 50 minutes and did not permit the use of a calculator. Part B contained 15 multiple choice items to be completed in 40 minutes and did permit the use of a graphing calculator. This section was marked using the same correction factor as the 1988 examination. Section 2 contained six free response problems to be completed in 90 minutes and did permit the use of a graphing calculator, although each question could have been answered without it. Equal weight was assigned to both sections of the examination and to each question within each section.

A Comparison of Section One of Each Examination

An examination of the items in Section 1 of the 1988 examination reveals that 29 of the 45 items (64%) could be classified as "plug and chug". These items require only the memorization of rules and skill in symbolic manipulation. An examination of the 1997 examination reveals that 16 of 25 items in Part A and 8 of 15 items in Part B could be similarly classified. This gives a total of 24 of 40 items (60%). If the graphing calculator is used where possible in Part B to perform the symbolic manipulation, the total drops to 22 of 40 items (55%).

A Comparison of Section Two of Each Examination

Neither of the six free response problems in Section 2 of the 1988 examination is an application -type problem. Problem 6 (the last and most difficult) of Section 2 of the 1997 examination deals with solving a differential equation that arises from the motion of a skydiver after her parachute opens. The differential equation is provided, along with its initial condition, and in part (a), students are directed to solve the equation using separation of variables. Part (b) requires that students find the terminal velocity of the skydiver using a limit at infinity. In part (c), the student must distinguish between speed and velocity and determine when a safe landing speed is reached.

Another evident difference between the two examinations is the amount of higher order thinking required by both examinations. In the 1988 examination, problems 1 to 4 and most of problem 5 contain questions that are very similar to the questions students have solved repeatedly following template examples provided in the text and require only the memorization of rules and expressions. Even question 6, supposedly the most difficult question on the examination, requires little beyond superficial understanding and skill in integration. The 1997 examination contains fewer questions of this type, replacing them with items that require a deeper understanding of the concepts in the course. Question 5 in particular can be answered only with a good understanding of derivatives and the Fundamental Theorem of Calculus and will almost certainly be novel to students. To further illustrate the difference in the amount of higher order thinking required by the 1997 examination, the questions on the one common topic tested in both examinations (rectilinear motion) will be analyzed.

The 1988 Examination: Section 2, Question 2

A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.
(b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.
(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

To complete part (a), the student needs to know:

(i) that the acceleration function is obtained from the derivative of the velocity function.

(ii) the Sum Rule

(iii) the derivative of a constant function

(iv) the derivative of the sine function.

Part (a) demands little in the way of higher order thinking as it only requires the memorization of one very basic calculus fact plus one general and two specific simple rules of differentiation.

To complete part (b), the student needs to:

(i) equate the given function to zero.
(ii) solve a simple trigonometric equation and check the solutions against the domain.

The completion of part (b) requires no knowledge beyond that gained from secondary school mathematics and physics.

To complete part (c), the student needs to know:

(i) that the position function is obtained from the antiderivative of the velocity function.

(ii) the antiderivative of a constant function.

(iii) the antiderivative of the sine function.

(iv) that substitution of the initial condition will provide the constant of integration.

Part (c) again only requires the memorization of one basic calculus fact and two of the simplest rules of integration. Even if the student assigns the values of the independent and dependent variables in reverse, correct manipulation of the corresponding equation will still result in an accurate value for the constant of integration.

Each part of this question can be completed competently with only a limited understanding of the concepts of calculus. Students only need good memorization skills and a facility with the pencil and paper manipulation of symbols. As well, some parts of the question seem to be designed to reduce the possibility of student error.

The 1997 Examination: Section 2, Question 1

A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \leq 0$.
(b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0,3].
(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

To complete part (a), the student needs to know:

(i) that the position function is obtained from the antiderivative of the velocity function.

(ii) the General Power Rule for Integration.

(iii) that substitution of the initial condition will provide the constant of integration.

This question requires little beyond that in part (c) of the 1987 examination except that the student must assign the independent and dependent values correctly to obtain an accurate value for the constant of integration.

To complete part (b), the student needs to:

(i) understand the difference between instantaneous and average velocity.

(ii) know how to obtain average velocity using distance and time.

(iii) solve a quadratic equation and test the values obtained against the domain.

Solving part (b) first and foremost requires an adequate, though perhaps not a thorough understanding of instantaneous and average rate of change. Without this knowledge, the student could obtain at most two-thirds of the marks for the entire question. The completion the remainder of part (b) requires no knowledge beyond that gained from secondary school mathematics and physics.

To complete part (c), the student needs to:

(i) understand the difference between the meaning of the value obtained by integrating the velocity function on the interval and the value obtained by integrating the absolute value of the velocity function on the interval.

(ii) set up and evaluate the proper integral, possibly using a graphing calculator.

Or

(i) realize that the direction of the motion of the particle may not be constant on the interval.

(ii) determine the times when the velocity is zero.

(iii) determine the position of the particle at these times

(iv) find the total distance traveled by the particle.

Solving part (c) using the first method requires a good understanding of what a definite integral involving velocity means and why using the velocity function itself as the integrand results in the displacement of the particle whereas using the absolute value of the velocity results in the distance the particle travels. Solving this part using the second method requires a basic understanding of rectilinear motion, particularly that linear motion is not restricted to one direction. Although method two is not as mathematically sophisticated as method one, both solutions require a greater understanding of calculus and rectilinear motion than the corresponding question on the 1987 examination.

Although parts of this question can also be completed competently with only a limited understanding of the concepts of calculus, some parts of some questions require that students have a much deeper understanding of some concepts than that required on the 1987 examination. This is the general trend throughout the 1997 examination. As well, the rectilinear motion question on the 1997 examination was the first of a series of increasingly difficult questions, which culminated with an application question in problem six.

Recent AP Calculus AB Examination Changes Enabled by the Increased Use of Graphing Calculators

The use of graphing calculators has enabled the following changes on more recent AP Calculus AB examinations.

1. A decrease in the number of multiple-choice questions that require "plug and chug" solutions. The percentage dropped from 64% on the 1987 examination to 55% on the 1997 examination.
2. An increase in the number of questions or part questions that require higher order thinking.

3. An increase in the number of application questions.

Students who write the AP Calculus AB examination receive a score from The College Board on a scale from 1 (no recommendation) to 5 (extremely well qualified). The overall increase in the difficulty of the AP calculus AB examination enabled by the use of graphing calculators may be seen as unfair by students as it lowers their chances of receiving university credit for AP Calculus AB. However, this problem has been offset somewhat by the decision of some mathematics departments to lower the benchmark used to determine university credit. For instance, in May 1999, the Department of Mathematics at Memorial University of Newfoundland lowered the minimum acceptable standard for university credit from 4 to 3. Despite this potential problem, more recent examinations allowing the use of graphing calculators seem to be preferable to students in Mathematics 4225 as 4 of the 6 completers indicated that they did not prefer the older examination format that prohibited the use of a graphing calculator.

ANALYSIS

Chapter Eleven: The Student Experience of Learning AP Calculus AB

Introduction

As most of the discussion of results occurred in the four results chapters, this chapter contains only a brief description of the student experience of learning early calculus concepts in a web-based environment and identifies some of the major sources of student frustration in the course.

The Student Experience of Learning AP Calculus AB

For the majority of students in Mathematics 4225, the experience of learning calculus in a web-based environment did not match their expectations. Some of the students appreciated the opportunity to have a new learning experience and to attempt a college-level mathematics course before completing secondary school but, for a variety of reasons, many students found the experience frustrating to the point that they withdrew from the course. The remainder of this chapter identifies some of the major contributors to students' frustration.

Sources of Student Frustration

One of the major underlying sources of the frustration experienced by students in Mathematics 4225 may have been a result of the behaviorist approach to teaching and learning in the course. Time restrictions and technological difficulties resulted in a tendency for the instructor to act as the disseminator of information rather than as a guide in the learning process. Consequently, students assumed a passive, reproductive role in the learning process as opposed to developing personal meaning in an active, exploratory learning environment with extended, relevant problem solving activities and student collaboration. For this reason, some students could not attain the conceptual understanding necessary to be successful in the course.

Other problems may have been caused by the incongruencies between the epistemological messages conveyed by the course text and by the course evaluation. The text may have led some students to form the opinion that success in calculus required memorizing rules and following template procedures. However, evaluation instruments often demanded conceptual understanding and the ability to form new knowledge. For those students who had yet to achieve epistemological autonomy, this disparity in messages may have been insurmountable.

Significant problems arose from the lack of telecommunication infrastructure to support synchronous communication. Bandwidth restrictions caused the audio during online classes to be haphazard at best, and video to be nonexistent. The result was a significant decrease in the instructor's synchronous contact time with students. This meant that students generally did not investigate concepts in depth and received less feedback on homework questions and evaluation instruments.

Since Mathematics 4225 was not required for secondary school graduation nor requisite for entrance into post secondary programs, some students viewed Mathematics 4225 as a desirable but dispensable luxury. Initially, all students were prepared to spend considerable time completing homework assignments and other course requirements but, as other courses and extracurricular activities became more demanding of their time, the commitment of some students to Mathematics 4225 diminished to the point where their effort was minimal and their achievement decreased to an unacceptable level. In particular, for those students with a performance orientation to learning, their low academic achievement may have reduced their self-esteem and influenced their decision to withdraw from the course.

Some students' lack of knowledge of the features of the Mathematics 4255 website and textbook, and of the software used in Mathematics 4225 meant that those students could not utilize a significant portion of the support structure incorporated into the course. Many students were unaware of the existence of the appendices of the text, the external links to tutorials, applications and symbolic manipulators at the website, and the online evaluation instruments. Students were generally unaware of the application-sharing feature of NetMeeting, and few utilized Derive. Hence students missed powerful tools for investigating and understanding calculus concepts.

The lack of applications of calculus may have been a source of frustration for some students. Those students who were unaware of the external links to calculus applications at the Mathematics 4255 website would not have been exposed to realistic applications until the project was assigned in the last month of the school year. Hence these students would have to learn concepts and theory in isolation without the opportunity to experience the application of these concepts and theories in real-life situations. This may have been a disappointment to those students who were beginning to make firm career choices and expected at least some application of calculus in those fields.

Finally, some students may have attained a level of understanding of prerequisite calculus knowledge sufficient to allow academic success in secondary school mathematics courses but not to permit equal success in a first calculus course without remedial support. However, the facets and layers of students' conceptual understanding were not determined previous to their participation in the course so remedial action did not occur. The gap between the actual and required understanding of prerequisite concepts such as variable and function meant that subsequent conceptual understanding may have been difficult or impossible.

The students who enrolled in Mathematics 4225 represent the most mathematically gifted segment of the secondary school population and the considerable attrition rate in this course indicates that significant improvements and fine tuning are needed if the number of successful participants is to increase and this type of learning environment is to become a viable alternative to the current distance education models used to deliver mathematics courses in Newfoundland and Labrador's secondary school system.

CONCLUSIONS AND RECOMMENDATIONS

Chapter Twelve: Conclusions and Recommendations

This thesis investigates the student experience of learning calculus at a distance within a constructivist framework. The aim of the thesis is to identify impediments to learning calculus in a web-based environment and to make recommendations for the improvement of learning in that context.

The sixteen recommendations in this chapter fall into three broad categories:

1. Administrative. These recommendations deal with the daily management of offering Mathematics 4225.
2. Technical/technological. These recommendations deal with the technological details of offering Mathematics 4225.
3. Learning. These recommendations deal with the factors that affect the cognitive processes used in learning calculus concepts.

It should be noted that some of these recommendations are not confined solely to one specific category as the effect of their implementation may impact upon or complement recommendations in one or both of the remaining categories.

Administrative Recommendations

These recommendations deal with the management of the details of offering Mathematics 4225. They arise not only from the student survey responses but also from data collected by the researcher during site visits.

Enrollment in Mathematics 4225 was on a volunteer basis. Students in Advanced Mathematics 3201 or Calculus Readiness 3105 were asked to indicate their interest in participating in a first calculus course with minimal examination of their mathematical background or academic maturity. Hence students were admitted into Mathematics 4225 with prerequisite skills that allowed them a degree of success in secondary school mathematics but not necessarily in a college-level, first calculus course. These students consequently experienced limited success and subsequently withdrew from Mathematics 4225.

Recommendation #1

Specify stringent enrolment criteria for Mathematics 4225. In the initial few years in which Mathematics 4225 is offered, only students with high achievement in advanced mathematics and a demonstrated commitment to learning should enroll in the course. This means that initial enrolments may be lower but a history of student success in Mathematics 4225 should be established. These enrolment criteria should also include the recommendation of the Mathematics 4225 instructor or a teacher with experience in teaching AP calculus.

Mathematics 4225 students were permitted to withdraw from the course at any time. This allowed students to continue to enjoy the prestige and privileges associated with participating in an advanced placement course without making any real commitment to the course. These students also placed an added strain on the already limited resources available to more committed students in the course. Students withdrawing at different times in the course created scheduling problems for administrators as these students consequently needed to join other courses that already had high enrollment. There was also the legal liability issue, as students who withdrew from the course and could not enroll in an alternate course had to be supervised when present in the school.

Recommendation #2

Implement a fee and withdrawal schedule similar to that in colleges and universities. Students should be required to pay the fee for their AP Calculus AB examination early in the year and definite deadlines for withdrawal with and without financial penalty should be established and enforced. This should help ensure that only students that are committed to the course will enroll.

Students indicated that a lack of time was a major concern for them. Some students did not have the opportunity to have their questions answered or to avail of the self-evaluation because of the demands of Mathematics 4225 or their other high school courses. They also indicated that they had little time to explore the Mathematics 4255 website and use its features. The amount of work and the level of knowledge required in the course was far above that of any normal secondary school course and the added time pressure imposed by having to write both a Mathematics 4225 final in early May and The College Board AP Calculus AB examination in mid-May as opposed to one month later suggest that only allotting time for the course equivalent to that of a two credit high school course creates an unmanageable situation for some students.

Recommendation #3

Allot time for Mathematics 4225 equivalent to that of four high school credits and raise the credit value of the course from the current two credits to four credits.

Scheduling difficulties caused one of the sites to adjust the offline class schedule such that students at that site were not scheduled for Mathematics 4225 at the same time as students at the other sites. This reduced the opportunity for interaction between the students at this site and the instructor and students at other sites as both were in other classes. It also posed difficulty in scheduling examinations during offline classes, as all sites could not write at the same time nor on the same day.

Recommendation #4

Online and offline class schedules should be identical for all sites. This provides more opportunity for students at different sites to interact and lessens the possibility of compromising the validity of examinations.

School district policy limits to two the number of examinations a student can write in one day. When a student is required to write more than two examinations in a day, it is much more difficult for the instructor to reschedule an examination at several remote sites than it is for an on-site teacher instructing in one school.

Recommendation #5

Develop and implement at the school district level a student evaluation policy that gives priority to the scheduling of distance education examinations.

The instructor made only two visits to each remote site during the nine-month duration of the course. In the survey, students reported a preference for greater face-to-face interaction with the teacher. While frequent visits to the sites may not be feasible, the need for greater interaction with a teacher/mentor can be addressed in two ways:

Recommendation #6

Provide an on-site teaching assistant, with mathematical training, to act as a liaison between the students and the instructor and as a mentor for students. This already occurs in many schools in the current distance education model and greatly facilitates student progress.

Recommendation #7

Arrange for the instructor to visit remote sites at least once every two weeks, plus schedule separate online conferences between the instructor and each site during the alternate weeks. This would allow greater personal contact between the instructor and students.

Technical/Technological Recommendations

These recommendations deal with the technological details of offering Mathematics 4225. They arise exclusively from the student survey responses.

For at least half of the year at one site, students in another course filled the computer lab to capacity during the site's offline AP calculus classes. This meant that the student AP computer was the only one available at that time and the initial enrollment of seven students at that site restricted student access to the Mathematics 4255 website during offline classes. The restricted access to a computer during the school day was especially detrimental to those students who did not have a computer with Internet access at home and who could not use the school computer facilities after school hours. These students did not have the opportunity to fully explore the website and take advantage of some of its features such as the self-evaluation, tutorials, and applications, nor did they have the opportunity to interact with other sites during offline classes.

Recommendation #8

During offline classes, ensure that AP students have access to Internet-ready computers in a two to one student-computer ratio.

The major source of frustration by far for most students was the unreliability of the audio. Some noncompleters suggested it was the main reason they withdrew from the course. The importance of the real-time audio in a web-based environment cannot be overrated, as it is the key component of the synchronous interaction between instructor and student and among students.

Recommendation #9

Provide the telecommunication infrastructure required to allow reliable, quality, real-time audio and video. Since this cannot be provided through the low speed modems currently in use, a system compatible with the Direct PC satellites dishes currently on Newfoundland and Labrador schools should be developed. However, these dishes are static and still require a telephone modem for outgoing signals. A more suitable option would be to install a dynamic satellite dish on those schools that have students enrolled in web-based courses. An alternative to this is to schedule separate online classes with each site, since the audio at least works acceptably when the instructor is connected to only one other site. This would mean an increase in the time assigned to the instructor for Mathematics 4225 but would satisfy the student desire for more online classes.

Most students in Mathematics 4225 were very adept in the use of graphing calculators. However, these students had little or no exposure to software packages such as Geometer's Sketchpad or Derive and actually requested more training on Microsoft NetMeeting in the student survey. Students were forced to learn the basic features of NetMeeting as it was used for synchronous interaction but they did not use the application-sharing feature of this software package. The large majority of students used neither Geometer's Sketchpad nor Derive and hence lost powerful tools for formulating and investigating conjectures plus alternate avenues for exploring word problems. While most students were familiar with the chat feature in NetMeeting, only the two students with previous distance education experience were familiar with the use of a whiteboard. As well, students reported they were unaware of some of the features of the text and those with ample access to computers still reported little knowledge of some of the features of the Mathematics 4255 website, such as the external hyperlinks.

Recommendation #10

Designate two days early in the school year for students to receive training in the various software packages and equipment used in Mathematics 4225, and to become familiar with the features of the text and Mathematics 4255 website. Even if Geometer's Sketchpad and Derive are not available at the student sites, they can still be used through the application-sharing feature of NetMeeting. The instructor can then insist that students use these packages by designing appropriate evaluation. An added benefit should be a decrease in the amount of pencil and paper symbolic manipulation that must be performed. Familiarity with the text and website will also facilitate the learning of calculus concepts and increase the comfort level of students as they enter the course.

Students who had Internet-ready computers at home had a definite advantage in Mathematics 4225, especially given the restricted access to computers in the often-crowded computer labs at school. Home computers gave students virtually unrestricted access to the Mathematics 4255 website and its features and permitted asynchronous interaction at any time between students at different sites. Students indicated that having a home Internet-ready computer enabled them to experience more success in the course.

Recommendation #11

Encourage students to have an Internet-ready computer at home with a suitable browser and appropriate software. This may mean that students will have to purchase some software but this cost can be minimized if student versions of the software can be purchased at educational prices.

Learning Recommendations

These recommendations deal with the factors that affect the cognitive processes used in learning calculus concepts. They arise not only from the student survey responses but also from current literature on constructivism, learning calculus and learning at a distance.

The earlier comparison of the current course text and the Consortium text reveals that the latter text conveys more appropriate epistemological messages to students. In comparison, the Larson text places more emphasis on learning calculus concepts in isolation, on the development of mechanical skills as opposed to the application of concepts, on formal as opposed to intuitive reasoning, and on the textual presentation of the material. It places less emphasis on the theory and history of mathematics and provides fewer representations of calculus material. For these reasons, the messages communicated to students about the origin and nature of mathematical knowledge and about which aspects of mathematics are important are less suitable.

Recommendation #12

Replace the current textbook with the Calculus Consortium text. This text is also more appropriate because it possesses many of the characteristics which constructivists and mathematics education researchers advocate are important in learning mathematics. The text also incorporates many of the suggestions for course improvement arising from the student survey. However, with respect to some of the differences identified in the earlier comparison of the two texts, the following cautions are necessary.

(a) The instructor must provide questions that require the proper formulation of a mathematical model as the vast majority of modeling questions provide the mathematical model. The ability to construct an accurate mathematical model is an important skill for those who will encounter real-life calculus applications. Creating a model demands a familiarity with basic exponential, logarithmic and trigonometric graphs. A review of these graphs occurs in the first chapter of the Consortium text.

(b) The instructor may need to provide supplemental questions requiring basic computation. This text contains less basic computation problems, but the student survey indicated that students considered these questions important, although possibly because the concepts were not covered well during online classes and their exposure to numerous questions of this type in their previous mathematics courses.

(c) It is still important that students have ability in pencil-and-paper manipulation of symbols, and memorize the basic rules of differentiation and integration, even though the number of questions in the text requiring symbolic manipulation has decreased significantly, as questions requiring the use of these facts and skills are still a significant part of the AP Calculus AB examination.

(d) Few questions in the text require student collaboration, so the instructor must ensure that opportunities for student interaction are maximized, as this interaction promotes deep learning.

(e) Even though the emphasis on proof has increased in this text, the instructor still needs to ensure that changes in the hypothesis in conjectures and theorems are investigated to allow the student to experience more than just the standard process of theorem-proof-application.

It is unreasonable to expect students to learn numerous calculus concepts in isolation and yet remain motivated and committed to the course. Students must receive exposure to the real-life application of those concepts.

Recommendation #13

Regularly present students with a variety of calculus applications, which require some time to investigate and demand collaboration. Enrolment makes it impossible to group a calculus class by discipline or area of interest, so a narrow disciplinary focus is impractical. If the interests and future plans of students are determined in advance and the appropriate applications provided, learning the terminology associated with an application will only be a minor obstacle. Extended investigations allow students to experience the entire problem solving process, including reflection and self-regulation.

Noncompleters were particularly adamant that there was too little interaction between students and the instructor (see Table 38). Interaction is valuable as it stimulates interest, learning, and commitment to the course, reduces feelings of isolation, and promotes the development of interpersonal communication skills. Equally important, interaction also promotes the development of the sense of community and belonging that may be vital to the success of small groups of students in geographically isolated locations.

Recommendation #14

Increase both synchronous and asynchronous interaction between teacher and students and between students. This can be achieved in several ways.

(a) Student-instructor collaboration in problem solving. This activity has the additional benefit of allowing students' insight into the mathematician's way of thinking as well as providing the instructor with information on student thought processes. Teacher insight into any incongruity between student concept image and the concept definition may facilitate the removal of the cognitive obstacles that prevent conceptual understanding.

(b) Calculus labs. There were external links at the Mathematics 4225 website that presented students with a variety of calculus laboratories. These labs again provided a good opportunity for teacher-student interaction and cooperative problem solving.

(c) Avenues for asynchronous communication such as the bulletin board, calendar, private e-mail, and fax. Many students in non-web-based distance education courses offered using the Telemedicine model find some of these avenues of communication invaluable. However, with the exception of one completer, students in Mathematics 4225 used these forms of asynchronous communication to a much lesser extent. Given the technical difficulties during online classes, students greatly limited their opportunities for interaction with the instructor and other students by not using these avenues of communication.

(d) Real-time video during online classes. This feature may help to personalize the instructor and students and foster a sense of community among participants in the course.

(e) Group presentations, especially those where group members belong to multiple sites. Class presentations may also be an avenue for fostering a sense of community among participants, and may be especially valuable for the isolated student. To promote writing in the course, a written report should also be submitted by each group.

(f) Learning conversations. These dialogues are valuable because they provide the student with feedback on the quality of their work and promote their understanding of concepts. They also provide insight into student thought processes.

(g) The chat feature in NetMeeting and the WebCT chat room. NetMeeting's chat feature was used during online classes to communicate with students when the audio failed. The WebCT chat room prevented unwanted outside input as it was password protected. Neither of these features required any special computer hardware or software.

(h) Study groups. These may be especially effective as participants in Mathematics 4225 have very similar mathematical backgrounds. They may also decrease feelings of isolation and anxiety for some students.

(i) Class socials. Students expressed a need for three to four class social events during the year.

The results of Table 56 indicate that noncompleters adopted a surface approach to learning. While this type of understanding may have allowed some success in previous mathematics courses, it will not result in the conceptual understanding necessary for success in a first calculus course, nor provide the foundation for further work in mathematics.

Recommendation #15

Promote a deep approach to learning. Such an approach to learning can be fostered through motivation, activity, interaction, and the creation of structured knowledge base in students. In addition to interaction with others, a deep approach to learning in Mathematics 4225 can be promoted in the following ways.

(a) Limit features such as *My Record* that may promote comparison of quantitative achievement among students. Students identified this particular feature as an important and useful one but aspects of the feature that allow comparison of quantitative achievement with other students may promote a surface level of learning.

(b) Design assessment that demands deep level processing. Such assessment simulates and supports reflection on calculus concepts and hence promotes deep learning. Such reflection can be achieved by requiring students to explain the meaning of their solutions and the logic used to obtain it, as well as encouraging them to examine the reasonableness of their answers.

(c) Increase the time available for student reflection on calculus concepts. Assessment items such as assignments should not be the only opportunity for student reflection on the ideas and concepts of calculus. However, the demands of other courses and extracurricular activities often allow fewer opportunities for student reflection than is optimal in a first calculus course. Students must be provided with time to reflect on their work if they are to develop a deep understanding of the ideas and concepts presented to them. This could be achieved by assigning to Mathematics 4225 the time equivalent to that of a four-credit high school course.

(d) Reduce behaviorist learning goals and objectives. Behavioral objectives may be necessary as they reflect some of the current expectations of AP mathematics. However, general learning abilities and self-awareness as a learner should also be explicit goals of the course.

(e) Improve advance organizers. Advance organizers may be crucial in establishing the appropriate mind set in students. Students did not assign great importance or usefulness to the advance organizers presented in the lesson notes. This indicates that this aspect of the notes needs to be reviewed and improved.

(f) Connect new knowledge to previous experience. The connection of previous knowledge and new ideas is necessary to form a structured knowledge base and to promote the reification of concepts necessary to expand or reconstruct existing schemata. This can be achieved if the ideas and concepts of calculus are connected to previous high school mathematics courses and if students are presented with real-life, relevant applications of calculus in areas where students have previous experience or interest. These applications can take the form of projects but they need to be more numerous and varied and not be restricted to the last month of the course, as was the case in Mathematics 4225.

(g) Increase the use of metacognition. Metacognition is important because it has the potential to encourage a deep approach to learning by making learning more meaningful to the student. Metacognition involves the beliefs and intuitions the student possesses about mathematics, the knowledge they have about their own thought processes, and their self-awareness as a learner.

These aspects of metacognition can be exercised in the attempt to collectively solve unfamiliar, realistic problems in mathematics where the teacher acts as guide instead of expert. As students and instructor solve these problems together, students should reconstruct beliefs about mathematics. The commonly held belief that mathematics consists of using memorized formulas in unrealistic situations can be reshaped if problems are related to real-life phenomenon. The participation in the problem solving process should also help students to develop a sense of what they know about their own thought processes. They should also be helped to develop their self-awareness as a learner by being guided to make sure they understand the problem before they attempt a solution, to develop a plan of attack for solving the problem, to monitor the effectiveness of their plan in solving the problem, and to allocate resources as they work on the solution to the problem.

(h) Construct a significant number of concept maps. Concept maps have the potential for inducing deeper level processing of content. However, the one concept map provided to students in Mathematics 4225 was insufficient to promote meaningful learning. Students in Mathematics 4225 should be required to produce at least one concept map per chapter, with a gradual increase in the autonomy expected in the construction of the concept map.

(i) Allow for greater student input into selecting and planning their learning. These opportunities help develop the student's intrinsic motivation which in turn fosters deep learning. It also increases student ownership of the course.

The actual topics to be covered in Mathematics 4225 are dictated by the AP calculus AB Topical Outline but students should be given input on the order in which those topics are studied and which topics receive greater emphasis. They should also be involved in the negotiation of the types and weights of evaluation instruments.

It should be noted that students in Mathematics 4225 were exposed to several inappropriate messages about the origin and nature of calculus knowledge. The implementation of recommendations 12-15, along with the replacement of the current textbook, should result in the conveyance of more appropriate epistemological messages and help students achieve epistemological autonomy.

When onsite noncompleters are removed from the results, Table 52 indicates that noncompleters did not receive enough support from the teacher. Given the difficulty encountered with the audio during online classes, this result is credible.

Recommendation #16

Increase the amount of teacher support in Mathematics 4225. The student surveys suggest specific ways in which this can be achieved.

(a) Provide more regular and extensive feedback on homework, assignments and examinations. Table 41 indicates that noncompleters in particular were not satisfied with the level of teacher feedback on evaluation instruments.

(b) Provide complete, detailed solutions for the evaluation instruments given to students. Students enrolled in distance education courses using the Telemedicine model receive complete solutions for all their evaluation instruments. The same provision should be made for Mathematics 4225 students.

(c) Provide a complete solution guide for students with detailed solutions for the homework questions. Students are currently required to purchase a student and solution guide that gives the detailed solutions to most of the even-numbered homework questions, but the solution for a significant number of homework questions do not appear in the guide. Solutions for these questions should be provided.

(d) More consistent evaluation. Evaluation instruments such as assignments and tests should be consistent in structure and length, and should reflect the expectations of the AP calculus AB examination.

Although the encapsulation of the concept of function is prerequisite to the conceptual understanding of calculus concepts, research by Mitchelmore (1996) and Mamona-Downs (1997) suggest that an understanding of variable at or near the point of encapsulation is itself necessary for an understanding of functions. Students' understanding of the variable and function concepts must be determined before enrolment in Mathematics 4225 and the appropriate level of understanding of these concepts must be attained in order to prevent the resulting cognitive obstacles from impeding conceptual understanding.

Recommendation #17

For each student who intends to enroll in Mathematics 4225, determine their depth of cognitive development for the facets of the variable concept, develop an individual genetic decomposition for the concept, and design the appropriate instruction to help the learner reach the appropriate level of understanding of the concept. This process should then be repeated for the function concept.

It should be noted that although the Larson and Consortium texts contain initial chapters that review the concept of function, both assume previous conceptual understanding of variable. Given the research findings of Mamona-Downs (1997), Mitchelmore (1996), Trigueros, Ursini and Reyes (1996) and Ursini and Trigueros (1997), this assumption may not be justified

References

Alarcón, F. E., & Stoudt, R. A. (1997). The rise and fall of a mathematica-based calculus curriculum reform movement. Problems, Resources and Issues in Mathematics Undergraduate Studies, 7 (1), 73-88.

Alberts, G. E. (1988). Mathematics as a client discipline. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 41). Mathematical Association of America.

Al-Kunifed, A., & Wandersee, J. H. (1990). One hundred references related to concept mapping. Journal of Research in Science Teaching, 27 (10), 1069-1075.

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. Journal of Educational Psychology, 84 (3), 261-271.

Anderson, R. D. (1988). Calculus in a large university environment. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 157-161). Mathematical Association of America.

Archer, R. D., & Armstrong, J. S. (1988). Calculus for physical sciences: Second discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 60-61). Mathematical Association of America.

Armstrong, G., Garner, L., & Wynn, J. (1994). Our experience with two reformed calculus programs. Problems, Resources and Issues in Mathematics Undergraduate Studies, 4 (4), 301-311.

Artigue, M. (1991). Didactical engineering, research and development: Some theoretical problems linked to this duality. For the Learning of Mathematics: An International Journal of Mathematics Education, 11 (1), 13-18.

Azcarate, C., & Espinoza, L. (1995). A study on the secondary teaching system about the concept of limit. Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education, Brazil, 2, 11-17.

Bailey, C., & Chambers, J. (1996). Interactive learning and technology in the US science and mathematics reform movement. British Journal of Educational Technology, 27 (2), 123-133.

Bangert-Drowns, R. L., Kulik, C. C., Kului, J. A., & Morgan, M. T. (1991). The instructional effect of feedback in test-like events. Review of Educational Research, 61 (2), 213-238

Barrett, L. K., & Teles, E. J. (1988). Objectives, teaching, and assessment: Third discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 84-85). Mathematical Association of America.

Bernard, R. M., & Naidu, S. (1992). Post-questioning, concept mapping and feedback: A distance education field experiment. British Journal of Educational Technology, 23 (1), 48-60.

Bezuidenhout, J., Human, P., & Olivier, A. (1998). Some misconceptions underlying first-year students' understanding of average rate and of average value. Proceedings of the Twenty-Second International Conference for the Psychology of Mathematics Education, South Africa, 2, 96-103.

Biggs, J. (1994). Student learning research and theory: Where do we currently stand? In G. Gibbs (Ed.), Improving Student Learning: Theory and Practice (pp. 1-19). Oxford: Oxford Center for Staff Development.

Bills, L. (1996). The use of examples in the teaching and learning of mathematics. Proceedings of the Twentieth International Conference for the Psychology of Mathematics Education, Spain, 2, 81-88.

Binns, J. E. (1994). One's company, two's a crowd - pupil's difficulties with more than one variable. Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education, Portugal, 2, 80-87.

Blunt, A., & Dent, B. (1999). Computer managed student assessment: A case study. The Canadian Journal for the Study of Adult Education, 13 (1), 41-60.

Boekaerts, M. (1997). Self-regulated learning: A new concept embraced by researchers, policy makers, educators, teachers and students. Learning and Instruction, 7 (2), 161-186.

Bogdan, R. C., & Biklen, S. K. (1982). Qualitative Research for Education: An Introduction to Theory and Methods. Boston: Allyn and Bacon.

Boondao, S. (1992). The effects of students' backgrounds, their cognitive styles and the use of assignments on mathematical achievements in a distance education program [Abstract]. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 127.

Bossert, W., & Chinn, W. G. (1988). Calculus for the life sciences. In L. Steen (Ed.), Calculus or a New Century: A Pump, Not a Filter (pp. 66-68). Mathematical Association of America.

Bradburn, J. (1988). A two-year college perspective. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 154-157). Mathematical Association of America.

Bressoud, D. M. (1996). Assessing student performance. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 66-73). Mathematical Association of America.

Brito, D. L., & Goldberg, D. Y. (1988). Calculus for business and social science students: First discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 68-70). Mathematical Association of America.Buccino, A., & Rosenstein, G., Jr. (1988). Objectives, teaching and assessment: First discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 81-83). Mathematical Association of America.

Buck, R. C. (1988). Computers and calculus: The second stage. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 168-172). Mathematical Association of America.

Bullen, M., (1998). Participation and critical thinking in online university distance education. Journal of Distance Education, 13 (2), 1-32.

Butler, D. L., & Winne, P. H. (1995). Feedback and self-regulated learning: A theoretical synthesis. Review of Educational Research, 65 (3), 245-281.

Carlson, D., & Roberts, W. (1996). Changing calculus: Its impact on the post-calculus curriculum. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 149-151). Mathematical Association of America.

Carr, R. (1992). Guided problem solving for first year tertiary students. Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia, Richmond, New South Wales, Australia, 189-196.

Chrobak, M.. (1988). The role of the calculator industry. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 43-44). Mathematical Association of America.

Cipra, B. A. (1988). Recent innovations in calculus instruction. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 95-103). Mathematical Association of America.

Commission on Standards for School Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Commission on Teaching Standards for School Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

Compton, W. D. (1988). Calculus for engineering practice. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 18-19). Mathematical Association of America.

Cornu, B. (1991). Limits. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 153-166). Boston, MA: Kluwer Academic Publishers.

Crawford, K., Gordon, S., Nicholas, J., Prosser, M. (1994). Students' reports of their learning about functions. Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education, Portugal, 2, 233-239.

Curtis, P. C. Jr., & Northcutt, R. A. (1988). Objectives, teaching, and assessment: Second discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 83-84). Mathematical Association of America.

Davis, R. M. (1988). Calculus: Changes for the better. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 44). Mathematical Association of America.

Deekers, J., Cuskelly, E., Kemp, N., & Phillips, J. (1992). Use of instructional materials by distance education students: Patterns and student perceptions. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 378-386.

DeMarois, P., & Tall, D. (1996). Facets and layers of the function concept. Proceedings of the Twentieth International Conference for the Psychology of Mathematics Education, Spain, 2, 297-304.

Dodge, W. R. (1988). Imperatives for high school mathematics. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 45). Mathematical Association of America.

Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 25-41). Boston, MA: Kluwer Academic Publishers.

Dreyfus, T., & Eisenberg, T. (1982). Intuitive functional concepts: A baseline study on intuitions. Journal for Research in Mathematics Education, 13, 360-380.

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 95-123). Boston, MA: Kluwer Academic Publishers.

Dubinsky, E. (1992). A learning theory approach to calculus. In Zaven A. Karian (Ed.), Symbolic Computation in Undergraduate mathematics Education (pp. 43-55). Mathematical Association of America.

Dubinsky, E., Czarnocha, B., Prabhu, V., & Vidakovic, D. (1999). One theoretical perspective in undergraduate mathematics education research. Proceedings of the Twenty-Third Annual Conference of the International Group for the Psychology of Mathematics Education, Israel, 1, 95-110.

Egerer, G., & Cannon, R. J. (1988). Calculus for business and social science students: Second discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 70-71). Mathematical Association of America.

Eisenberg, T. (1991). Functions and associated learning difficulties. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 140-152). Boston, MA: Kluwer Academic Publishers.

Erdman, C. A., & Malone, J. J. (1988). Calculus for engineering students: Third discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 65-66). Mathematical Association of America.

Esquinca, M. S., & Osorio, F. C. (1996). The role of the visual and analytic acts in the students' understanding of linear differential equations. Proceedings of the Eighteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 65.

Ferrina-Mundy, J., & Graham, K. G. (1994). Research in calculus learning: Understanding of limits, derivative and integrals. In J. Kaput and E. Dubinsky (Eds.), Research Issues in Undergraduate Mathematics Learning. MAA Notes No. 34. Washington, DC: Mathematical Association of America.

Ferritor, D. E. (1988). A Chancellor's Challenge. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 27-29). Mathematical Association of America.

Finlow-Bates, K. (1994). First year mathematics students' notions of the role of informal proof and examples. Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education, Portugal, 2, 344-351.

Flashman, M. (1996). Planning and change: Experience reports. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 41-51). Mathematical Association of America.

Frid, S. D. (1992). Calculus students' sources of conviction. Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia, Richmond, New South Wales, Australia, 294-304.

Fulton, J. D. (1988). An effective class size for calculus. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 46-47). Mathematical Association of America.

Garland, M. R. (1995). Helping students achieve epistemological autonomy. In D. Stewart (Ed.), One World Many Voices: Quality in Open and Distance Learning. Proceedings of the Seventeenth World Conference for Distance Education, 2, 77-80.

Gehrke, M., & Pengelley, D. (1996). Towards active processes for teaching and learning. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 20-23). Mathematical Association of America.

Gillman, L. (1988). Two proposals for calculus. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 216-218). Mathematical Association of America.

Glaser, B. G., & Strauss, A. L. (1967). The Discovery of Grounded Theory: Strategies for Qualitative Research. Chicago: Aldine Publishing Company.

Glesne, C., & Peshkin, A.(1992). Becoming Qualitative Researchers: An Introduction. New York: Longman.

Goodson-Espy. T. (1998). The roles of reification and reflective abstraction in the development of abstract thought: Transitions from arithmetic to algebra. Educational Studies in Mathematics, 36 (3), 219-245.

Government of Newfoundland and Labrador. (1997). The senior high school program: New directions for the 21st century. St. John's, Newfoundland, Canada: Author.

Gray, E. M., & Tall, D. O. (1991). Duality, ambiguity and flexibility in successful mathematical thinking. Proceedings of the Fifteenth International Conference for the Psychology of Mathematics Education, Italy, 2, 722-729.

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity and flexibility: A proceptual view of simple arithmetic. Journal of Research in Mathematics Education, 26 (2), 115-141.

Haines, C. W., & Boutilier, P. O. (1988). Calculus for engineering students: Second discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 63-65). Mathematical Association of America.

Hallett, D. H. (1996). Constructing a new calculus course. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 28-30). Mathematical Association of America.

Harvey, J. G. (1988). The role of placement testing. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 135-140). Mathematical Association of America.

Hasemann, K., & Mansfield, H. (1995). Concept mapping in research on mathematical knowledge development: Background, methods, findings and conclusions. Educational Studies in Mathematics, 29, 45-72.

Heeren, E., & Collis, B. (1993). Design considerations for telecommunications-supported cooperative learning environments: Concept mapping as a "telecooperation support tool". Journal of Educational Multimedia and Hypermedia, 4 (2), 107-127.

Henri, F., & Hotte, R. (1992). Improving CMC strategies in distance education [Abstract]. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 252.

Henri, F., & Kaye, A. (1993). Problems of distance education. In K. Harry, J. Magnus, & D. Keegan (Eds.), Distance Education: New Perspectives (pp. 25-32). New York: Routledge.

Hillman, D. C., Willis, D. J., & Gunawardena, C. N. (1994). Learner-interface interaction in distance education: An extension of contemporary models and strategies for practitioners. The American Journal of Distance Education, 8 (2), 31-42.

Hiltz, S. R. (1993). Correlates of learning in a virtual classroom. International Journal of Man-Machine Studies, 39 (1), 71-98.

Hodgson, B. R. (1988). Evolution in the teaching of calculus. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 49-50). Mathematical Association of America.

Hollar, J. C. (1996). Function understanding in a graphing approach curriculum. Proceedings of the Eighteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 207-208.

Holmberg, B. (1995). Theory and Practice of Distance Education. New York: Routledge.

Horn, H. S. (1988). Calculus in the biological sciences. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 20-21). Mathematical Association of America.

Hughes, R. J. (1988). Calculus reform and women undergraduates. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 125-129). Mathematical Association of America.

International Commission on Mathematical Instruction. (1998). ICMI study on the teaching and learning of mathematics at the university level. Educational Studies in Mathematics, 36, 91-101.

Joughin, G., Lai, T., & Cottman, C. (1992). Distance learners' approaches to studying: The nature of deep and surface approaches reconsidered . (ERIC Document Reproduction Service No. ED 357 226)

Kaput, J. J., & Thompson, P. W. (1994). Technology in mathematics education research: The first twenty-five years in JRME. Journal for Research in Mathematics Education, 25 (6), 676-684.

Kasworm, C. E. (1997). Adult meaning making in the undergraduate classroom . (ERIC Document Reproduction Service No. ED 410 778)

Kasworm, C. E., & Yao, B. (1992). The development of adult learner autonomy and self-directedness in distance education . (ERIC Document Reproduction Service No. ED 355 453)

Kearsley, G. (1995). The nature and value of interaction in distance learning. In M. F. Beaudoin (Ed.), Distance Education Symposium 3: Instruction, Selected Papers Presented at the Third Distance Education Symposium (pp. 83-92). Pennsylvania State University, The American Center for the Study of Distance Education.

Kenelly, J. (1996). Technology in mathematics instruction. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 24-27). Mathematical Association of America.

Kenelly, J. W., & Eslinger, R. C. (1988). Computer algebra systems: First discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 78-79). Mathematical Association of America.

Knight, G. M. (1988). Collective dreaming or collaborative planning? In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 51). Mathematical Association of America.

Kolata, G. B. (1988). Calculus reform: Is it needed? Is it possible? In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 91-94). Mathematical Association of America.

Kulhavy, R. W. (1977). Feedback in written instruction. Review of Educational Research, 47, 211-232.

Lathrop, K. D. (1988). Calculus for engineering. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 104-106). Mathematical Association of America.

Lauder, W. (1996). Constructing meaning in the learning experience: The role of alternative theoretical frameworks. Journal of Advanced Nursing, 24, 91-97

Lauzon, A. C. (1992). Integrating computer-based construction with computer conferencing: An evaluation of a model for designing online education. American Journal of Distance Education, 6 (2), 32-46.

Layton, K. P. (1988). Perspective from high schools. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 149-153). Mathematical Association of America.

Levin, S. A. (1988). Calculus for the biological sciences. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 116-121). Mathematical Association of America.

Lovelock, D., & Newell, A. C. (1988). A calculus curriculum for the nineties. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 162-168). Mathematical Association of America.

Lyons, J. J. (1988). Innovation in calculus textbooks. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 145-149). Mathematical Association of America.

Malcom, S. M., & Treisman, U. (1988). Calculus success for all students. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 129-134). Mathematical Association of America.

Mamona-Downs, J. (1997). Students' dependence on symbolic variables in functions. Proceedings of the Twenty-First International Conference for the Psychology of Mathematics Education, Finland, 1, 245.

Martin, T. S., (1996). First-year calculus students' procedural and conceptual understandings of geometric related rate problems. Proceedings of the Eighteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 47-52.

Mathematical Sciences Education Board. (1990). Reshaping school mathematics: A philosophy and framework for curriculum. Washington, DC: National Academy Press.

Meel, D. E. (1997). Mis-generalizations in calculus: Searching for the origins. Proceedings of the Nineteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 23-29.

Miller, N. C. (1995). Women in consortium calculus. The Mathematics Teacher, 88 (7), 546-549.

Miller W. W., & Webster, J. K. (1997). A comparison of interaction needs and performance of distance learners in synchronous and asynchronous classes. (ERIC Document Reproduction Service No. ED 415 411)

Mochon, S., & Godinez, J. (1996). A study about the methods used by college students to model real situations. Proceedings of the Eighteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 53-57.

Morgan, A. R. (1995). Improving student learning in distance education: Theory, research and practice. European Journal of Psychology of Education, 10 (2), 121-130.

Moskowitz, H. (1988). Calculus for management: A case study. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 21-23). Mathematical Association of America.

Naidoo, R. (1998). Errors made in differential calculus by first year technikon students. Proceedings of the Twenty-Second International Conference for the Psychology of Mathematics Education, South Africa, 4, 288.

Naidu, S. (1990). Concept Mapping: Student Workbook. (ERIC Document Reproduction Service No. ED 329 247)

Naidu, S., & Bernard, R. M. (1992a). Research on learning strategies in distance education: Implications for instruction [Abstract]. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 144.

Naidu, S., & Bernard, R. M. (1992b). Enhancing academic performance in distance education with concept mapping and inserted questions. Distance Education, 13 (2), 218-233.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

National Science Foundation. (1988). NSF Workshop on Undergraduate Mathematics. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 221-223). Mathematical Association of America.

Newman, R. J., & Poiani, E. L. (1988). Encouraging success by minority students. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 74-76). Mathematical Association of America.

Novak, J. D., Gowin, D. B., & Johansen, G. T. (1983). The use of concept mapping and knowledge vee mapping with junior high science students. Science Education, 67 (5), 625-645.

O'Meara, T. (1988). Calculus – a call to arms. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 52). Mathematical Association of America.

Orton, A. (1984). Understanding rate of change. Mathematics in School, 13 (5), 23-26.

Ostebee, A. (1993). Symbolic computation systems and algebraic, graphical, and numerical viewpoints in elementary calculus. In D. L. Ferguson (Ed.), Advanced Educational Technologies for Mathematics and Sciences, NATO ASI Series, Series F: Vol. 107. Computer and Systems Sciences (pp. 567-583). New York: Springer-Verlag.

Persons, H., & Griffiths, C. (1992). Developing higher order thinking skills in distance education students through telephone seminars [Abstract]. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 190.

Porzio, D. T. (1997). Effects of different instructional approaches on calculus students' understanding of the relationship between slope, rate of change, and the first derivative. Proceedings of the Nineteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 37-44.

Prichett, G. D. (1988). Calculus in the undergraduate business curriculum. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 112-116). Mathematical Association of America.

Priestley, W. M. (1988). Surprises. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 53). Mathematical Association of America.

Ralston, A. (1988). Calculus and computer science. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 23-24). Mathematical Association of America.

Rasmussen, C. L. (1998). Learning obstacles in differential equations. Proceedings of the Twenty-Second International Conference for the Psychology of Mathematics Education, South Africa, 4, 25-32.

Rassian, S., & Vinner, S. (1997). Images and definitions for the concept of even/odd function. Proceedings of the Twenty-First International Conference for the Psychology of Mathematics Education, Finland, 4, 41-48.

Rassian, S., & Vinner, S. (1998). Images and definitions for the concept of increasing/decreasing functions. Proceedings of the Twenty-Second International Conference for the Psychology of Mathematics Education, South Africa, 4, 33-40.

Redish, E. F. (1988). The coming revolution in physics instruction. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 106-112). Mathematical Association of America.

Reed, M.C. (1988). Now is your chance. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 30-32). Mathematical Association of America.

Risnes, M. (1997). Do the concepts of derivative and marginal cost make sense for business students? Proceedings of the Twenty-First International Conference for the Psychology of Mathematics Education, Finland, 1, 260.

Robler, M., Edwards, J., & Havriluk, M. (1997). Integrating Educational Technology into Teaching. New York: Prentice Hall/Merrill College Publishing.

Rodriguez, D. E. (1995). Interaction in the ITESM's distance education system. In E. D. Wagner & M. A. Koble (Eds.), Distance Education Symposium 3: Course Design, Selected Papers Presented at the Third Distance Education Symposium (pp. 93-102). Pennsylvania State University, The American Center for the Study of Distance Education.

Ross, S. C. (1996). Visions of calculus. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 8-15). Mathematical Association of America.

Rowntree, D. (1992). Exploring Open and Distance Learning. London: Kogan Page.

Rumble, G. (1992). The management of distance learning systems. UNESCO International Institute for Educational Planning, Paris.

Ryan, L. (1992). The utilization of feedback for maintaining the academic quality of post-graduate distance education programs. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 147.

Ryan, J. (1992). Integrating computers into the teaching of calculus: Differentiating student needs. Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia, Richmond, New South Wales, Australia, 478-487.

Salamon, L. B. (1988). Involvement in calculus learning. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 32-35). Mathematical Association of America.

Schoenfeld, A. H. (1995). A brief biography of calculus reform. UME Trends, 6 (6), 3-5.

Scholdt, G. P., Zhang, S., & Fulford, C. P. (1995). Sharing Across Disciplines – Interaction Strategies in Distance Education Part I: Asking and Answering Questions. University of Hawaii. (ERIC Document Reproduction Service No. ED 383377)

Schuell, T. J. (1986). Cognitive conceptions of learning. Review of Educational Research, 56 (4), 411-436.

Schuemer, R. (1993). Some Psychological Aspects of Distance Education . (ERIC Document Reproduction Service No. ED 357 266)

Schunk, D. H., & Zimmerman, B. J. (Eds.), (1994). Self-Regulation of Learning and Performance: Issues and Educational Applications. Hillsdale, NJ: Erlbaum.

Seaton, W. J. (1993). Computer-mediated communication and self-directed learning. Open Learning, 8 (2), 49-54.

Selden, A., & Selden, J. (1990). Constructivism in mathematics education: A view of how people learn. UME Trends, 2 (1), 18-25.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. Educational Studies in Mathematics, 22, (1), 1-36.

Shama, G., & Movshovitz-Hadar, N. (1994). Is infinity a whole number? Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education, Portugal, 4, 265-272.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77, 20-26.

Skemp, R. R. (1987). The Psychology of Learning Mathematics. Hillsdale, NJ: Erlbaum.

Small, D. B. (1988). Transition from high school to college calculus. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 224-229). Mathematical Association of America.

Smith D. A. (1996). Thinking about learning, learning about thinking. In A. W. Roberts (Ed.), Calculus: The Dynamics of Change (pp. 31-37). Mathematical Association of America.

Starr, S. F. (1988). Calculus in the core of liberal education. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 35-37). Mathematical Association of America.

Steen, L. A. (Ed.). (1988a). Calculus for a New Century: A Pump, Not a Filter. Mathematical Association of America.

Steen, L. A. (1988b). Calculus today. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 10-13). Mathematical Association of America.

Steen, L. A. (1988c). Who still does math with paper and pencil? In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 231-232). Mathematical Association of America.

Stevenson, J. R. (1988). Calculus for the physical sciences. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 25-26). Mathematical Association of America.

Strang, G. (1988). Calculus for a purpose. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 54). Mathematical Association of America.

Tall, D. (1987). Constructing the concept image of tangent. Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education, Canada, 3, 69-75.

Tall, D. (1989). Concept images, generic organizations, computers, and curriculum change. For the Learning of Mathematics: An International Journal of Mathematics Education, 9 (3), 37-42.

Tall, D. (1991). The psychology of advanced mathematical thinking. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 3-21). Boston, MA: Kluwer Academic Publishers.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 495-511). New York: Macmillan Publishing Company.

Tall, D. (1995). Cognitive growth in elementary and advanced mathematical thinking. Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education, Brazil, 1, 61-75.

Tall, D. (1999). Reflections on APOS theory in elementary and advanced mathematical thinking. Proceedings of the Twenty-Third Annual Conference of the International Group for the Psychology of Mathematics Education, Israel, 1, 111-118.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. Educational Studies in Mathematics, 12 (2), 151-169.

Thomas, M. O. J., & Hong, Y. Y. (1996). The Riemann integral in calculus: Students' processes and concepts. Proceedings from the Nineteenth Conference of the mathematics Education Research Association of Australasia, Melbourne, Victoria, Australia, 572-579.

Thorpe, M. (1995a). Reflective learning in distance education. European Journal of Psychology of Education, 10 (2), 153-167.

Thorpe, M. (1995b). Bringing learner experience into distance education. Proceedings of the Seventeenth World Conference for Distance Education, 1, 364-367.

Tirosh, D. (1991). The role of students' intuitions in teaching the cantorian theory. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 199-214). Boston, MA: Kluwer Academic Publishers.

Trigueros, M., Ursini, S., & Reyes, A. (1996). College students' conception of variable. Proceedings of the Twentieth International Conference for the Psychology of Mathematics Education, Spain, 4, 315-322.

Tsamir, P., & Tirosh, D. (1992). Students' awareness of inconsistent ideas about actual infinity. Proceedings of the Sixteenth International Conference for the Psychology of Mathematics Education, USA, 3, 90-97.

Tucker, A. C., & Leitzel, J. R. C. (1995). Assessing Calculus Reform Efforts: A Report to the Community. Mathematical Association of America.

Uegama, W., Bullen, M., & Neufeld, R. (1992). The cultivation of critical thinking and independent learning in the undergraduate curriculum [Abstract]. Proceedings of the Sixteenth World Conference of the International Council for Distance Education, Nonthaburi, Thailand, 1992.

Ursini, S., & Trigueros, M. (1997). Understanding of different uses of variable: A study with starting college students. Proceedings of the Twenty-First International Conference for the Psychology of Mathematics Education, Finland, 4, 254-261.

Vidakovic, D. (1996). Learning the concept of inverse function. Journal of Computers in Mathematics and Science Teaching, 15 (3), 295-318.

Vidakovic, D., & Czarnocha, B. (1996). Mental constructions used in understanding the chain rule. Proceedings of the Eighteenth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, USA, 1, 58-62.

Villarreal M. E., & Borba, M. C. (1998). Conceptions and graphical interpretations about derivative. Proceedings of the Twenty-Second International Conference for the Psychology of Mathematics Education, South Africa, 4, 316.

Vinner, S. (1982). Conflicts between definitions and intuitions: The case of the tangent. Proceedings of the Sixth International Conference for the Psychology of Mathematics Education, Belgium, 24-28.

Vinner, S. (1983). Concept definition, concept image and the notion of function. International Journal of Mathematical Education in Science and Technology, 14, 239-305.

Vinner, S. (1992). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 65-81). Boston, MA: Kluwer Academic Publishers.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. Journal for Research in Mathematics Education, 20, 356-366.

von Glassersfeld, E. (1983). Learning as a constructivist activity. Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Conference for the Psychology of Mathematics Education, Canada, 41-63.

Watkins, S. A. (1988). Yes, Virginia .... In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (p. 55). Mathematical Association of America.

Watnick, R. (1993). Calculus teaching methods developed from studies in educational psychology. Problems, Resources and Issues in Mathematics Undergraduate Studies, 3 (2), 124-132.

Wells, C. (1999). Handbook of Mathematical Discourse [On-line]. Available: http://www.cwru.edu/artsci/math/wells/pub/abouthbk.htm

White, R. M. (1988). Calculus of reality. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 6-9). Mathematical Association of America.

Wilson J. M., & Albers, D. J. (1988). Calculus for physical sciences: First discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 59-60). Mathematical Association of America.

Winne, P. (1995). Inherent details in self-regulated learning. Educational Psychologist, 30 (4), 223-228.

Young, G. (1988). Present problems and future prospects. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 172-175). Mathematical Association of America.

Young, G., & Marks-Maran, D. (1998). Using constructivism to develop a quality framework for learner support: A case study. Open Learning, 13 (2), 30-37.

Young, P., & Blumenthal, M. (1988). Calculus for computing science students. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 71-74). Mathematical Association of America.

Zorn, P. (1988). Computing in undergraduate mathematics. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 233-239). Mathematical Association of America.

Zorn, P., & Viktora, S. S. (1988). Computer algebra systems: Second discussion session. In L. Steen (Ed.), Calculus for a New Century: A Pump, Not a Filter (pp. 79-81). Mathematical Association of America.

Appendix A: Student Survey

Please remove this page from the remainder of the survey and refer to the table as you complete Part 1 of the survey. For each item, indicate how **frequently** you accessed/used each feature, how **useful** each feature was to you, and how **important** each feature was to you by circling the appropriate number. Any information you give is confidential.

Frequency of Access/Use	**5** means **more than 6 times per week**	**4** means **5-6 times per week**	**3** means **3-4 times per week**	**2** means **1-2 times per week**	**1** means **less than once per week**
Usefulness to You	**5** means **extremely useful**	**4** means **very useful**	**3** means **fairly useful**	**2** means **somewhat useful**	**1** means **not at all useful**
Importance to You	**5** means **extremely important**	**4** means **very important**	**3** means **fairly important**	**2** means **somewhat important**	**1** means **not at all important**

Note: 1. The completion of this survey is entirely optional. It will take some time to complete and refusal to complete the questionnaire will in no way affect your evaluation in Mathematics 4225. However, the names of those students who complete the survey will be entered into a draw for two free movie passes.

2. To complete the survey you will require Internet access to the Mathematics 4255 website, and a copy of the text and student solution guide.

Name ____________________ School ____________________

Please answer the following questions carefully and accurately. The information you give will be used to improve Mathematics 4225 for future AP mathematics students. In the space below each item, please write any comments that you feel are important and that relate to that item. Also include any strengths or weaknesses of the feature in this space. **All information is confidential!!** Thank You.

Beginning of Part 1 of the Survey

1. Why did you enroll in Mathematics 4225?

2. Were you enrolled in any distance education course(s) before enrolling in Mathematics 4225? (Circle Y or N) Y N

3. Did you write the AP mathematics final examination in May 11^{th}, 2000? (Circle Y or N) Y N

4. If not, explain briefly below why you did not write this examination.

5. Did you write the Mathematics 4225 final examination on May 5^{th}, 2000? (Circle Y or N) Y N

6. If not, explain briefly below why you did not write this examination.

7. Did you withdraw from this course before its completion? (Circle Y or N) Y N

8. If YES, give the approximate date when you withdrew from the course.

____

9. If YES, give the reason(s) why you withdrew from the course.

10. While enrolled in the course, approximately how many hours per week did you spend studying Mathematics 4225? _______________

11. Approximately what percentage of the homework questions did you complete?

____

12. What post-secondary career did you plan to pursue before enrolling in Mathematics 4225?

13. Have you post-secondary career plans changed since enrolling in Mathematics 4225? (Circle Y or N) Y N

14. If so, explain why they have changed.

15. Will you continue to enroll in other mathematics courses in the post secondary institution you plan to attend? (Circle Y or N) Y N

16. What technologies did you use in the course? (You can circle more than one letter.)

(a) Computer
(b) Graphing Calculator
(c) Geometer's Sketchpad
(d) Derive
(e) Other(s) Please list it/them.

17. Was a computer with Internet access usually available to you at school during your offline classes? (Circle Y or N) Y N

18. Was a computer with Internet access usually available to you at home? (Circle Y or N) Y N

19. Please indicate (by circling the appropriate number in the table below) how **frequently** you used a computer with Internet access during offline classes, how **useful** this computer was to you during your offline classes, and how **important** it was to you to be able to use this computer during your offline classes. (Refer to the table on page 1.)

Access to Internet-Ready Computer During Offline Classes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

20. Please indicate (by circling the appropriate number in the table below) how **frequently** you used a home computer with Internet access for Mathematics 4225, how **useful** this computer was to you, and how **important** it was to you to be able to use a home computer for Mathematics 4225. (Refer to the table on page 1.)

Access to Internet-Ready Computer at Home

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

1. Features of the Mathematics 4225 Website:

Note: The following items refer to the features of the Mathematics 4255 website. You should be connected to this website when you complete this section of the survey. If you have forgotten the login information you can use *Student* for the user name and *education* for the password. Refer to the table on page 1 as you complete each item.

(A) The Online Calendar

I. Dates for Online Classes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

II. Dates for Offline Classes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

III. Dates for Examinations

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IV. Assignment Due Dates

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

V. Dates for Extra Classes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VI. Special Notices (e.g.: A notice that the instructor would be unavailable for a class or that dates in the Calendar had changed.)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VII. Calendar Compile Option

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(B) My Record

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(C) Lessons

I. Curriculum Outcomes (see the top right frame of each lesson)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

II. Text Reference Pages for Curriculum Outcomes (see the end of each curriculum outcome)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

III. Advance Organizer for Each Lesson (The sentence in the notes that begins with "In this lesson you will …")

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IV. Online Notes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

V. Links to Illustrations Within Some Lesson Notes (e.g.: Section 6.2, Lesson #1)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VI. Links to Other Resources Within Some Lesson Notes (e.g.: See Section 6.2, Lesson #1)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VII. Brief Summary at the End of the Lesson Notes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VIII. Homework Questions Assigned at the End of the Lesson Notes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IX. Reading Assignment at the End of the Lesson Notes

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

X. Online Multiple Choice Quiz at the End of Each Section

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XI. Online Solutions to the Multiple Choice Quizzes at the End of Each Section

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XII. Online Multiple Choice Test at the End of Each Topic

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XIII. Online Solutions to the Multiple Choice Test at the End of each Topic

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XIV. Online Long Answer Test at the End of Each Topic

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XV. Online Solutions to the Long Answer Test at the End of Each Topic

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(D) Concept Map

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(E) Course Outline

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(F) Outcomes Linked to Lessons

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(G) Course Evaluation

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(H) External Links

I. Ask Mr. Calculus

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

II. UCalc

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

III. The Calculus Toolkit

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IV. Some Real World Applications for Differential Equations

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

V. Online Integrator

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VI. Mathematical Models

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VII. Student Calculus Projects

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VIII. Calculus Labs

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IX. Academic Assistance

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

X. Calculus Tutorials

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XI. Mathematics Applets

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(I) Bulletin Board

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(J) Chat

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(K) Private E-mail

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

(L) Password

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

Write your overall opinion of the Mathematics 4255 website in the space below. Please identify aspects of the website that you felt were valuable and those that you felt needed improvement. Briefly support your views.

1. Features of the AP Mathematics Text Book

Note: The following items refer to the features and aspects of the textbook and student solution guide. You should refer to the text/solution guide as you complete this part of the survey. Refer also to the table on page 1 as you complete each item.

I. Basic Differentiation Rules Inside the Front Cover or on p. 155 or on p. 376

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

II. Basic Integration Rules Inside the Front Cover or on p. 263 or on p. 384

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

III. Formulas From Geometry Inside the Front Cover

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IV. Rules of Trigonometry Inside the Back Cover

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

V. Rules of Algebra Inside the Back Cover

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VI. Colour Computer Graphics

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VII. Technology Tips (e.g.: pp. 194 or 195)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

VIII. Historical Notes (e.g.: p. 297 or p. 283)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

IX. Summaries and Guidelines (e.g.: p. 166 or p. 226)

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

X. Remarks (e.g.: p. 155 or p. 296)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XI. Exercises Using the Graphing Calculator (e.g.: #21, p. 240)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XII. Challenging Homework Problems (e.g.: #54 p. 234)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XIII. A Variety of Homework Problems (e.g.: #'s 8, 90, 94, 103, 107-109, pp. 155-157)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XIV. Many Basic Computation Problems (e.g.: #'s 7-30 and #'s 41-74 pp. 155-156)

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XV. Appendix A: Proofs of Selected Theorems

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XVI. Appendix B: Basic Differentiation Rules for Elementary Functions

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XVII. Appendix C: Integration Tables

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XVIII. Appendix D: Real Numbers and the Real Line

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XIX. Answers to the Odd-Numbered Exercises

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XX. The Inclusion of a Prerequisites Chapter

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

XXI. Study and Solutions Guide for the Text

Frequency of Access/Use	5	4	3	2	1
Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

Do you feel that the textbook and student solution guide are appropriate for this course?
(Circle Y or N) Y N

Write your overall opinion of the AP mathematics textbook/student solution guide in the space below. Please identify aspects of the text/student solution guide that you felt were valuable and those that you felt needed improvement. Briefly support your views.

2. Features of the Online Classes

The following items refer to features and aspects of the online classes.

1. What do you think is the appropriate number of online classes per week in this course? (Give the number of classes and the number of minutes for each class.)

   ____

2. Approximately how many online classes did you miss this year?

   ____

3. What was the **main** reason why you did not attend these online classes?

   (a) I was absent from school.
   (b) Our site was having technical problems.
   (c) I was attending another function in the school.
   (d) I didn't need to go to those classes.
   (e) Other (Please explain in the space below)

4. What would you prefer that the instructor do during online classes? (i.e.: What should happen during an online class?) You can make more than one selection.

(a) Explain the big picture. (e.g. Explain why derivatives are worth studying.)
(b) Explain major concepts. (e.g. Derivatives)
(c) Show how the concept can be applied. (e.g. Explain how the derivative is used in optimization problems.)
(d) Use the concept in problem solving. (e.g. Solve optimization problems)
(e) Other (Please explain)

5. What did the instructor actually do during online classes?

(a) Explain the big picture
(b) Explain major concepts.
(c) Show how the concept can be applied.
(d) Use the concept in problem solving.
(e) Other (Please explain)

6. In the space below, describe what typically occurred in an online class. If you need extra space, please continue on the back of this sheet.

7. How important are online classes to you? (Circle one letter)

(a) Extremely Important
(b) Very Important
(c) Fairly Important
(d) Somewhat Important
(e) Not Important At All

8. How important are extra online classes to you? (Circle one letter)

(a) Extremely Important
(b) Very Important
(c) Fairly Important
(d) Somewhat Important
(e) Not Important At All

9. How important are offline classes to you? (Circle one letter)

(a) Extremely Important
(b) Very Important
(c) Fairly Important
(d) Somewhat Important
(e) Not Important At All

For items 10-14, please indicate the usefulness, and importance, of each feature of the online classes. Refer to the table on page 1 as you complete each item.

10. Audio

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

11. Whiteboard

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

12. Chat

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

13. Video

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

14. Ability to Save Whiteboard Files

Usefulness	5	4	3	2	1
Importance	5	4	3	2	1

Please write any comments that you feel are important in the space below.

Write your overall opinion of the online classes in the space below. Please identify aspects of the online classes that you felt were valuable and those that you felt needed improvement. Briefly support your views.

4. Other Features of the Course

Note: The following items refer to student evaluation, collaboration, and social interaction.

(A) Evaluation

1. Please indicate the percentage weights that you feel should be assigned to each major evaluation instrument. Your percentages must add to 100.

(a) Topic Tests __________
(b) Topic Assignments __________
(c) Midterm __________
(d) Mathematics 4225 Final __________
(e) Special Project __________

2. Please list any other methods of evaluation that you feel should be used in Mathematics 4225. Give their percentage weights.

3. Do you feel that the **number** of topic examinations was appropriate for this course? (Circle Y or N) Y N

4. If NO, please indicate the appropriate number of examinations. __________

5. Do you feel that the **length** of the topic examinations was appropriate in this course? (Circle Y or N) Y N

6. If NO, please indicate if the topic examinations should increase or decrease in length. __________

7. Do you feel that the **difficulty** of topic examinations was appropriate for this course? (Circle Y or N) Y N

8. If NO, please indicate if you consider the topic examinations too difficult or too easy. __________

9. Do you feel that the **number** of assignments was appropriate for this course? (Circle Y or N) Y N

10. If NO, please indicate the appropriate number of assignments. __________

11. Do you feel that the **length** of the assignments was appropriate in this course? (Circle Y or N) Y N

12. If NO, please indicate if the assignments should increase or decrease in length.

    ____

13. Do you feel that the **difficulty** of assignments was appropriate for this course? (Circle Y or N) Y N

14. If NO, please indicate if you considered the assignments too difficult or too easy.

    ____

Write your overall opinion of the course evaluation in the space below. Please identify aspects of the course evaluation that you felt were valuable and those that you felt needed improvement. Briefly support your views.

(B) Collaboration (Working, studying, and learning with others)

1. Please indicate how many times you collaborated with another student at your site this year. __________

2. How important was this type of collaboration?
(a) Extremely Important
(b) Very Important
(c) Fairly Important
(d) Somewhat Important
(e) Not Important At All

3. Please indicate how many times you collaborated with another student at another site this year. __________

4. How important was this type of collaboration?
   (a) Extremely Important
   (b) Very Important
   (c) Fairly Important
   (d) Somewhat Important
   (e) Not Important At All

5. Did you form a study group with other members in the course? (Circle Y or N) Y N

6. If no, do you think you would have benefited from a study group? (Circle Y or N) Y N

7. Did you receive help from anyone other than the instructor or another student in the course? (Circle Y or N) Y N

Write your overall opinion of student-student and teacher-student collaboration in the space below. Please identify aspects of collaboration that you felt were valuable and those that you felt needed improvement. Briefly support your views.

(C) Social Interaction of Students

1. Do you feel that the gathering of all AP students in Clarenville in June 1999 was worthwhile and important? Please explain.

2. Do you feel that the gathering of all AP students in Clarenville in September 1999 was worthwhile and important? Please explain.

3. How many times during the year do you feel the AP students should have such a gathering? ____

Write your overall opinion of the student-student and student-teacher social interaction in the space below. Please identify aspects of the interaction that you felt were valuable and those that you felt needed improvement. Briefly support your views.

End of Part 1 of the Survey

Please remove this page from the remainder of the survey and refer to it as you complete items 1-79 of Part 2 of the survey.

For the statement in each table in Part 2, please indicate if you **(4) strongly agree, (3) agree, (2) disagree** or **(1) strongly disagree** by circling the appropriate number in the table.

Beginning of Part 2 of the Survey

1. There was a lack of mathematical modeling in the course. (e.g.: See question 34, p. 249. The equation $C = \frac{80{,}000p}{100-p}$, $0 \le p \le 100$ is an example of a mathematical model.)			
4	3	2	1

2. I would have preferred to work with models I developed myself rather than models already given in the text.			
4	3	2	1

3. The class size was too large.			
4	3	2	1

4. The class size was too small.			
4	3	2	1

5. There was too much material to cover in Mathematics 4225.			
4	3	2	1

6. There were too many word problems in the course.			
4	3	2	1

7. There were too few word problems in the course.			
4	3	2	1

8. There were too many template word problems, which followed the examples in the book and required only straightforward calculations, instead of nonstandard word problems, which required a lot of thought and discussion to solve.			
4	3	2	1

9. I was not required to justify my answers enough in Mathematics 4225.			
4	3	2	1

10. I view calculus as a collection of rules and procedures, which must be memorized in order to pass the course.			
4	3	2	1

11. Mathematics 4225 required a large amount of symbolic manipulation. (i.e. There were many problems where I just had to match the question to a basic rule of limits, differentiation or integration and work out the answer.)			
4	3	2	1

12. You could do well in this course by being good at symbolic manipulation (i.e.: being able to manipulate symbols and expressions without understanding) instead of having a good understanding of the material in the course.			
4	3	2	1

13. I expected that I would have to memorize a lot of rules and procedures when I first entered Mathematics 4225.			
4	3	2	1

14. I felt the teacher and I should have worked together more in the course.			
4	3	2	1

15. I had too little interaction with the teacher.			
4	3	2	1

16. I had too many interactions with the teacher.			
4	3	2	1

17. There was not enough writing in Mathematics 4225.			
4	3	2	1

18. There was too much writing in Mathematics 4225.			
4	3	2	1

19. The word problems in Mathematics 4225 were relevant and realistic.			
4	3	2	1

20. Emphasis was placed on getting the correct answer rather than knowing what the answer meant.			
4	3	2	1

21. Emphasis was placed on determining the reasonableness of the answer obtained.			
4	3	2	1

22. Emphasis was placed on getting the correct answer the first time I attempted a question.			
4	3	2	1

23. I would have preferred that more emphasis be placed on how the topics covered can be applied in my chosen career.			
4	3	2	1

24. I would have preferred being exposed to a wide range of applications of the topics covered.			
4	3	2	1

25. I received enough good feedback from the instructor on my homework.			
4	3	2	1

26. I received prompt feedback from the instructor on my homework.			
4	3	2	1

27. I received enough good feedback from the instructor on my assignments.			
4	3	2	1

28. I received prompt feedback from the instructor on my assignments.			
4	3	2	1

29. I received enough good feedback from the instructor on my tests.			
4	3	2	1

30. I received prompt feedback from the instructor on my tests.			
4	3	2	1

31. I would have preferred more face-to-face meetings with the teacher.			
4	3	2	1

32. There was not enough time to cover all the topics in the course so I understood them well.			
4	3	2	1

33. The pace of the course was too fast.			
4	3	2	1

34. The pace of the course was too slow.			
4	3	2	1

35. The standards and expectations of the course were too high.			
4	3	2	1

36. The standards and expectations of the course were too low.			
4	3	2	1

37. The standards and expectations of the course were higher than those of the tests.			
4	3	2	1

38. The standards and expectations of the course were lower than those of the tests.			
4	3	2	1

39. The standards and expectations of the course were higher than those of the assignments.			
4	3	2	1

40. The standards and expectations of the course were lower than those of the assignments.			
4	3	2	1

41. The standards and expectations of the course were higher than those of the final examination written on May 5.			
4	3	2	1

42. The standards and expectations of the course were lower than those of the final examination written on May 5.			
4	3	2	1

43. The standards and expectations of the course were higher than those of the AP Calculus AB examination written on May 11.			
4	3	2	1

44. The standards and expectations of the course were lower than those of the AP Calculus AB examination written on May 11.			
4	3	2	1

45. There were too many proofs in the course.			
4	3	2	1

46. There were too few proofs in the course.			
4	3	2	1

47. The course placed too little emphasis on numerical methods (e.g.: Finding the answer using the Trapezoid Rule).			
4	3	2	1

48. I would have liked to have had more opportunities to give presentations to the class.			
4	3	2	1

49. I felt that the course was not realistic (i.e.: It was not connected to things in the real world).			
4	3	2	1

50. The graphing calculator helped me understand the concepts of the course.			
4	3	2	1

51. Geometer's Sketchpad helped me understand the concepts of the course.			
4	3	2	1

52. Derive helped me understand the concepts of the course.			
4	3	2	1

53. Calculus was linked to other disciplines such as biology, physics and chemistry.			
4	3	2	1

54. In this course we used technology to perform all the symbolic manipulation and focused instead on the understanding and application of concepts.			
4	3	2	1

55. This web-based course suited the way I learn.			
4	3	2	1

56. Taking Mathematics 4225 helped me become a better learner.			
4	3	2	1

57. I needed to be an independent learner to be successful in this course.			
4	3	2	1

58. I needed to take responsibility for my own learning in order to be successful in Mathematics 4225.			
4	3	2	1

59. I felt isolated in this course.			
4	3	2	1

60. I had enough support from the teacher in this course.			
4	3	2	1

61. The material I had to learn was connected to my previous experience and knowledge.			
4	3	2	1

62. The material was discussed until everyone understood it.			
4	3	2	1

63. I feel that participation in other distance education courses helped me determine what would be required of me in this course.			
4	3	2	1

64. I was initially apprehensive about taking this course because I knew I would often have to use a computer.			
4	3	2	1

65. I did the work in this course because I was interested in it.			
4	3	2	1

66. I did the work in this course because it had to be done in order to go on in the course.			
4	3	2	1

67. I tried to figure out what the work meant as I did it.			
4	3	2	1

68. I tried to figure out how the work I was doing fit into the overall scheme of things.			
4	3	2	1

69. I could relate the things I was learning to the things I already knew and to my previous experiences.			
4	3	2	1

70. I relied on memorization in this course.			
4	3	2	1

71. I worried about the time it would take to do the work in this course.			
4	3	2	1

72. I would prefer to write AP Calculus examinations like those given in 1985 and 1988, where only a scientific calculator was permitted but there were less application problems, as opposed to the 2000 Calculus AB examination where the use of a graphing calculator was permitted and there were more application problems.			
4	3	2	1

73. I was often working 2 or 3 sections behind the instructor.			
4	3	2	1

74. This course has improved my ability to think.			
4	3	2	1

75. I found it difficult to apply the things I learned to application problems.			
4	3	2	1

76. I found the mathematical language used in the course confusing.			
4	3	2	1

77. I believe that having someone at my site to compete against helps me work harder.			
4	3	2	1

78. The topics chosen for the project were relevant and interesting.			
4	3	2	1

79. If I could, I would participate in another distance education course like Mathematics 4225.			
4	3	2	1

80. Would you have liked to have spent more time on any particular section(s) of the text? (Circle Y or N)
Y N

81. If yes, please list the section(s).

82. What topics did you find confusing? (List them in the space below.)

83. What should this course emphasize?

(a) Manipulation and Procedure
(b) Problem Solving
(c) Theory and Proof
(d) Other Area(s) Please list it/them

84. Would you have liked more face-to-face meetings with the instructor? (Circle Y or N)
Y N

85. If yes, please indicate what you would have liked to have done in these face-to-face meetings.

86. In the space below list the ways in which this course can be changed to improve your learning?

87. Write your overall opinion of Mathematics 4225 in the space below. Please identify aspects of the course that you felt were valuable and those that you felt needed improvement. Briefly support your views.

End Part 2 of the Survey

Appendix B: Student Information Sheet/Registration Form

Mathematics 4225

Student Information Sheet/Registration Form

Student's Name ______________________________

Student's School ______________________________

Grade Level in Sept. ______________________________

Mailing Address ______________________________

City/Town ______________________________

Postal Code ______________________________

Home Phone ______________________________

E-mail Address ______________________________

Do you have access to a computer at home? (Yes/No) __________

Do you have Internet access at home? (Yes/No) __________

Describe your present interest in using computers.

____

____

Describe your present skill level with computers, including types of software used.

____

____

List all mathematics courses completed in Level I-III (include approximate mark achieved).

____

____

Appendix C: Sample Consent Forms

DIRECTOR CONSENT FORM

Dear Director:

I am a graduate student in the Faculty of Education at Memorial University of Newfoundland. I will be surveying and interviewing students at ______________________ to investigate the factors that affect a student's learning of mathematics while that student is enrolled in an advanced placement calculus course which is being taught at a distance using the Internet. This research is being supervised by Dr. Ken Stevens, Chair, Centre for TeleLearning and Rural Education, Memorial University of Newfoundland and by Dr. David Reid, professor of mathematics education, Acadia University. I am requesting your permission for students in your school district to take part in this study.

The student's participation will consist of completing a survey which asks them to indicate their frequency of use, the usefulness, and the importance of the features of the:

(i) Mathematics 4255 website.
(ii) course textbook
(iii) online classes
(iv) course evaluation
(v) collaboration in the course
(vi) interaction of students in the course

The student will also be asked to indicate their degree of agreement with statements about a number of factors which research literature indicates are important in the learning of calculus and in learning at a distance. After the surveys have been analyzed, it may become necessary to ask students to participate in a group interview which will expand and clarify only their responses in the survey. At the start of both the survey and the interview, it will be made very clear that the student can withdraw from the survey or interview process at any time without prejudice and/or refrain from answering whatever questions he or she prefers to omit. The survey and interview should take approximately one hour each. The names of students who participate in the survey and interview process will be entered into a draw for two free movie passes.

All information gathered in this study is strictly confidential and at no time will individuals be identified. I am interested in how well students feel they learned calculus in a web-based format and how this learning can be improved. I am not interested in any individual student's performance. Participation is voluntary and you may withdraw your permission at any time. This study has received the approval of the Faculty of Education's Ethics Review Committee. The results of my research will be made available to you upon request.

The completed surveys, along with any audio or video recordings, or written notes made during the interview, will be held in a secure location and destroyed upon completion of the study.

If you are in agreement with having students in your school district participate in this study, please sign, date, and return page 3 of this letter. Pages 1 and 2 are for your records. If you have any questions or concerns please do not hesitate to contact me at 709 467-2717 (W) or 709 467-4355 (R). If at any time you wish to speak with a resource person not associated with the study, please contact Dr. Clar Doyle, Dean (acting), Graduate Programs and Research, Faculty of Education, Memorial University of Newfoundland (Tel: 709-737-8588).

I would appreciate it if you would please return this sheet to me by

__________________________.

Yours sincerely,

I ____________________________(Director) hereby give permission for students in this school district to take part in a study on the factors that affect the learning of calculus in a web-based environment being undertaken by Dean C. Holloway. I understand that participation is voluntary and that my permission can be withdrawn at any time. All information is strictly confidential and no individual will be identified.

______________________ ____

Date Director's Signature

PRINCIPAL CONSENT FORM

Dear Principal:

I am a graduate student in the Faculty of Education at Memorial University of Newfoundland. I will be surveying and interviewing students at ________________________ to investigate the factors that affect a student's learning of mathematics while that student is enrolled in an advanced placement calculus course which is being taught at a distance using the Internet. This research is being supervised by Dr. Ken Stevens, Chair, Centre for TeleLearning and Rural Education, Memorial University of Newfoundland and by Dr. David Reid, professor of mathematics education, Acadia University. I am requesting your permission for students in your school to take part in this study.

The student's participation will consist of completing a survey which asks them to indicate their frequency of use, the usefulness, and the importance of the features of the:

- (i) Mathematics 4255 website.
- (ii) course textbook
- (iii) online classes
- (iv) course evaluation
- (v) collaboration in the course
- (vi) interaction of students in the course

Students will also be asked to indicate their degree of agreement with statements about a number of factors which research literature indicates are important in the learning of calculus and in learning at a distance. After the surveys have been analyzed, it may become necessary to ask students to participate in a group interview which will expand and clarify only their responses in the survey. At the start of both the survey and the interview, it will be made very clear that students can withdraw from the survey or interview process at any time without prejudice and/or refrain from answering whatever questions he or she prefers to omit. The survey and interview should take approximately one hour each. The names of students who participate in the survey and interview process will be entered into a draw for two free movie passes.

All information gathered in this study is strictly confidential and at no time will individuals be identified. I am interested in how well students feel they learned calculus in a web-based format and how this learning can be improved. I am not interested in any individual child's performance. Participation is voluntary and you may withdraw your permission at any time. This study has received the approval of the Faculty of Education's Ethics Review Committee. The results of my research will be made available to you upon request.

The completed surveys, along with any audio or video recordings, or written notes made during the interview, will be held in a secure location and destroyed upon completion of the study.

If you are in agreement with having your students participate in this study, please sign, date, and return page 3 of this letter. Pages 1 and 2 are for your records. If you have any questions or concerns please do not hesitate to contact me at 709 467-2717 (W) or 709 467-4355 (R). If at any time you wish to speak with a resource person not associated with the study, please contact Dr. Clar Doyle, Dean (acting), Graduate Programs and Research, Faculty of Education, Memorial University of Newfoundland (Tel: 709-737-8588).

I would appreciate it if you would please return this sheet to me by

__________________________.

Yours sincerely,

I ___________________________(principal) hereby give permission for students at this school to take part in a study on the factors that affect the learning of calculus in a web-based environment being undertaken by Dean C. Holloway. I understand that participation is voluntary and that my permission can be withdrawn at any time. All information is strictly confidential and no individual will be identified.

______________________ ____

Date Principal's Signature

PARENTAL CONSENT FORM

Dear Parent or Guardian:

I am a graduate student in the Faculty of Education at Memorial University of Newfoundland. I will be surveying and interviewing students at ________________________ to investigate the factors that affect a student's learning of mathematics while that student is enrolled in an advanced placement calculus course which is being taught at a distance using the Internet. This research is being supervised by Dr. Ken Stevens, Chair, Centre for TeleLearning and Rural Education, Memorial University of Newfoundland and by Dr. David Reid, professor of mathematics education, Acadia University. I am requesting your permission for your child to take part in this study.

Your child's participation will consist of completing a survey which asks them to indicate their frequency of use, the usefulness, and the importance of the features of the:

(i) Mathematics 4255 website
(ii) course textbook
(iii) online classes
(iv) course evaluation
(v) collaboration in the course
(vi) interaction of students in the course

Your child will also be asked to indicate their degree of agreement with statements about a number of factors which research literature indicates are important in the learning of calculus and in learning at a distance. After the surveys have been analyzed, it may become necessary to ask your child to participate in a group interview which will expand and clarify only their responses in the survey. At the start of both the survey and the interview, it will be made very clear that your child can withdraw from the survey or interview process at any time without prejudice and/or refrain from answering whatever questions he or she prefers to omit. The survey and interview should take approximately one hour each. The names of students who participate in the survey and interview process will be entered into a draw for two free movie passes.

All information gathered in this study is strictly confidential and at no time will individuals be identified. I am interested in how well students feel they learned calculus in a web-based format and how this learning can be improved. I am not interested in any individual child's performance. Participation is voluntary and you may withdraw your child at any time. This study has received the approval of the Faculty of Education's Ethics Review Committee. The results of my research will be made available to you upon request.

The completed surveys, along with any audio or video recordings, or written notes made during the interview, will be held in a secure location and destroyed upon completion of the study.

If you are in agreement with having your child participate in this study, please sign, date, and return page 3 of this letter. Pages 1 and 2 are for your records. If you have any questions or concerns please do not hesitate to contact me at 709 467-2717 (W) or 709 467-4355 (R). If at any time you wish to speak with a resource person not associated with the study, please contact Dr. Clar Doyle, Dean (acting), Graduate Programs and Research, Faculty of Education, Memorial University of Newfoundland (Tel: 709-737-8588).

I would appreciate it if you would please return this sheet to me by

________________________.

Yours sincerely,

I ___________________________(parent/guardian) hereby give permission for my son/daughter _____________________(student name) to take part in a study on the factors that affect the learning of calculus in a web-based environment being undertaken by Dean C. Holloway. I understand that permission to conduct the study has been received from the director of the school district and the principal of the school which my son/daughter attends. I understand that participation is voluntary and that my child and/or I can withdraw permission at any time. All information is strictly confidential and no individual will be identified.

______________________ ____

Date Parents/Guardian's Signature

STUDENT CONSENT FORM

Dear Student:

I am a graduate student in the Faculty of Education at Memorial University of Newfoundland. I will be surveying and interviewing students at ______________________ to investigate the factors that affect a student's learning of mathematics while that student is enrolled in an advanced placement calculus course which is being taught at a distance using the Internet. This research is being supervised by Dr. Ken Stevens, Chair, Centre for TeleLearning and Rural Education, Memorial University of Newfoundland and by Dr. David Reid, professor of mathematics education, Acadia University. I am requesting that you agree to take part in this study.

Your participation will consist of completing a survey which asks you to indicate your frequency of use, the usefulness, and the importance of the features of the:

(i) Mathematics 4255 website.
(ii) course textbook
(iii) online classes
(iv) course evaluation
(v) collaboration in the course
(vi) interaction of students in the course

You will also be asked to indicate your degree of agreement with statements about a number of factors which research literature indicates are important in the learning of calculus and in learning at a distance. After the surveys have been analyzed, it may be necessary to ask you to participate in a group interview which will expand and clarify only your responses in the survey. At the start of both the survey and the interview, it will be made very clear that you can withdraw from the survey or interview process at any time without prejudice and/or refrain from answering whatever questions you prefer to omit. The survey and interview should take approximately one hour each. The names of students who participate in the survey and interview process will be entered into a draw for two free movie passes.

All information gathered in this study is strictly confidential and at no time will individuals be identified. I am interested in how well students feel they learned calculus in a web-based format and how this learning can be improved. I am not interested in any individual student's performance. Participation is voluntary and you may withdraw at any time. This study has received the approval of the Faculty of Education's Ethics Review Committee. The results of my research will be made available to you upon request.

The completed surveys, along with any audio or video recordings, or written notes made during the interview, will be held in a secure location and destroyed upon completion of the study.

If you agree to participate in this study, please sign, date, and return page 3 of this letter. Pages 1 and 2 are for you. If you have any questions or concerns please do not hesitate to contact me at 709 467-2717 (W) or 709 467-4355 (R). If at any time you wish to speak with a resource person not associated with the study, please contact Dr. Clar Doyle, Dean (acting), Graduate Programs and Research, Faculty of Education, Memorial University of Newfoundland (Tel: 709-737-8588).

I would appreciate it if you would please return this sheet to me by

__________________________.

Yours sincerely,

I ____________________________(student) hereby agree to take part in a study on the factors that affect the learning of calculus in a web-based environment being undertaken by Dean C. Holloway. I understand that permission to conduct the study has been received from the director of the school district and the principal of the school which I attend. I understand that participation is voluntary and that I can withdraw at any time. All information is strictly confidential and no individual will be identified.

______________________ ____

Date Student's Signature