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“A SEMIPARAMETRIC IV ESTIMATION OF THE
GOVERNMENT DEBT/GDP-GROWTH RELATIONSHIP
FOR OECD COUNTRIES”

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1 Introduction

In the aftermath of the great recession (2008), rising debt-to-GDP levels across OECD countries triggered a growing interest from researchers to explore the effect that high government indebtedness potentially has on economic growth. In their AER proceeding paper, C. Reinhart and Rogoff (2010b) report that advanced countries with debt-to-GDP levels exceeding 90% are associated with an average and median growth rate reduction of 2.5 and 1 percentage points respectively. RR explore the debt/GDP-growth relationship using descriptive statistics and the categorization of the debt-to-GDP distribution in four categories: 0-30, 30-60, 60-90 and 90+%. In an attempt to replicate RR's study, Herndon, Ash, and Pollin (2013) find that the 90+% category is associated with an average growth rate of 2.2%, instead of the -0.1% level previously proposed by RR. In opposition to the sharp decline in growth rates, this rather suggests a linear and negative effect of high debt-to-GDP levels on growth. The authors justify the result difference with the identification of methodological errors in RR's work. Even though RR's methodological approach is restrictive by design and potentially error-ridden, their work have had a significant impact on the research field that specifically focussed on the analysis of the debt/GDP-growth relationship in advanced countries.

The theoretical and empirical bodies of literature, related to this field, show that a valid estimation of the relationship between the debt-to-GDP and growth variables requires an empirical methodology that takes into account three important characteristics: potential cross-country heterogeneity, statistical endogeneity and non-linearity in the functional form.

Among those, cross-country heterogeneity is perhaps the most important and challenging characteristic to deal with when empirically assessing the debt/GDP-growth relationship. Tolerance to government indebtedness varies strongly across advanced economies (C. Reinhart and Rogoff (2003)). For example, markets' reactions to a rise of the debt-to-GDP levels in the United-States is expected to be drastically different compared to a similar fiscal movement in countries forming the original PIIGS (Portugal, Italy, Ireland, Greece and Spain) group. Moreover, De Grauwe (2012) finds evidence that being part of a monetary union changes the markets' sentiments towards the ability of a country to repay its sovereign debt.

Even inside the European Monetary Union, Codogno, Favero, and Missale (2003) identify that the risk of default is an important components of yield differentials across different government bonds. Emphasizing on cross-country heterogeneity is also the principal conclusion of Panizza and Presbitero (2013)'s extensive survey of the theoretical and empirical debt/GDP-growth literature. Therefore, in order to account for the presence of strong cross-country heterogeneity throughout this empirical study, we include country specific effects in our model specification and analyze the relationship for sub-groups of countries that share more homogenous traits.

A second characteristic identified relevant to the estimation of this relationship is the presence of statistical endogeneity in the standard bi-variate and contemporaneous regression model, potentially leading to biased estimates. As detailed in C. Reinhart and Rogoff (2011) and C. Reinhart, V. Reinhart, and Rogoff (2012), the likely source of endogeneity in the debt/GDP-growth link mainly consists of problems related to reverse causality and simultaneity statistical issues. The impact of automatic fiscal stabilizers or countercyclical policies implies that fiscal policy can also react to cyclical movements of GDP growth rates. This potential fiscal response to production cyclical movements supports the presence of a reverse causality pattern in the relationship.

Moreover, DeLong, Summers, and Feldstein (2012) link positive economic outcomes to such fiscal reactions in the context of low interest rates. Consequently, the link between debt-to-GDP levels and growth could indeed be bi-directional, what would mitigate the identification of the causality of interest. Additionally, a simultaneous effect on the debt-to-GDP and growth variables can arise in consequence of an external shock, such as a banking crisis, leading to irrelevant correlation for the identification of the desired causal link. In order to deal with the problems related to statistical endogeneity and to reduce the estimation bias, the empirical literature mostly employed the instrumental variable approach, as well as a predetermined growth specification, a methodological approach we replicate in the context of a nonparametric specification.

Thirdly, given the strong support for the presence of non-linearities in the functional form of the debt/GDP-growth relationship by important contributors to the concerned theoretical and empirical literature, such as Ball and Mankiw

(1995), Minea and Parent (2012), and Baum, Checherita-Westphal, and Rother (2013), we think the estimation method should allow for a flexible specification. In fact, 50% of the reviewed empirical body of literature post-C. Reinhart and Rogoff (2010b) finds evidence for a non-linear functional form, using threshold regression frameworks for the most part. Theoretically, the non-linear aspects of debt-growth relationship are supported by the views of Ball and Mankiw (1995) and Krugman (1988). The authors propose that if creditors lose confidence in the ability of a government to pay back its debt through future revenues, the markets gradually perceive this signal, such that an investment “crowding-out” dynamic gets triggered and negative economic outcomes potentially follow. Although Panizza and Presbitero (2013) don’t find quantitative evidence for this investment “crowding-out” theoretical channel, in our opinion, empirical investigations of the debt-growth relationship should employ estimation methods with the ability to depict some non-linearities. So far, the empirical literature have mainly applied IV, regime-switching (threshold regression) or GMM estimators, leaving important research opportunities for the application of more flexible estimation strategies.

In this study, we investigate the functional form of the government debt/GDP-growth relationship for OECD countries using a semiparametric IV estimator. In our opinion, this estimation method can account for the three characteristics mentioned above, namely the presence of cross-country heterogeneity, statistical endogeneity and non-linearities in the functional form. Using this method, we investigate the functional form of the relationship through a growth model specification based on the empirical work of Cecchetti, Mohanty, and Zampolli (2011). We estimate the relationship for different sub-groups of OECD countries over the 1971-2007/2010 period: 1) 22 OECD countries, 2) 15 EU countries, 3) the PIIGS group, 4) OECD countries excluding the PIIGS group and 5) EU countries excluding the PIIGS group. We find strong heterogeneity in the estimated functional forms across the different sub-groups. Complex non-linearities best describe the relationship for both sub-groups of 22 OECD countries and 15 EU countries, while a clear inverted U-shaped functional form is depicted for the PIIGS sub-group. When the PIIGS countries are excluded from the OECD and EU country samples, the non-linearities in the estimated functional form are almost absent, such that

the debt/GDP-growth relationship is best described as slightly U-shaped or linear and positive. Moreover, the functional form of the PIIGS sub-group accounts for the most part of the non-linear patterns of the estimated relationship for the OECD and EU samples. Based on our estimation results, we find that describing the debt/GDP-growth relationship for OECD countries is more complex than comparing the debt levels to a unique threshold based on pooled analyses. Relating to Ghosh et al. (2013), highly indebted countries with low fiscal space, i.e., the distance between the actual debt level and the modelled debt limit, tend to experience rapidly decreasing growth rates. This results does not hold for countries with larger fiscal space, which according to our estimates, could still experience positive debt/GDP-growth links at high debt-to-GDP levels. In order to assess the suitability of the semiparametric IV estimator for the analysis of this specific relationship, we adapt and apply two specification tests that evaluate the necessity of the IV and nonparametric approaches separately. The test results suggest the use of both aspects and therefore support the choice of the semiparametric IV estimator.

The paper is structured in the following way: section 2 presents the theoretical background that underlies the debt/GDP-growth relationship; section 3 details the empirical review; section 4 briefly describes our dataset and variable selection; section 5 presents our models and estimation methods; section 6 describes two specification testing procedures; section 7 reports the estimation and test results; section 8 closes with a short discussion on the implication of our results for economic policies.

2 Theoretical background

In this section, we outline the principal channels identified by the theoretical literature in support of a potential link between government debt and economic growth. First, we briefly analyze the channels underlying potential economic outcomes from fiscal policy and secondly, we present the theoretical views of the recent literature in support, or not, of a relationship between the level of government debt-to-GDP and economic growth. We distinguish the effect of the fiscal policy and government debt-to-GDP variables on growth in order to isolate the channels specific

to the debt/GDP-growth relationship, our principal research inquiry. Using this theoretical framework, we try to understand the prerequisites for a valid statistical investigation of the relationship and identify the weaknesses or strength of the empirical literature accordingly.

2.1 Government debt

Diamond (1965) analyzes the effect of government debt on the long-run competitive equilibrium, in the context of a neoclassical growth model framework with overlapping-generations and infinite horizon time specification. Before introducing government debt in the model, the author compares the solutions from the centrally planned and competitive economies. He finds that a competitive solution can be efficient if the interest rate and the population growth rate are equal. But given that the interest rate also depends on additional parameters that vary across economies, he finds that an inefficient equilibrium is also possible. Therefore, if the competitive equilibrium of the model is inefficient, such that the capital-labor ratio is different from the one prescribed by the golden rule, a change in the level of relative capital towards the efficient ratio can increase the level of economic welfare. The author then introduces government debt in the form of a fiscal stimulus that finances public expenditures with no permanent effect on future generations, which are later paid by taxes on the younger cohorts. He finds that internal or external debt reduces the savings of the younger generations, leading to a lower aggregate level of capital and a greater pressure on the interest rate. The resulting outcome of a government debt issuance on the welfare of the economy differs according to the state of equilibrium. If the competitive equilibrium of the economy is efficient, government debt reduces the welfare by moving the capital-labor ratio away from the golden rule state. However, if there is capital over-accumulation in the economy, with an interest rate lower than the population growth rate, government debt lowers the capital-labor ratio and increases the interest rate. Consequently, this converges the economy towards the golden rule level and increases economic welfare. However, a fiscal stimulus can also move an equilibrium of capital under-accumulation further away from the golden rule, leading to an even lower level of welfare. Blanchard (1985) aggregates Diamond (1965)'s framework with finite

horizon and continuous time specification. In this model, agents have a constant probability of death throughout their life, declining labor income as they get older, different ages and varying levels of wealth, but face the same horizon and propensity to consume. Using this framework, the author finds the possibility for dynamically inefficient growth paths in the economy without fiscal policy, supported by a decreasing level of capital accumulation and declining labor income. When fiscal policy is introduced in the model, in the form of a government debt issuance and proportional taxes on agents, the level of capital and foreign assets accumulation are reduced, what consequently moves the equilibrium to lower consumption and capital levels. Therefore, if the equilibrium of the economy without fiscal policy is in a state of capital over-accumulation, the government debt issuance moves the equilibrium towards the efficient path and increases economic welfare. In an extension of Blanchard (1985)'s model, Saint-Paul (1992) shows that the conclusion of a possible increase in the welfare through government spending, found in the two previous papers, does not hold in an endogenous growth model framework. The author introduces an externality, implying constant returns to capital at the aggregate level. By doing so, the model yields different social and private rates of return. He then demonstrates that the social rate of return is unconditionally higher than the population growth rate, such that any equilibrium growth path is necessarily dynamically production-efficient. In this context, public debt issuance always leads the economy to production-inefficient growth paths.

Aside from the formal model frameworks of Diamond (1965), Blanchard (1985) and Saint-Paul (1992), Ball and Mankiw (1995) describe what they perceive as the potential channels that underly and justify the effects of fiscal policy on economic growth. Using national accounting identities derived from $(Y = C + I + G + NX)$, they show that the loss of national savings, following government budget deficits, is linked with a proportionate reduction in investments and net exports. They identify the channels underlying these effects as the changes brought by budget deficits on the interest and exchange rates. The authors argue that, when the government issues debt, it competes with firms and households on the funds market. The higher demand for loanable funds raises the equilibrium domestic interest rate and consequently have a negative impact on the investment level in the economy. Additionally, a higher interest rate attracts a greater demand for domestic

assets by foreign investors, what potentially increases the pressure on the domestic currency exchange rate and leads to a fall in net exports. On a long term perspective, the authors identify that sustained budget deficits and debt accumulation can result in negative outcomes on the economy's output and wealth. When experienced on an extended period of time, decreasing national savings and investments reduce the capital accumulation rate, resulting in smaller production capacities and a lower economic growth rate. Also, the increasing proportion of domestic assets held by foreign investors reduces the wealth of residents, given that a greater portion of the income from domestic production goes abroad in the form of interest, rent and profit. The authors propose two strategies in order for a government to cope with sustained deficits, defined as lasting a decade or more. When the debt comes due, the government may raise taxes and cut transfers in order to pay for the interests and debt capital. On this matter, Elmendorf and Mankiw (1999) advance that if Ricardian equivalences don't hold, the short run positive stimulus of an increased government spending would not be compensated in the future by private savings and thus, the strategy of future tax payments on households could still produce a shortage of national savings in the long run. However, in the context of low interest rates and a depressed economic state, DeLong, Summers, and Feldstein (2012) argue that fiscal policy could produce positive outcomes on economic growth and that in this case, the additional government debt is likely to be self financing, overriding the pattern of low national savings in the long run. Ball and Mankiw (1995) propose a second approach for the government to deal with sustained deficits, which consists of rolling over its debt, such that the actual interest costs are paid by the issuance of additional debt. As long as the production growth rate is higher than the interest rate, this strategy can be sustainable. However, history provides several examples of sudden variations in interest rates and production levels, such that rolling the debt over is somewhat of a gamble for the government. If played right, the government can avoid the economic disagreements linked with taxes and spending cuts, but if it does not, rising debt-to-GDP levels may affect the economy through other channels than previously covered for the fiscal policy. The next subsection reviews the theoretical works that specifically explored how the debt-to-GDP levels of a country could affect its economic growth rate.

2.2 Government debt-to-GDP

In addition to the effects that sustained government debt and deficits potentially have on economic growth, when a country's debt-to-GDP level rises, investors become increasingly concerned about its solvency and may stop financing its public debt, or even withdraw their current assets. Krugman (1988) defines the concept of "debt overhang" in order to characterize the increasing uncertainty from investors when a developing country's debt accumulation is perceived larger than the present value of its future income. In the context of uncertain government solvency, he argues that creditors are left with two options. They can additionally finance the country in hope that it will be able to restore the health of its public finance and get fully repaid afterwards, or they can forgive a part of their actual assets and be partly repaid in the short term. The potential conflict of interests between the collective benefit of financing and the individual advantage of withdrawing the assets creates ambiguity in the signal perceived by the market. He argues that if provided with sufficient evidence that the country only faces a problem of liquidity, creditors may get sufficient collective financing momentum. However, his analysis shows that a pure liquidity problem is unlikely and that it must originate from solvency issues. If the markets perceive a structural problem of solvency, the likely outcome of bankruptcy may wary creditors and potentially lead to an investment "crowding-out" phenomenon. How creditors perceive the uncertainty of a country's solvency relates to its "debt intolerance" measure. C. Reinhart and Rogoff (2003) introduce this notion in order to characterize the difficulty a developing country faces when dealing with a debt-to-GDP ratio that would be easily manageable by advanced economies. The authors propose that such intolerance levels are characterizable by a small number of variables related to factors like repayment history, indebtedness level, and history of macroeconomic stability.

Even though Krugman (1988) and C. Reinhart and Rogoff (2003) explore the debt problems of developing countries, their analyses are adaptable to the risk that unsustainable debt-to-GDP levels may have on production growth in advanced economies. Following this idea, Ball and Mankiw (1995) analyze precisely how a loss of confidence from investors could impact economic growth in advanced countries facing an increasing risk of "debt overhang". The fact that national debt

in advanced economies is often largely owned by residents, whereas foreign debt accounts for a greater part of the developing countries' indebtedness, makes a government from the former less likely to default on its debt. Instead, it may employ alternative debt management strategies, such as partially defaulting on its bonds or taxing other assets held by domestic and foreign investors, what may delay the market's reaction to a signal of insolvency. Nonetheless, advanced countries with insolvency issues and deteriorating net-foreign-asset positions can still face a gradual decrease in the demand for their debt titles, potentially leading to a confidence collapse from creditors. In this case, the increasing risk on the country's assets is likely to induce pressure on the domestic interest rates and government bond yields. This may lower domestic investments from firms and households, as well as reducing the incentive for public spendings by the government, such that the capital level in the economy is expected to fall. Additionally, the rising cost of financing for the government makes its payments on the debt grow faster, what further deteriorates its debt-to-GDP ratio and accentuates the investment "crowding-out" phenomenon. Secondly, the authors view the investment "crowding-out" as a source for declining domestic asset prices, given its increased supply in the market. This directly lowers the wealth of assets holders and exacerbates the risk of firms bankruptcy, two risky ingredients for the build up of a banking crisis. Thirdly, as domestic assets are exchanged for foreign funds, the increased supply of domestic currency in the market pushes its relative value to fall. Consequently, the trade balance moves towards surplus and leads to an outflow of capital, as well as an increasing pressure on domestic inflation as imports become costlier. The change in trade balance may be sufficient to generate a sectorial shock, subsequently shifting the production from non-tradeable to tradeable industries and causing a rise in the structural unemployment rate. In the view of the authors, these channels, namely the rising domestic interest rate, the fall in domestic asset prices and the fall in the currency exchange rate, link rising debt-to-GDP levels and a potential investment "crowding-out" phenomenon to negative impacts on economic growth.

This brings to the issue that determining a "debt intolerance" measure for advanced countries is more challenging than doing so for developing economies. Generally speaking, advanced countries have more stable macroeconomic backgrounds and repayment histories given that the national debt is mostly internally

held. Empirical evidences from Bohn (2005) and Mendoza and Ostry (2008) illustrate that governments in advanced economies behave responsibly in general, usually generating fiscal surpluses to counter rising debt services. Nonetheless, this group of countries also illustrates divergence in their relative “debt intolerance” levels. On this account, Ghosh et al. (2013) assess the debt sustainability of 23 advanced countries using the theoretical framework of a stochastic model of sovereign default. Investors lend to a government that faces an endogenous debt limit, beyond which fiscal solvency is in doubt. Employing this framework and an empirical estimation of the necessary parameters, they determine the countries’ “fiscal space”, defined as the gap between the current debt ratio and indebtedness limit, a point upon which a government cannot roll its debt over anymore. Using data covering the period 1970-2007, they find that the fiscal space vary considerably across advanced economies. According to their results, Greece, Iceland, Italy, Japan and Portugal have limited or no available fiscal manoeuvre, the UK and US have fiscal constraints, and Australia, Korea and the nordic countries have ample fiscal space.

Up to this point, the covered theoretical insights propose that if a country, with limited fiscal space, reaches debt-to-GDP levels that exceed its debt intolerance measure, investors may perceive a signal suggesting a risk of debt overhang, such that a gradual investment crowding out may trigger negative effects on GDP growth. This suggests the presence of a rapidly declining effect of additional government debt on growth, past a given critical debt-to-GDP level. In other words, this theoretically justifies the non-linear, often described as inverted U-shaped, or threshold effect, in the debt/GDP-growth functional form.

An interesting approach for fiscal policy is to evaluate what is the optimal debt-to-GDP level that governments should target in order to maximize long-term growth. Checherita-Westphal, Hallett, and Rother (2012) explore this question in the context of an endogenous growth model, with a government that contracts debt only to finance public investments, while current spendings must equal current revenues. They find that the government level of debt that maximizes steady state growth is a function of the output elasticity of public capital stock, which they empirically estimate using data for OECD countries. Their results suggest an inverted U-shaped debt/GDP-growth functional form, with an optimal debt level

at 67% of GDP for OECD countries and 50% for the euro area. Greiner (2012) generalizes their endogenous growth model by allowing alternative government fiscal policies, such that deficits are not necessarily proportionate to public investment. This updated model finds no inverted U-shaped functional form, but rather a monotonic relationship with higher growth rates for lower government deficit and debt levels. Additionally, the author demonstrates that the endogenous growth model in Checherita-Westphal, Hallett, and Rother (2012) is structurally the same as a tax-growth model without government debt, which also results in an inverted U-shaped tax-growth relationship. The fact that the non-linear effect may not be attributable to government debt-to-GDP itself undermines the previous findings.

Other theoretical works, as identified by Panizza and Presbitero (2013), propose alternatives to the inverted U-shaped functional form of the debt/GDP-growth relationship, supported by the investment “crowding-out” concept. based on the analysis of the government debt valuation equation, Cochrane (2011) finds no justification for a threshold or inverted U-shaped relationship and rebuts the link between economic growth and the “crowding-out” theory. In his view, given that the equation only proposes that inflation rises contemporarily if agents think that future debt-to-GDP levels will grow uncontrollably, if agents are convinced that the government will generate future budget surpluses when faced with a rising debt-to-GDP level, carrying a large government debt burden is simply not problematic. Also, based on the absence of an investment “crowding-out” mechanism in the government debt valuation equation, the author proposes that long term nominal rates only reflect expected inflation and a risk premium for government debt. In his opinion, interest rates are rather determined by the ability for the government to run surpluses relative to the public debt level. Alternatively, Teles and Mussolini (2014) augment the model of Saint-Paul (1992) with the introduction of productive government spendings and find that a rising debt-to-GDP level affects growth through decreasing marginal productivity of public spendings. This inclusion changes the conclusion of Saint-Paul (1992), proposing that government debt unconditionally reduces growth. Instead, the updated model finds a positive link between government debt and growth under certain fiscal conditions and parameter values.

The analysis of the theoretical literature concerned with debt/GDP-growth

relationship does not provide a clear “take home” conclusion regarding the characteristics of its functional form. Some recent theoretical developments come up with conclusions that challenge the standard investment “crowding-out” channel and the threshold or inverted U-shaped functional form. Given the complexities required by theoretical models to account for the multiple characteristics that link government debt-to-GDP and growth in reality, empirical explorations of the subject may lead to more conclusive findings. The next section covers the empirical studies concerned with the analysis of the debt/GDP-growth relationship in advanced economies.

3 Empirical review

The body of empirical literature that specifically analyzes the effect of government debt-to-GDP levels on growth in advanced economies, follows for the most part C. Reinhart and Rogoff (2010b), which is based on the findings from their NBER working paper C. Reinhart and Rogoff (2010a). RR significantly contributed to the research field by collecting and making available a new database on public debt relative to GDP, covering over 200 years and 70 countries. In their study, they explore the debt/GDP-growth relationship by classifying year-country observations, according to their government indebtedness levels, and comparing the average and median growth rates of each category. The annual observations from 20 advanced countries, covering the 1946-2009 period, are classified in four categories. Observations with debt-to-GDP ratios in the range 0-30% are in category 1, 30-60% are in category 2, 60-90% are in category 3 and 90%+ are in category 4. For each category, they calculate the average and median growth rates and compare the statistics accordingly. The growth statistics are relatively stable across the three first categories. However, the authors find that average economic growth is more than 3 percentage points lower in the 90%+ range, compared to the other categories, while the median growth rate illustrates the reduction of only one percentage point. RR’s findings triggered a growing interest from researchers to verify and further explore their empirical evidences, supporting a drastic fall in the growth rates of highly indebted countries and suggesting a threshold or inverted U-shaped effect.

Herndon, Ash, and Pollin (2013) is the paper that most specifically tackles RR's findings by attempting to identically reproduce the article's results, using the same database and methodology. Their replication contradicts the previous conclusions, which supported a sharp decline in economic growth rates for countries with debt-to-GDP ratio above the 90% threshold. Instead, Herndon, Ash, and Pollin (2013) find an average GDP growth rate of 2.2% for the 90%+ debt-to-GDP category, compared with the -0.1% level previously found by RR. This contrasting new average growth rate provides evidence against a threshold effect or inverted U-shaped functional form of the debt/GDP-growth relationship. Rather, it suggests a slightly negative correlation as the debt-to-GDP ratio goes beyond the 90% level. They justify the divergence of their new results with those of RR with the identification of several errors in the methodology of the previous study. The first error consists of data exclusion from three countries: Australia (1946-1950), New-Zealand (1946-1949) and Canada (1946-1950). In particular, data from New-Zealand (1946-1949), which contains four observations with debt-to-GDP ratios above 90%, such that only its exclusion lowers the average growth rate of 0.3 percentage point. Secondly, a coding error excludes five countries from the statistics computation: Australia, Austria, Belgium, Canada and Denmark. Again, this exclusion itself causes a -0.3 percentage point variation of the average growth statistic for the 90%+ category. Thirdly, RR calculate the average and median growth rates with country weighted statistics, instead of using country-year weights. Accordingly, a country observed only one period inside a given debt-to-GDP category is given the same weight as a country observed more than one period in the same category. Once corrected for these errors, Herndon, Ash, and Pollin (2013) not only refute the 90% debt-to-GDP threshold effect on growth, but also find evidence for non-linearities in the debt/GDP-growth relationship inside the 0-30% category, using a locally smoothed nonparametric estimator on the raw dataset.

The methodological approach of Herndon, Ash, and Pollin (2013), that is to apply a nonparametric estimator to the empirical analysis of the debt/GDP-growth relationship, is a good starting point. In fact, this estimation strategy allows to depict any non-linearities in the relationship and spares the need to impose hypotheses on the model specification. Moreover, given that C. Reinhart and Rogoff

(2010b) is a pivotal paper in the concerned literature and finds evidence for a non-linear functional form of the debt/GDP-growth relationship, further research should employ estimation methods that suits a non-linear specification. However, Herndon, Ash, and Pollin (2013)'s bi-variate nonparametric specification does not account for likely endogeneity in the debt/GDP-growth model, what may have increased the risk of a biased estimation. The first source of endogeneity in their model specification is the lack of control for covariates, potentially resulting in a problem of omitted variables. Secondly, as mentioned in the introduction, the fact that fiscal policy can react to the cyclicity of GDP growth rates, through automatic fiscal stabilizers and countercyclical policies, potentially introduces reverse causality in the debt-growth relationship. Also, a shock in the economy, such as a banking crisis, can lead to a simultaneous effect on both variables and cause endogeneity of the regressors. Reverse causality and simultaneity were identified to be relevant to the debt/GDP-growth relationship by C. Reinhart, V. Reinhart, and Rogoff (2012) and C. Reinhart and Rogoff (2011), but were not accounted for in the nonparametric regression of Herndon, Ash, and Pollin (2013). In the next subsections, we review the empirical literature post-RR and analyze how studies have dealt with issues related to endogeneity and non-linearity.

3.1 Endogeneity

Endogeneity is perhaps the most challenging statistical issue that economists face when trying to identify causality from one variable to another. The study of the debt/GDP-growth relationship makes no exception and several methodological approaches have been implemented to minimize the related estimation bias. In order to control for omitted variable bias and to partly reduce the reverse causality effect, Cecchetti, Mohanty, and Zampolli (2011), Woo and Kumar (2010), Checherita-Westphal and Rother (2012) and Panizza and Presbitero (2014) estimate the relationship through the following model specification:

$$\begin{aligned} \overline{growth}_{i,t+1,t+n} &= \overline{debt/GDP}_{i,t}\phi + X_{i,t}\beta + \mu_i + \gamma_t + \epsilon_{i,t,t+n}, \\ &\text{for } i = 1, \dots, n; t = 1, \dots, T. \end{aligned}$$

In the expression, $\overline{growth}_{i,t+1,t+n}$ is the n period moving average of the annual real GDP growth variable, $X_{i,t}$ is a set of control variables thought to explain growth, $debt/GDP_{i,t}$ is the debt-to-GDP ratio, while μ_i and γ_t are country and period specific effects respectively. In this model specification, all independent variables are growth predetermined. Usually, n is set to 5 so to reduce the potential reverse causality in the debt/GDP-growth relationship, again, potentially arising from the contemporaneous effects of business fluctuations and their related automatic fiscal stabilizers and countercyclical policies responses. The option of using a 5-year non-overlapping moving average comes at the cost of significantly reducing the sample size, such that the estimation is left with less degrees of freedom. An alternative is to use an overlapping moving average specification, which spares the lost of observations, but introduces an autocorrelation process in the error terms, potentially leading to incorrect parameter standard errors. All cited studies did correct for the autocorrelation error process with robust standard errors and corresponding parameter significance testing procedure. A 5-year moving average specification also changes the scope of analysis towards the effect of government debt-to-GDP levels on medium/long term economic growth. We will cover this aspect in more details in section 3.4. Additionally, augmenting the debt/GDP-growth model with control variables thought to explain growth (Barro and Sala-i Martin (2004)), as well as with country and period specific effects, reduces the potential for omitted variable bias and partly accounts for cross-country heterogeneity. Cecchetti, Mohanty, and Zampolli (2011) estimate this model using a 5-year overlapping moving average specification, for a panel of 18 OECD countries over the period 1980-2006. They find a significant and negative effect of the debt-to-GDP variable on long term economic growth, such that a 10 percentage point increase in government indebtedness is associated with a 17-18 basis point reduction in 5-year average growth. They compute the Hubert-White sandwich standard errors and find they are larger than otherwise, consequently suggesting the presence of heteroskedastic error terms.

Woo and Kumar (2010), Checherita-Westphal and Rother (2012) and Panizza and Presbitero (2014) further tackle the endogeneity issue by instrumenting the debt-to-GDP ratio, using variables thought to be uncorrelated to the error terms and strongly linked to government indebtedness. Woo and Kumar (2010) find that

the system GMM estimator, which uses lagged values and differences of the endogenous regressor as instruments, best accounts for endogeneity in the estimation of the last model. The SGMM estimation results are similar to those of Cecchetti, Mohanty, and Zampolli (2011) (*debt/GDP* coefficient: -0.02; significance level: ** 5%) and are robust to different time periods, samples, specifications and control variables. Panizza and Presbitero (2013) think that the results similarities between both papers can be explained by two possible outcomes: whether the debt-to-GDP is not endogenous in the concerned model specification, or that SGMM is not an appropriate estimation strategy to account for it. However, Panizza and Presbitero (2013)'s analysis do not expose the fact that Woo and Kumar (2010) also obtain quite different results between the FE and SGMM estimations of the complete model, which includes period specific effects (*debt/GDP* coefficient: -0.004 vs. -0.02**).

Instead of using the SGMM approach, Checherita-Westphal and Rother (2012) and Panizza and Presbitero (2014) choose to use external instruments for the debt-to-GDP variable. Covering a panel of 12 euro countries over the period 1970-2008, Checherita-Westphal and Rother (2012) instrument the debt ratio of a country i at period t , with the debt-to-GDP average of countries $j : 1, \dots, 12; j \neq i$, for the same period. The validity condition of this instrumental variable supposes that the debt-to-GDP ratio of other euro countries does not affect the economic growth of another country, also part of the monetary union. This consists of a strong assumption, which is particularly hard to defend in the context of a study that specifically estimates the relationship between debt-to-GDP and growth levels for countries in the euro area. Checherita-Westphal and Rother (2012) augment their model specification with a squared debt-to-GDP variable. This specification allows them to estimate a quadratic functional form of the debt/GDP-growth relationship, using the FE, 2SLS and GMM estimators. They find a significant inverted U-shaped effect of the debt-to-GDP variable on growth, with a relationship maximum around the debt ratio of 90-100%. The results are robust to annual, 5-year overlapping and 5-year non-overlapping growth variable specifications, as well as for different country and time samples. They also identify the channels underlying the non-linear effect of the debt/GDP-growth relationship to be the impact of the public debt variable on private savings, public investments and total

factor productivity. Panizza and Presbitero (2014) use the same model specification and dataset as Cecchetti, Mohanty, and Zampolli (2011), and instrument the debt-to-GDP ratio with the valuation effects from the interaction between the government debt labelled in foreign currency and movements in the exchange rate:

$$VE_{i,t} = \frac{\sum_{j=1}^N D_{ij,t}(e_{ij,t+1} - e_{ij,t})}{\sum_{j=1}^N D_{ij,t}}.$$

In the expression, $D_{ij,t}$ is the stock of debt of country i denominated in currency of country j and $e_{ij,t}$ is the currency exchange rate between countries i and j , at period t . The authors then show that this instrumental variable is relevant to the debt-to-GDP variable, given that it provides a strong explanatory power in the first step regression and rejects several weak instrument tests. However, the model is exactly identified, such that a test of exclusion restriction is not possible. They discuss the conditions under which the instrument is likely to be valid. These consist of augmenting the model with the variables that represent the channels through which the valuation effects can affect growth. They find that the share of debt labelled in foreign currency and the trade-weighted effective exchange rate are both correlated with the instrument and the dependent variable. In order for the exclusion restriction to be valid, they augment their model with both variables, which consists of almost the same elements used to generate the instrument. Furthermore, Panizza and Presbitero (2014) mathematically demonstrate that the bias related to the simultaneity and reverse causality effects in the relationship is likely to be negative. They confirm this finding by comparing the debt-to-GDP variable coefficients from their OLS and IV estimations. Indeed, instrumenting the endogenous variable changes the debt-to-GDP estimated coefficient from negative (-1.796**) to positive (0.322) and makes it lose its significance, thus confirming the theoretical derivation that suggested a negative OLS biased estimate. The next subsection covers how the empirical literature specifically explored the presence of non-linearities in the debt/GDP-growth relationship.

3.2 Non-linearity

Minea and Parent (2012) investigate the debt/GDP-growth threshold effect previously identified by RR with an up-to-date econometric method. They use the Panel Smooth Threshold Regression (PSTR), a method developed by Gonzales et al. (2005), which allows to specify endogenous thresholds with gradual transitional effects:

$$growth_{i,t} = \alpha_i + \beta_1 debt/GDP_{i,t} + \beta_2 debt/GDP_{i,t} \Gamma(debt/GDP_{i,t}, T, \gamma) + \epsilon_{i,t},$$
$$\Gamma(debt/GDP_{i,t}, T, \gamma) = [1 + \exp(-\gamma \prod_{h=1}^H (debt/GDP_{i,t} - t_h))]^{-1}.$$

In the expression, $\Gamma(\cdot)$ is the logistic function that weights, according to the smoothing parameter γ , the distance between the regime switching variable and the thresholds $T : t_1, \dots, t_H$. In this case, the regime switching variable is also the debt-to-GDP variable. This method spares the need for hypothesis on exogenous thresholds, it provides a testing procedure for the presence of non-linearities and allows for smooth transitions between regimes. The authors determine that a three threshold specification, such that $H = 3$, provides the best model fit. Using the same country and time sample as RR, but from a different database, they find a debt/GDP-growth relationship with complex non-linearities, what contradicts the unilateral decline of growth for countries with debt-to-GDP ratios above 90%. Their model estimation identifies two significant thresholds, at the 90 and 115% debt-to-GDP levels. Between both, the debt/GDP-growth effect is negative and switches to positive past the 115% point. This functional form is robust to a longer time span, covering 1880-2009, for which the model finds a slightly different second threshold at 130%, instead of 115%. The authors find no significant difference between the debt/GDP-growth effect in the 0-90% and the 115%+ regimes, what provides evidence against a drastically different functional form for highly indebted countries.

Several articles assessed the non-linear functional form of the debt/GDP-growth relationship using non-dynamic and dynamic “standard” threshold regression models. The non-dynamic methodological framework was proposed by Hansen (1999)

and allows to estimate a threshold model for panel data with individual-specific effects. In the context of a multivariate debt/GDP-growth model, it can be characterized by the following two-regime equation:

$$g_{i,t} = \alpha_i + \delta' X_{i,t} + \beta_1 d_{i,t} I(q_{i,t} \leq T) + \beta_2 d_{i,t-1} I(q_{i,t} > T) + \epsilon_{i,t}.$$

In the expression, $g_{i,t}$ is the real GDP annual growth of country i at time t , $X_{i,t}$ is a set of growth related explanatory variables, which can include the debt-to-GDP or real GDP variables, and $q_{i,t}$ is the regime switching variable, usually the government debt variable. The testing procedure estimates the model for a pre-determined set of thresholds and finds T^* that minimizes the sum of squared residuals. Using this framework, Égert (2012) estimates a bivariate debt/GDP-growth model and selects the debt-to-GDP ratio as the regime switching variable in a three-regime specification. For advanced countries, over the 1790-2009 period, he finds two optimal thresholds at the debt-to-GDP levels of 20.38% and 55.35%, which define positive and significant coefficients for the first two regimes, and a positive, but non-significant, third regime coefficient. For a shorter time span, he finds that a two-regime model is more appropriate, with a single estimated threshold at the same 20.38% level and two significant coefficients that change signs from positive to negative accordingly. The results somehow indicate a debt/GDP-growth effect that changes from positive to null or negative as the ratio of debt-to-GDP increases and suggest thresholds at much lower levels than previously suggested. One important drawback of this study is the lack of specific threshold significance testing, which is indirectly replaced by tests against the null hypothesis of equality between the parameters of the different regimes.

On this account, Hansen (2000) proposes a correct procedure for testing the significance of a threshold in the previous regression framework, which necessitates to approximate the critical value of the test statistic using a bootstrap method, known as the Supremum F-, LM- or Wald-test. The Supremum F-test compares the F^* -statistic of the optimal threshold (T^*) model to the approximated distribution of a predetermined number of F_{null}^* -statistics, each obtained from specific bootstrap samples of the model under the null hypothesis of linearity. The threshold (T^*) is significant if the F^* -statistic is higher than the 5% critical-value of

the approximated F_{null}^* -statistic distribution. Afonso and Jalles (2013) apply this threshold significance procedure to a two-regime threshold regression model of the debt/GDP-growth relationship, using data for 155 developing and advanced countries over the 1980-2008 period. In their model specification, $X_{i,t}$ includes the initial real GDP, population growth, trade openness, education and investment variables, such that the debt-to-GDP ratio is only the regime switching variable. Similarly to Minea and Parent (2012), they find that GDP growth rates for low (<30%) and high (>90%) indebted OECD countries are statistically not different. They also find a significant optimal threshold at the 59% debt-to-GDP level, with an approximated p-value of 0.079 according to a Supremum Wald-test procedure.

The last covered papers analyze the non-linear functional form of the contemporaneous effect of government indebtedness on growth, which results may not hold if transposed to a dynamic specification. Following this idea, Baum, Checherita-Westphal, and Rother (2013) estimate a two-regime dynamic instrumental variable threshold model, a methodological approach previously proposed by Caner and Hansen (2004) and later adapted to panel data by Kremer, Bick, and Nautz (2013). The equation of this model is the following:

$$g_{i,t} = \alpha_i + \delta_1 g_{i,t-1} + \delta_2' X_{i,t} + \beta_1 d_{i,t-1} I(d_{i,t-1} \leq T) + \beta_2 d_{i,t-1} I(d_{i,t-1} > T) + \epsilon_{i,t},$$

$$g_{i,t-1} = \xi' Z_i + u_{i,t},$$

where $g_{i,t-1}$ and $d_{i,t-1}$ are lagged values of the growth and debt-to-GDP variables, while Z_i a set of instruments for $g_{i,t-1}$. Baum, Checherita-Westphal, and Rother (2013) choose to specify the growth variable as endogenous, instead of the debt-to-GDP ratio, using its lagged values as instruments in Z_i . The authors characterize the short-term dynamic effect of their specification as near contemporaneous. They estimate their model using the GMM estimator for 12 euro countries, covering the 1990-2007/2010 period, and specify 3 control variables in $X_{i,t}$: a measure of trade openness, the ratio of gross capital formation to GDP and a dummy variable indicating the European Monetary Union membership. Omitting the great recession in the time sample, they find an optimal threshold at the 67% debt-to-GDP level, with a Supremum-F test p-value of 0.078, such that it defines a significant and positive debt/GDP-growth effect in the first regime and a non-

significant near null effect in the second regime. This suggests that low indebted euro countries potentially face a positive, but rapidly decreasing, growth effect for additional debt-to-GDP increments. For the 1990-2010 sample, the optimal threshold is closer to RR’s findings, at the 95% debt level (p-value : 0.098), which now defines a significant and negative short term impact of government debt-to-GDP on growth for highly (95%+) indebted euro countries.

Kourtellos, Stengos, and Tan (2013) explore the relationship using the structural threshold regression (STR) model, a methodological approach they propose in Kourtellos, Stengos, and Tan (2015), which is built on the framework of Hansen (1999) and Caner and Hansen (2004). Their contribution to the standard multi-regime panel model consists of specifying the threshold and the regime switching variable as endogenous. The model equation specified in Kourtellos, Stengos, and Tan (2013) is the following:

$$\begin{aligned}
 growth_{i,t} &= \beta' X_{i,t} + \delta' X_{i,t} I(q_{i,t} \leq T) + \kappa \lambda_{i,t}(\gamma) + \epsilon_{i,t}, \\
 I(q_{i,t} \leq T) &= \begin{cases} 1, & \text{if } q_{i,t} \leq T \\ 0, & \text{if } q_{i,t} > T \end{cases}, \\
 q_{i,t} &= \pi' Z_{i,t} + v_{i,t},
 \end{aligned}$$

where $X_{i,t}$ is a set of variables thought to explain growth that includes the debt-to-GDP ratio, $q_{i,t}$ is the regime switching variable, T is the threshold to be optimized, $Z_{i,t}$ is a set of instruments for $q_{i,t}$, while $\lambda_{i,t}(\gamma)$ is a scalar variable that restores the conditional mean of errors. They estimate their model using 15 different variables for $q_{i,t}$, on a 10-year average non-overlapping panel dataset that covers 85 countries over the 1980-2009 period. By comparing the J-statistics, a measure of model validity, from the GMM estimations of all 15 models, they find the democracy variable to be the most relevant regime switching variable. According to their testing procedure, the debt-to-GDP variable is not a significant regime switching variable, in opposition to Minea and Parent (2012), Afonso and Jalles (2013) and Égert (2012)’s modelling approach. The estimation of their “optimal” model, including the democracy measure as the regime switching variable, results in a significant and negative effect of the debt-to-GDP ratio on economic growth for low

democracy countries and a positive, but non-significant, debt/GDP-growth relationship for high democracy countries. Accordingly, government debt in advanced countries, identified to have relatively high democracy levels, is growth neutral. However, employing a 10-year non-overlapping data specification, such that every country gets represented through only three periods, can undermine the ability of the model to represent the temporal aspect of the relationship.

Some of the papers that primarily focus on dealing with endogeneity through their model specification, covered in section 3.1, also partly account for a potential non-linear functional form. Checherita-Westphal and Rother (2012) employ the simple method of adding a quadratic debt-to-GDP term to the multivariate equation. As mentioned in the last section, they find a significant and negative quadratic coefficient, providing evidence for an inverted U-shaped relationship with a tipping point inside the 90-100% debt-to-GDP range. Woo and Kumar (2010) augment their multivariate specification with pre-determined debt-to-GDP categories, which they include in the model as corresponding dummy variables. They replicate the same debt-to-GDP categories as in C. Reinhart and Rogoff (2010b) and justify the presence of non-linearities in the debt/GDP-growth relationship on the basis of a single significant dummy coefficient for the >90% debt-to-GDP category. Cecchetti, Mohanty, and Zampolli (2011) also explore the presence of a threshold effect in the relationship using Hansen (1999)'s method and find that a debt-to-GDP level of 96% maximizes the LR statistic of the threshold augmented model. However, they do not provide a testing procedure for the threshold's significance. Lastly, Panizza and Presbitero (2012) estimate a two regime threshold regression model, for which they find two stable and significant regime coefficients. However, the difference between both coefficient values is not statistically significant, thus indicating the absence of a turning point that could justify a non-linear functional form. Table 4 of the Annex section reports a brief summarized synthesis of the covered papers, which include a formal model or method, from the theoretical and empirical bodies of literature.

3.3 Contribution to the literature

Most covered studies in the empirical literature suggest alternative findings to the sharp decline in economic growth rates for advanced countries with debt-to-GDP levels above the 90% threshold, previously proposed by C. Reinhart and Rogoff (2010b). Evidence for an inverted U-shaped functional form of the debt/GDP-growth relationship is perhaps the most common result of this literature. However, many alternative findings are suggested, such as the presence of complex non-linearities, strong heterogeneity or even evidence for a linear effect. Therefore, the analysis of the empirical literature, similarly to the theoretical part, contains too much ambiguity for concluding on a robust description of the debt/GDP-growth relationship. In our opinion, an important drawback of most reviewed methodological approaches for evaluating the presence of a non-linear pattern in the functional form is the limited specification that dynamic and non-dynamic threshold regression models offer. Aside from the quadratic modelling of Checherita-Westphal and Rother (2012), which specification flexibility could have been increased by employing a box-cox regression framework instead, and the simplistic bivariate nonparametric model of Herndon, Ash, and Pollin (2013), all reviewed empirical papers tested a linear specification against a threshold alternative. A threshold effect is a somewhat restrictive specification, which may not be adequate to model certain observed non-linear processes. Given the divergent estimation results by the reviewed threshold regression models, we share the opinion of Minea and Parent (2012) regarding the existence of more complex non-linearities in the relationship, which can only be captured by more flexible estimation strategies. Additionally, given the potential for reverse causality and simultaneity bias, an instrumental variable approach seems advisable in order to reduce the related endogeneity bias. The importance of such approach is illustrated by the change of debt-to-GDP coefficient signs and significance levels when instrumenting the variable in Panizza and Presbitero (2014) and Woo and Kumar (2010). On the basis of the theoretical and empirical literature analysis, we think that a flexible estimation strategy that minimizes the parametric constraints on the model specification and allows for an instrumental variable approach is suitable for the investigation of the debt/GDP-growth functional form. In order to account for these methodological

requirements, we propose to estimate the relationship using the semiparametric IV estimator. First, it allows to specify the debt-to-GDP variable nonparametrically and to control for omitted variables by augmenting the bivariate debt/GDP-growth relationship with a set of linearly specified variables thought to explain growth. Secondly, the debt-to-GDP variable can be instrumented in order to reduce the potential for reverse causality and simultaneity bias. However, compared to the parametric class, this estimator requires more data for a similar degree of precision, because less information is provided by the model specification. Also, interpreting its estimation results can be more challenging given the lack of parameter significance tests. In addition, we apply two specification tests in order to assess the suitability of the semiparametric IV estimator: a nonparametric test of exogeneity; a parametric model against a nonparametric alternative test, with identification through instrumental variables. We are not aware of other empirical work that analyzed the debt/GDP-growth relationship using this estimation strategy.

3.4 Moving average specification and consideration for heterogeneity

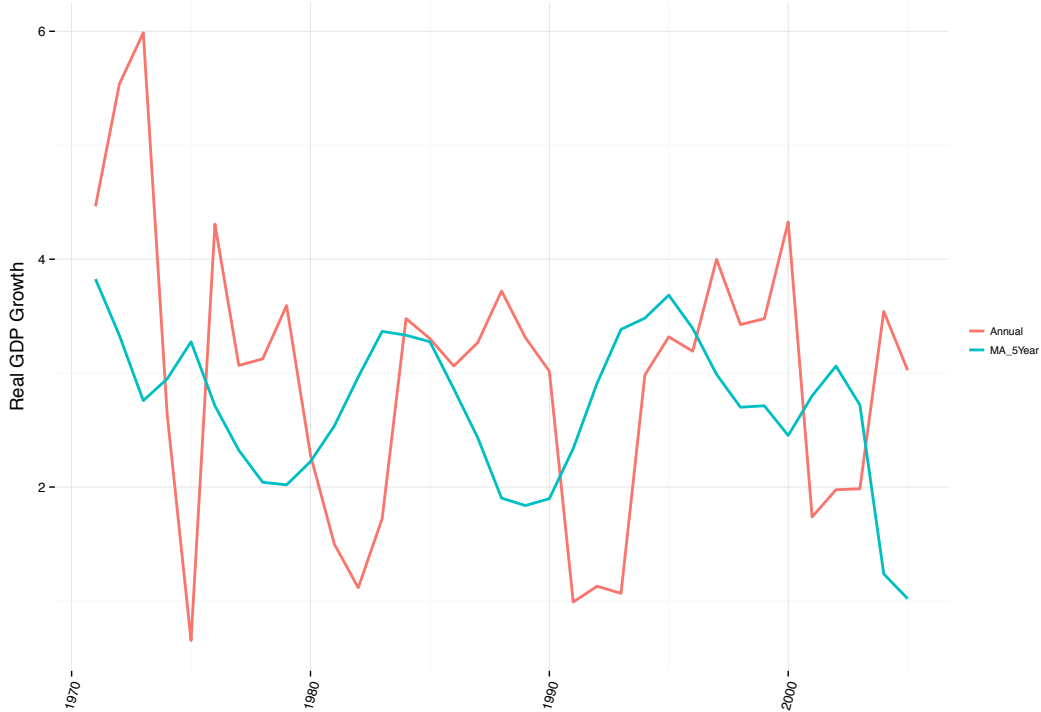
Following Cecchetti, Mohanty, and Zampolli (2011) and Panizza and Presbitero (2014), we choose to specify our growth variable as a 5-year overlapping moving average. As mentioned in the former study, a 5-year MA specification is common in the growth literature. Annual production growth is potentially constituted of important cyclical movements, which may bring irrelevant information for the debt/GDP-growth analysis. In fact, as covered in the theoretical review, the relationship of interest is likely characterized by a “gradual” propagation feature, such that a short-term study may not be valid for the purposes of a growth analysis. Specifying the dependent variable as a moving average reduces the cyclicity (see figure 1 (a)) and allows the model to grasp a medium/long-term impact of government debt-to-GDP levels. Figure 1 (b) reports the debt/GDP-growth bivariate observational plot, with annual, 2 and 5-year moving averages of the annual GDP growth variable on the y-axis and the growth predetermined debt-to-GDP variable on the x-axis. The figure depicts varying distribution characteristics across the different MA orders, such that estimation results may be conditional on the choice

of the GDP growth variable specification. Using an overlapping moving average specification introduces an autocorrelation process in the error term, which could invalidate standard parameter significance tests. This does not affect our conclusions based on semiparametric estimations, given that no significance testing procedures are provided. We use a robust method for the computation of correct estimated parameters' standard errors (HAC) for the OLS and IV estimates.

As mentioned in the introduction, cross-country heterogeneity is an important issue to account for in the empirical analysis of the debt/GDP-growth relationship. This represents the principal recommendation of Panizza and Presbitero (2013)'s survey of the concerned literature. In fact, it is logical to think that debt intolerance levels are not stable across countries and depend on specific factors such as repayment history, indebtedness level, and history of macroeconomic stability C. Reinhart and Rogoff (2003). This notion partially invalidates the methodological approach with the objective of finding a unique pooled debt-to-GDP threshold. We partly account for cross-country heterogeneity with the inclusion of country specific effect dummies in the model specification and with the estimation of the relationship for logical sub-samples of countries that share more homogenous traits.

Figure 1

(a) Cross-country average growth variable: annual and 5-year MA specifications

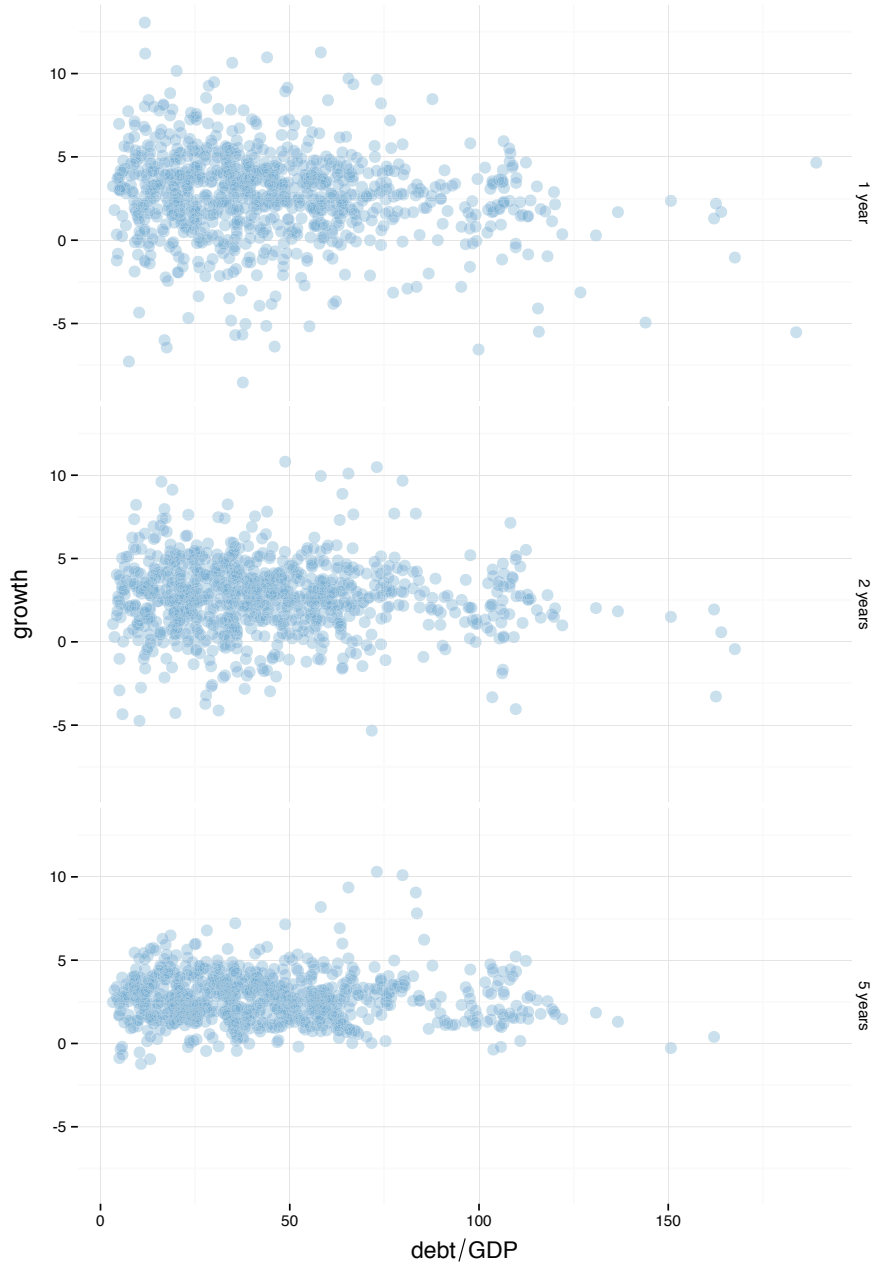


4 Data

Our dataset covers 23 advanced OECD countries over the period 1971-2010. The variable selection is based on Cecchetti, Mohanty, and Zampolli (2011), although from slightly different sources in order to increase the number of countries and the time span previously studied. It includes a total of 9 variables : GDP annual growth rate (*growth*), central government debt-to-GDP ratio (*debt/GDP*), log of real GDP (*RGDP*), gross domestic savings as % of GDP (*ngs*), annual population growth rate (*pop*), average year of total schooling (*school*), trade as % of GDP (*openc*), age dependency ratio (*dep*) and a banking crisis dummy variable (*crisis*). Table 1 reports the countries included in our dataset, with their *growth* and *debt/GDP* mean, median, standard deviations and scale comparable density statistics.

Figure 1











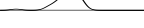
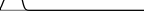

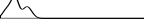








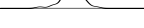

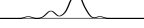
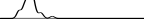
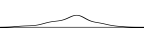







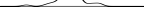
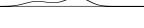


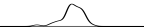
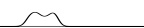






(b) Debt/GDP-growth relationship : annual, 2 and 5-year moving average



The debt-to-GDP and crisis dummy variables are from Reinhart and Rogoff's database¹, the population growth rate from the OCDE's database, the average year of total schooling variable from Barro-Lee's dataset and the real GDP variable from the Penn World Table 8.1. The annual GDP growth rate, gross domestic savings, trade and age dependency ratio variables are from the World Bank's database. We choose to keep the aggregated measure of the GDP annual growth rate and the debt-to-GDP variables so to have comparable results to C. Reinhart and Rogoff (2010b) and the mainstream conclusions.

¹<http://reinhartandrogoff.com/data/browse-by-topic/topics>

Table 1 – Country summary statistics

| Country (ISO 3166-2) | Growth | | | | Debt/GDP | | | |
|----------------------|--------|--------|------|---|----------|--------|-------|---|
| | Mean | Median | SD | Density | Mean | Median | SD | Density |
| AT | 2.52 | 2.48 | 1.92 |  | 41.45 | 46.35 | 17.99 |  |
| AU | 3.21 | 3.75 | 1.53 |  | 15.59 | 15.65 | 7.20 |  |
| BE | 2.26 | 2.34 | 1.87 |  | 85.31 | 95.35 | 26.46 |  |
| CA | 2.89 | 3.15 | 2.14 |  | 55.49 | 54.40 | 15.61 |  |
| CH | 1.59 | 1.95 | 2.22 |  | 17.64 | 15.40 | 7.31 |  |
| DE | 2.05 | 1.89 | 2.03 |  | 23.13 | 20.80 | 11.01 |  |
| DK | 1.90 | 2.22 | 2.22 |  | 46.43 | 51.80 | 24.12 |  |
| ES | 2.87 | 2.96 | 2.28 |  | 30.95 | 34.80 | 15.38 |  |
| FI | 2.71 | 3.17 | 3.22 |  | 27.36 | 15.95 | 20.38 |  |
| FR | 2.30 | 2.25 | 1.83 |  | 41.51 | 35.60 | 19.96 |  |
| GB | 2.36 | 2.85 | 2.33 |  | 41.12 | 41.00 | 9.18 |  |
| GR | 2.47 | 3.02 | 3.55 |  | 71.65 | 90.10 | 40.11 |  |
| IE | 4.61 | 4.69 | 3.67 |  | 66.36 | 72.70 | 27.79 |  |
| IS | 3.51 | 4.20 | 3.84 |  | 37.75 | 33.00 | 21.70 |  |
| IT | 2.06 | 1.86 | 2.31 |  | 86.09 | 97.70 | 27.76 |  |
| JP | 2.77 | 2.82 | 2.79 |  | 74.95 | 54.60 | 53.53 |  |
| NL | 2.48 | 2.54 | 1.84 |  | 58.27 | 56.40 | 13.25 |  |
| NO | 3.11 | 3.39 | 1.88 |  | 25.27 | 25.90 | 6.37 |  |
| NZ | 2.42 | 2.70 | 2.51 |  | 42.27 | 41.00 | 14.51 |  |
| PT | 2.95 | 2.81 | 3.20 |  | 32.20 | 24.55 | 24.16 |  |
| SE | 2.12 | 2.52 | 2.21 |  | 46.63 | 47.85 | 18.63 |  |
| TR | 4.28 | 5.15 | 4.25 |  | 32.50 | 28.40 | 13.83 |  |
| US | 2.92 | 3.41 | 2.17 |  | 52.30 | 56.10 | 15.69 |  |

5 Methodology

5.1 Models

Our general modelling specification replicates the approach taken by Cecchetti, Mohanty, and Zampolli (2011). In contrast of their methodology, we keep the variables of interest aggregated, such that our growth equation specifies the population growth rate (*pop*) as exogeneous, which gets incorporated in the model as a predetermined variable. In the debt/GDP-growth empirical literature, both aggregated and per capita specifications are used for the growth variable. We opt for the aggregated measure in order to challenge our results to the mainstream conclusions of C. Reinhart and Rogoff (2010b) and Herndon, Ash, and Pollin (2013). Even though we account for the population growth rate as exogenous in the control variable selection, we are aware that specifying our growth measure as aggregated changes the scope of analysis slightly compared to the works of reference. Following Cecchetti, Mohanty, and Zampolli (2011), we specify the dependent variable as a 5-year overlapping moving average of the annual GDP growth rate:

$$\overline{growth}_{i,t+1,t+6} = \frac{1}{5} \sum_{j=t+1}^{t+6} growth_{i,j}.$$

All independent variables (including the country and period specific effects) are growth predetermined, such that their time specification is one period lagged relative to the growth moving average. We estimate the following linear, IV, semiparametric and semiparametric IV models:

(A) Linear model:

$$\overline{growth}_{i,t+1,t+6} = debt/GDP_{i,t}\phi + X_{i,t}\beta + \mu_i + \gamma_t + \epsilon_{i,t,t+6}.$$

for $i = 1, \dots, n$; $t = 1, \dots, T$.

(B) IV model:

$$\begin{aligned}\bar{growth}_{i,t+1,t+6} &= debt/GDP_{i,t}\phi + X_{i,t}\beta + \mu_i + \gamma_t + \epsilon_{i,t,t+6}, \\ debt/GDP_{i,t} &= Z_{i,t}\psi + v_{i,t},\end{aligned}$$

for $i = 1, \dots, n; t = 1, \dots, T$.

(C) Semiparametric model:

$$\begin{aligned}\bar{growth}_{i,t+1,t+6} &= g(debt/GDP_{i,t}) + X_{i,t}\beta + \mu_i + \gamma_t + \epsilon_{i,t,t+6}.\end{aligned}$$

for $i = 1, \dots, n; t = 1, \dots, T$.

(D) Semiparametric IV model:

$$\begin{aligned}\bar{growth}_{i,t+1,t+6} &= g(debt/GDP_{i,t}) + X_{i,t}\beta + \mu_i + \gamma_t + \epsilon_{i,t,t+6}, \\ debt/GDP_{i,t} &= G(Z_{1i,t}) + Z_{2i,t}\psi + v_{i,t},\end{aligned}$$

for $i = 1, \dots, n; t = 1, \dots, T$.

In the expressions, $g(\cdot)$ and $G(\cdot)$ are unknown nonparametric functions, while $Z = \{Z_1, Z_2\}$ is a set of instrumental variables for $debt/GDP_{i,t}$. The matrix of predetermined regressors $X_{i,t}$ contains the following variables: the log of real GDP ($-RGDP_{i,t}$) accounts for the catch-up effect of the economy to its steady state, the gross domestic savings as % of GDP ($ngs_{i,t}$) and the annual population growth rate ($pop_{i,t}$) account respectively for the positive and negative effects on the steady state in the classical growth model of Solow, while the average year of total schooling ($school_{i,t}$), trade as % of GDP ($openc_{i,t}$), the age dependency ratio ($dep_{i,t}$) and a banking crisis dummy variable ($crisis_{i,t}$) represent a description of the technology and preferences of the countries. μ_i and γ_t are country and period specific sets of dummy variables.

The variable selection replicates Cecchetti, Mohanty, and Zampolli (2011) and Panizza and Presbitero (2014), two important articles of the empirical literature, in order for our model estimations to have comparable bases. We are confident that the potential for omitted variable bias is largely reduced by the account of

country and period specific effects in the specification. Moreover, our model of interest additionally reduces the risk of estimation bias by using the instrumental variable approach.

5.2 Estimators

The estimation structure of the models in section 5.1 proceeds in the following order: OLS, IV, semiparametric and semiparametric IV. In this section, we briefly recall the theoretical background of nonparametric and semiparametric estimators, in order to have the necessary notions required by the nonparametric IV and semiparametric IV estimators.

5.2.1 Nonparametric

The nonparametric estimators we cover here are the local constant, local linear and semiparametric kernel types. This subsection (5.2.1) follows Li and Racine (2011). The structure underlying nonparametric kernel estimators is to find a weighted average of the dependent variable, based on weights that represent the distance between the corresponding independent evaluation point and sampled observations. But before jumping into the derivation of the nonparametric regression estimator, let's start by recalling the notions of nonparametric density estimation.

Density estimation

The easiest way to illustrate the underlying mechanism of the nonparametric kernel framework is with the empirical estimation of a cumulative distribution function (CDF):

$$\hat{F}(x) = \frac{1}{n} \sum_{i=1}^n G\left(\frac{x - x_i}{h}\right) \quad (3)$$

where, $G(\cdot)$ is a weight function, commonly the CDF of the normal distribution, and h is a positive smoothing parameter known as the bandwidth. The selection of the smoothing parameter is critical. In fact, different values of h lead to quite divergent estimation results for a fixed dataset. This point is best illustrated by

evaluating the smoothing parameter at its limits:

$$\lim_{h \rightarrow 0} : G(\pm\infty) = \begin{cases} 0, & \text{if } x < x_t \\ 1, & \text{if } x > x_t \end{cases}$$

$$\lim_{h \rightarrow \infty} : G(0) = 0.5.$$

Here, we suppose that $G(\cdot) = \Phi$, the cumulative normal distribution. We see that, as h approaches ∞ , the estimate is fixed for all observation points, while as h goes to 0, the estimate is different for each x . Therefore, the variance of the estimated function is a negative function of the smoothing parameter h .

Nonparametric estimation of a probability density function (PDF) is nested in the regression framework, required to estimate a conditional expectation. The empirical nonparametric estimator of a PDF follows from (3):

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^n k\left(\frac{x - x_i}{h}\right) \quad (4)$$

where $k(\cdot) = \frac{\partial G(z)}{\partial z}$, commonly the PDF of the normal distribution. Finding an optimal smoothing parameter for this specific nonparametric estimator can be achieved by minimizing its integrated mean squared error (IMSE):

$$\min_h \int [\hat{f}(x) - f(x)]^2 dx,$$

where $f(x)$ is the true distribution of the underlying data generating process and $\hat{f}(x)$ consists of its nonparametric estimate. Given that $f(x)$ is unknown in practice, we can use a cross-validation technique, which finds the optimal smoothing parameter by numerically minimizing the IMSE. Using some manipulations, the CV technique discards the unknown $f(x)$ and minimizes the following expression using numerical search algorithms:

$$\min_h CV_f(h) = \frac{1}{n^2 h} \sum_{i=1}^n \sum_{j=1}^n \bar{k}\left(\frac{x_i - x_j}{h}\right) - \frac{2}{n(n-1)h} \sum_{i=1}^n \sum_{j \neq i, j=1}^n k\left(\frac{x_i - x_j}{h}\right)$$

where $\bar{k}(v)$ is the twofold convolution kernel based on $k(v)$. If $k(v) = \frac{e^{-\frac{v^2}{2}}}{\sqrt{2\pi}}$, a normal PDF, then the twofold convolution kernel is $\bar{k}(v) = \frac{e^{-\frac{v^2}{4}}}{\sqrt{4\pi}}$. The CV technique has the advantage to be entirely data-driven.

Multivariate density estimation is straightforward. The estimator of the multivariate PDF $f(x) = f(x_1, \dots, x_q)$ is:

$$\hat{f}(x) = \frac{1}{nh_1 \dots h_q} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right),$$

where $K\left(\frac{x - x_i}{h}\right) = k\left(\frac{x_1 - x_{i,1}}{h_1}\right) \times \dots \times k\left(\frac{x_q - x_{i,q}}{h_q}\right)$

Local constant kernel estimator

We now present the nonparametric regression local constant estimator, which uses the notions of nonparametric density estimation just covered. We start with the following model:

$$y = g(x) + u,$$

where $g(x)$ is an unknown function that represents:

$$E[y|x]. \tag{5}$$

In an idealized situation, we would possess a sample of observations $\{y_j, x\}_{j=1}^m$, such that the sample conditional expectation comes down to the following equivalence:

$$E[y|x] \equiv \frac{1}{m} \sum_{j=1}^m y_j,$$

which would lead to a consistent estimation as $m \rightarrow \infty$. However in practice, chances are that $x_i \neq x, \forall i$. The alternative consists of providing higher weights to observations that are closer to the evaluation point in the averaging process.

The local constant nonparametric estimator of (5) is:

$$\begin{aligned}
E[y|x] &= \int \frac{yf_{y,x}(x,y)}{f(x)} dy, \tag{6} \\
\hat{f}_{y,x}(x,y) &= \frac{1}{nh_0h_1 \dots h_q} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) k\left(\frac{y-y_i}{h_0}\right), \\
\text{where } K\left(\frac{x-x_i}{h}\right) &= k\left(\frac{x_1-x_{1i}}{h_1}\right) \times \dots \times k\left(\frac{x_q-x_{qi}}{h_q}\right), \\
\int y\hat{f}_{y,x}(x,y) dy &= \frac{1}{nh_0h_1 \dots h_q} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) y_i.
\end{aligned}$$

Expression (6) comes down to the following local constant kernel estimator when its elements are substituted by their empirical kernel estimates:

$$\hat{g}(x) = \int \frac{y\hat{f}_{y,x}(x,y) dy}{\hat{f}(x)} = \frac{\sum_{i=1}^n y_i K\left(\frac{x-x_i}{h}\right)}{\sum_{i=1}^n K\left(\frac{x-x_i}{h}\right)}. \tag{7}$$

The least squares cross-validation procedure that finds the optimal bandwidth parameters h_1, \dots, h_q in (7) have the objective to solve the following minimization problem:

$$\min_{h_1, \dots, h_q} CV_{lc}(h_1, \dots, h_q) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{g}_{-i}(x_i))^2 M(x_i),$$

where $\hat{g}_{-i}(x_i) = \frac{\sum_{l=1, l \neq i}^n y_l K\left(\frac{x_i-x_l}{h}\right)}{\sum_{l=1, l \neq i}^n K\left(\frac{x_i-x_l}{h}\right)}$ is the leave one out estimator of $g(x_i)$, and $0 \leq M(\cdot) \leq 1$ is a weight function that corrects for slow convergence issues. See Li and Racine (2011) for a more extensive discussion on the cross-validation technique. Again, this minimization is solved using numerical search algorithms.

Local linear kernel estimator

The local linear kernel estimator is an extension of the local constant and corrects for potential bias around the distribution tails. In order to get an expression in the likes of (7) for this estimator, it is useful to note that the $\hat{g}(x)$ found in (7) is

also the solution of the following optimization problem:

$$\min_a \sum_{i=1}^n (y_i - a)^2 K\left(\frac{x - x_i}{h}\right), \quad (8)$$

where \hat{a} solves for a , an equivalent of $\hat{g}(x)$ found in (7). The local linear kernel estimator solves an extension of (8) with a second order polynomial specification:

$$\min_{\{a,b\}} \sum_{i=1}^n (y_i - a - (x - x_i)'b)^2 K\left(\frac{x - x_i}{h}\right) \quad (9)$$

where \hat{a} and \hat{b} are equivalents of $\hat{g}(x)$ and $\hat{g}^{(1)}(x) = \frac{\partial g(x)}{\partial x}$ respectively. We can define our parameters of interest as $\delta = (a, b)'$, the dependent variable as Y , the $n \times (1 + q)$ set of explanatory variables as X , with $(1, (x - x_i)')$ as its i th row element and the $n \times n$ diagonal matrix as \mathcal{K} , with $K(\frac{x-x_i}{h})$ as its i th diagonal element. Using these elements, (9) can be rewritten in matrix notation as:

$$\min_{\delta} (Y - X\delta)' \mathcal{K} (Y - X\delta),$$

which takes the form of a generalized least square problem. From the GSL derivation, we know the solution of this minimization is:

$$\begin{aligned} \hat{\delta}(x) &= (X' \mathcal{K} X)^{-1} X' \mathcal{K} Y \\ &= \left[\sum_{i=1}^n K\left(\frac{x - x_i}{h}\right) \begin{pmatrix} 1 \\ (x - x_i)' \end{pmatrix} (1, (x - x_i)') \right]^{-1} \\ &\quad \times \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right) \begin{pmatrix} 1 \\ (x - x_i)' \end{pmatrix} y_i \end{aligned} \quad (10)$$

The least squares cross-validation procedure that finds the optimal bandwidths in (10) minimizes the following expression:

$$\min_h CV_{ll}(h_1, \dots, h_q) = \frac{1}{n} \sum_{i=1}^n [y_i - \hat{g}_{-i,L}(x_i)]^2 M(x_i),$$

where $\hat{g}_{-i,L}(x)$ is the leave one out local linear estimator of $g(x_i)$ and $M(\cdot)$ is a weight function.

Semiparametric estimator

Semiparametric estimators extend the nonparametric type by allowing to augment a nonparametric relation with parametrically specified variables. This approach is particularly handy for reducing the risk of potential curse of dimensionality in multivariate nonparametric estimations. In this paper, we present and estimate the debt/GDP-growth relationship with the simplest semiparametric method, the Robinson (1988)'s estimator, in the following model framework:

$$y_i = x_i' \beta + g(z_i) + u_i, \quad (11)$$

for $i = 1, \dots, n$.

The following procedure describes how to identify $\hat{\beta}$ and $\hat{g}(z)$ according to this method:

- First take the expectation of (11) conditional on z_i :

$$E(y_i|z_i) = E(x_i|z_i)' \beta + g(z_i). \quad (11.1)$$

- Then subtract (11.1) from (11):

$$y_i - E(y_i|z_i) = (x_i - E(x_i|z_i))' \beta + u_i. \quad (11.2)$$

- From (11.2), we can estimate $\hat{y}_i = E(y_i|z_i)$ and $\hat{x}_i = E(x_i|z_i)$ with local kernel estimators (constant or linear):

$$\hat{y}_i = \frac{\sum_{j=1}^n y_j K\left(\frac{z_i - z_j}{h}\right)}{\sum_{j=1}^n K\left(\frac{z_i - z_j}{h}\right)},$$

$$\hat{x}_i = \frac{\sum_{j=1}^n x_j K\left(\frac{z_i - z_j}{h}\right)}{\sum_{j=1}^n K\left(\frac{z_i - z_j}{h}\right)},$$

where $K\left(\frac{z_i - z_j}{h}\right) = \prod_{s=1}^q k\left(\frac{z_{is} - z_{js}}{h_s}\right)$.

- Using \hat{y}_i and \hat{x}_i , we can replace $\tilde{y}_i = y_i - E(y_i|z_i) = y_i - \hat{y}_i$ and $\tilde{x}_i = x_i - E(x_i|z_i) = x_i - \hat{x}_i$ in (11.2):

$$\tilde{y}_i = \tilde{x}_i' \beta + u_i,$$

from which we can obtain a consistent estimate of β with the OLS estimator:

$$\hat{\beta} = (\tilde{X}' \tilde{X})^{-1} \tilde{X}' \tilde{Y}.$$

- Having identified an estimator for β , we can now find a consistent estimator for $g(z)$ in the following new model:

$$y_i - x_i' \hat{\beta} = g(z_i) + u_i,$$

using the local constant or linear kernel estimator:

$$\hat{g}(z_i) = \frac{\sum_{j=1}^n (y_j - x_j' \hat{\beta}) K\left(\frac{z_i - z_j}{h}\right)}{\sum_{j=1}^n K\left(\frac{z_i - z_j}{h}\right)}. \quad (11.3)$$

The bandwidths h_1, \dots, h_q for (11.3) must solve the following problem:

$$\min_h \sum_{i=1}^n [y_i - x_i' \hat{\beta} - \hat{g}_{-i}(z_i)]^2$$

where $\hat{g}_{-i}(z_i)$ is the leave one out estimator of $g(z_i)$.

5.2.2 Nonparametric IV

Nonparametric IV estimators allow to estimate nonparametric functions of regressors that are specified as endogenous. The method has the objective to provide an estimate of $g(\cdot)$ in the following model:

$$y_i = g(x_i, w_i) + u_i, \quad (12)$$

for $i = 1, \dots, n$,

where $E(u|x) \neq 0$. x is a set of explanatory variables that are potentially endogenous and w is a set of instrumental variables for x . Estimating this model with standard nonparametric methods would yield biased estimates linked to problems related to regressors endogeneity. A nonparametric IV estimator uses the set of instrumental variables w to reduce the distance between the estimate and the true value of the nonparametric function.

Before presenting the simpler nonparametric IV estimator we choose to apply in our empirical work, namely the “control function model”, it is useful to present a brief overview of the “standard method”, which is based on an intuitive moment condition:

$$E(u|w) = 0. \tag{13}$$

The problem with this “standard” estimator is that its derivation provides its share of complexity, originating from an ill-posed inverse problem, i.e., small increments in the series expansion, used to derive the estimator, may result in large estimate variations. This problem requires a calibration step in order to choose the optimal series expansion level (Newey and Powell (2003)).

We choose to estimate the nonparametric IV model using the “control function model” estimator, a method that is empirically comparable to the “standard” method and most importantly, which does not imply the ill-posed inverse problem. The control function model is based on different and perhaps less intuitive moment conditions:

$$x = m(w) + v, \tag{14}$$

$$E(v|w) = 0, \tag{15}$$

$$E(u|w, v) = E(u|v), \tag{16}$$

where (15-16) are related to the validity of instruments, but are not equivalent to (13). The control function model was developed by Newey, Powell, and Vella (1999) and extended for local polynomial methods by Su and Ullah (2008). It is worth mentioning that the two estimators from (13) and (14-15-16) are from non-nested models and yield different estimation results.

We now derive an implementable procedure for this nonparametric IV estimator. Using (16) and the absence of (13) in the control function model, we start with the expectation of y , conditional on x , w and v :

$$E(y|x, w, v) = g(x) + E(u|x, w, v).$$

Given we suppose our instruments are valid:

$$\begin{aligned} &= g(x) + E(u|x - g(w), w, v), \\ &= g(x) + E(u|w, v), \\ &= g(x) + E(u|v), \end{aligned} \tag{17}$$

$$E(y|x, w, v) = \mu(x, v),$$

We can use (17) to estimate $g(x)$, by defining F_v as the CDF of v and the function:

$$\begin{aligned} g_{F_v}(x) &= \int \mu(x, v) dF_v(v) \\ &= \int g(x) + E(u|v) dF_v(v) \\ &= g(x) + c \end{aligned} \tag{18}$$

where c is a constant:

$$c = \int E(u|v) dF_v(v).$$

Therefore, we can use (17-18) to approximate $g(x)$ up to a constant. The following steps describe an implementable procedure of the nonparametric IV estimator using data from a finite sample:

Step 1. Obtain a consistent estimate of $m(w)$ in (14) using a consistent nonparametric estimator and get an estimate of v :

$$\hat{v}_i = x_i - \hat{m}(w_i).$$

Step 2. Estimate the function $\mu(\cdot)$ in the following model, using a consistent non-parametric estimator :

$$y_i = \mu(x_i, \hat{v}_i) + u_i.$$

Step 3. Get a consistent estimate of $g(x)$ with a discrete integration of \hat{v} from $\hat{\mu}(\cdot)$:

$$\hat{g}(x_i) = \frac{1}{n} \sum_{j=1}^n \hat{\mu}(x_i, \hat{v}_j). \quad (19)$$

If the local linear kernel estimator is used to estimate $\mu(\cdot)$ in step 2, then a consistent estimator of the slope of $g(x)$ is:

$$\hat{g}^{(1)}(x_i) = \frac{1}{n} \sum_{j=1}^n \hat{\mu}^{(1)}(x_i, \hat{v}_j).$$

Expression (19) consists of the discrete equivalent of (18), giving each observation z_i , $\forall i$, an equal probability over its CDF.

The control function model can also be applied to estimate a **semiparametric IV model** of the type:

$$y_i = x_i' \beta + g(z_i) + u_i, \quad (20)$$

$$z_i = w_{1i}' \gamma + m(w_{2i}) + v_1, \quad (21)$$

$$\text{for } i = 1, \dots, n,$$

where x is a set of k exogenous regressors, z is a set of potentially endogenous variables, while w_1 and w_2 are l_1 and l_2 instrumental variables, which possibly include x . Let $w = [w_1, w_2]$ and assume the same moment conditions than (15-16), required by the nonparametric IV “control function model”, then the conditional

expectation of y becomes:

$$\begin{aligned} E(y|x, z, w, v) &= x\beta + g(z) + E(u|x, z, w, v), \\ &= x\beta + g(z) + E(u|v), \\ E(y|x, z, w, v) &= \mu(x, z, v). \end{aligned}$$

Estimating $g(z)$ up to a constant, using data from a finite sample, can be achieved with the following procedure:

Step 1. Estimate (21) with a consistent estimator, such as the Robinson (1988)'s method described in section 5.2.3, and obtain an estimate of v :

$$\hat{v}_i = z_i - w'_{1i}\hat{\gamma} - \hat{m}(w_{2i}).$$

Step 2. Get an estimate of β in (20) using a mixture of the Robinson (1988)'s and Su and Ullah (2008)'s method. First, get consistent nonparametric estimates of $m_x(z_i, \hat{v}_i)$ and $m_y(z_i, \hat{v}_i)$:

$$\begin{aligned} x_i &= \hat{m}_x(z_i, \hat{v}_i) + \epsilon_{x,i}, \\ y_i &= \hat{m}_y(z_i, \hat{v}_i) + \epsilon_{y,i}. \end{aligned}$$

Then integrate out \hat{v}_i of $\hat{m}_x(x_i, \hat{v}_i)$ and $\hat{m}_y(x_i, \hat{v}_i)$, following Su and Ullah (2008):

$$\begin{aligned} \hat{m}_x(z_i) &= \frac{1}{n} \sum_{j=1}^n \hat{m}_x(z_i, \hat{v}_j), \\ \hat{m}_y(z_i) &= \frac{1}{n} \sum_{j=1}^n \hat{m}_y(z_i, \hat{v}_j). \end{aligned}$$

According to the Robinson (1988)'s procedure, define:

$$\begin{aligned} \tilde{x}_i &= x_i - \hat{m}_x(z_i), \\ \tilde{y}_i &= y_i - \hat{m}_y(z_i), \end{aligned}$$

and estimate β :

$$\hat{\beta} = (\tilde{X}'\tilde{X})\tilde{X}'\tilde{Y}.$$

Step 3. Build $\check{y}_i = y_i - x_i'\hat{\beta}$ and estimate $g(z_i, \hat{v}_i)$:

$$\check{y}_i = \hat{g}(z_i, \hat{v}_i) + u_i.$$

Following Su and Ullah (2008), integrate out \hat{v}_i of $\hat{g}(z_i, \hat{v}_i)$:

$$\hat{g}(z_i) = \frac{1}{n} \sum_{j=1}^n \hat{g}(z_i, \hat{v}_j)$$

This procedure is an extension of the partially linear model described in Su and Ullah (2008).

6 Specification testing procedure

In this section, we detail two specification tests. First, we describe the procedure of a nonparametric test of exogeneity, adapted from Blundell and Horowitz (2007). This test evaluates the necessity of the IV approach in a given nonparametric model. Secondly, we describe the procedure of a parametric model against a nonparametric alternative test, with identification through instrumental variables, adapted from Horowitz (2006). This test evaluates whether a parametric or nonparametric specification is advisable in the context of an endogenous model. We propose two procedures that are implementable in object oriented softwares, using finite sample data. One important advantage of the two specification tests consists of the lack of a nonparametric IV estimation inside their respective procedure. They are therefore not affected by the related ill-posed inverse problem, i.e., the risk of large variation of the estimates as the series approximation, required by the standard nonparametric IV estimator, expands. For practical purposes, we present simplified versions of both tests that assume a single explanatory variable x , a single instrumental variable w and the explained variable y . We use

this simplification in order to minimize the potential for “curse of dimensionality” problems, linked to the multivariate nonparametric density estimations required by both tests. As we recall from section 5.2.1, a multivariate nonparametric density estimation is determined by the product of the distances between the observations and the evaluation point, given a smoothing parameter, for all variables. Therefore, as the number of variables increases, the risk that the distance product gets affected by extreme values is greater and potentially leads to imprecise estimates.

6.1 A nonparametric test of exogeneity

6.1.1 Test statistic

We start with the definition of the model setting, where $g(\cdot)$ is an unknown nonparametric function, identified with the following moment condition:

$$E[y - g(x)|w] = 0.$$

Define the function $G(x) = E[y|x]$, such that x is exogenous if:

$$g(x) = G(x).$$

Testing the exogeneity of x is described by the following hypotheses test:

$$H_0 : E(y - G(x)|w) = 0$$

$$H_1 : E(y - G(x)|w) \neq 0.$$

x is endogenous if we reject H_0 . Intuitively, the null hypothesis represents the case for which the information brought by the instrument does not affect the conditional expectation significantly, such that the IV approach is not necessary. In the case of a rejection of the null hypothesis, the information brought by the instrument is significant, such that without it, the conditional expectation is statistically biased. Blundell and Horowitz (2007) show that we can test H_0 with a statistic that uses

the sample analog of:

$$S(x) = E\{[y - G(x)] \times f_{xw}(x, w)\},$$

with a sample average for $E\{\cdot\}$ and leave-one-out kernel estimators for $G(\cdot)$ and $f_{xw}(\cdot)$:

$$\hat{G}^{-i}(x_i) = \frac{\sum_{j=1, j \neq i}^n y_j k\left(\frac{x_i - x_j}{h}\right)}{\sum_{j=1, j \neq i}^n k\left(\frac{x_i - x_j}{h}\right)},$$

$$\hat{f}_{xw}^{-i}(x_i, w_i) = \frac{1}{nh_1 h_2} \sum_{j=1, j \neq i}^n k\left(\frac{x_i - x_j}{h_1}\right) k\left(\frac{w_i - w_j}{h_2}\right).$$

The sample analog of $S(x)$ is:

$$S_n(x_i) = \frac{1}{\sqrt{n}} \sum_{j=1}^n [y_j - \hat{G}^{-j}(x_j)] \hat{f}_{xw}^{-j}(x_i, w_j).$$

The test statistic is:

$$\tau_n = \sum_{i=1}^n S_n(x_i)^2,$$

and we reject H_0 if τ_n is larger than the critical value described in section 6.1.2.

6.1.2 Critical value

The statistic τ_n cannot be tabulated because it is not asymptotically pivotal. The critical value to which the test compares τ_n is obtained using the $1 - \alpha$ quantile of its approximated distribution under H_0 . Blundell and Horowitz (2007) show we can approximate the τ_n distribution with the following expression:

$$\hat{\tau}_n = \sum_{j=1}^L \hat{\omega}_j \chi_{(1)j}^2,$$

where $\chi_{(1)j}^2$ is a random sample of n observations from a chi-squared distribution with one degree of freedom. L determines the accuracy of the approximation and

is chosen arbitrarily. $\hat{\omega}_j$ are the eigenvalues of the matrix:

$$\hat{\Omega}_{L \times L} = \Phi' \Upsilon \Phi.$$

Υ denotes the $n \times n$ diagonal matrix with (i, i) elements being \hat{V}_i^2 , where:

$$\hat{V}_i = y_i - \hat{G}^{-i}(x_i).$$

Φ denotes the $n \times L$ matrix with (i, j) elements being:

$$\Phi_{ij} = \frac{1}{\sqrt{n}} \sum_{k=1}^L \left[\hat{d}_{jk} \phi_k(\mathcal{W}_i) - \frac{\hat{a}_{jk} \phi_k(\mathcal{X}_i)}{\hat{f}_x(x_i)} \right],$$

where $\hat{d}_{jk} = \frac{1}{n^2} \sum_{i=1}^n \sum_{J=1}^n \hat{f}_{xw}(x_i, w_J) \phi_j(\mathcal{X}_i) \phi_k(\mathcal{W}_J),$

$$\hat{a}_{jk} = \frac{1}{n^2} \sum_{i=1}^n \sum_{J=1}^n \hat{t}(x_{1i}, x_{2J}) \phi_j(\mathcal{X}_{1i}) \phi_k(\mathcal{X}_{2J}).$$

The set $\{\phi_j : j = 1, 2, \dots\}$ is a complete orthonormal basis for $L_2[0, 1]^{p+r}$, where $p \geq 1$ and $r \geq 0$. In application, such basis is satisfied by the Fourier basis series, and its implementation can be achieved with the R function *bf* from the *ldr* package. Define

$$\hat{t}(x_{1i}, x_{2J}) = \frac{1}{n} \sum_{j=1}^n \hat{f}_{xw}(x_{1i}, w_j) \hat{f}_{xw}(x_{2J}, w_j)$$

$$\mathcal{X}_i = \frac{x_i - \min(X)}{\max(X) - \min(X)}$$

$$\mathcal{W}_i = \frac{w_i - \min(W)}{\max(W) - \min(W)}.$$

6.2 Parametric model against a nonparametric alternative test, with identification through instrumental variables

6.2.1 Test statistic

We start with the model setting of the test:

$$E[y - g(x)|w] = 0,$$

where $g(\cdot)$ is an unknown nonparametric function. Testing a parametric model against a nonparametric alternative, with identification through instrumental variables, is described by the following hypothesis:

$$H_0 : E(y - G(x, \theta)|w) = 0,$$

$$H_1 : E(y - G(x, \theta)|w) \neq 0.$$

The null hypothesis, H_0 , requires that:

$$g(x) = G(x, \theta),$$

must hold for some $\theta \in \Theta$ and a known function $G(\cdot)$. In this case, the test does not reject the parametric specification in the context of an identification through instrumental variables. The alternative hypothesis, H_1 , represents the absence of $\theta \in \Theta$ that makes the last equality hold, or in other words, it suggests the rejection of the parametric for the nonparametric specification. Intuitively, the null hypothesis represents the case for which the parametric restriction does not change the conditional expectation significantly. If the null hypothesis is not rejected, we prioritize a given parametric specification because it increases the information provided to the model. However, if the parametric restriction causes a significant change in the conditional expectation, the alternative of a nonparametric specification is advisable in order to reduce the potential estimation bias. Similarly to the previous test, Horowitz (2006) shows that testing H_0 uses the sample analog

of the following expression:

$$S(x) = E\{[y - G(x, \theta)] \times f_{xw}(x, w)\},$$

with a sample average for $E\{\cdot\}$, any known parametric function for $G(\cdot)$ and a leave-one-out kernel estimator for $f_{xw}(\cdot)$:

$$\hat{f}_{xw}^{-i}(x_i, w_i) = \frac{1}{nh_1h_2} \sum_{j=1, j \neq i}^n k\left(\frac{x_i - x_j}{h_1}\right)k\left(\frac{w_i - w_j}{h_2}\right).$$

The sample analog of $S(x)$ is:

$$S_n(x_i) = \frac{1}{\sqrt{n}} \sum_{j=1}^n [y_j - \hat{G}(x_j, \hat{\theta})] \hat{f}_{xw}^{-j}(x_i, w_j).$$

The test statistic is:

$$\tau_n = \sum_{i=1}^n S_n(x_i)^2,$$

and we reject H_0 if τ_n is large.

6.2.2 Critical value

τ_n is not asymptotically pivotal and obtaining a critical value for the test requires to approximate its distribution under H_0 . Horowitz (2006) shows the approximated critical value of test is the $1 - \alpha$ quantile of the following approximated distribution of τ_n under H_0 :

$$\hat{\tau}_n = \sum_{j=1}^L \hat{\omega}_j \chi_{(1)j}^2,$$

where $\chi_{(1)j}^2$ is a random sample of n observations from a chi-squared distribution with one degree of freedom. L determines the accuracy of the approximation and

is chosen arbitrarily. $\hat{\omega}_j$ are the eigenvalues of the following matrix:

$$\Omega_{L \times L} = D\Phi M \Upsilon M' \Phi' D'.$$

Υ denotes the $n \times n$ diagonal matrix with (i, i) elements being \hat{V}_i^2 , where:

$$\hat{V}_i = y_i - \hat{G}(x_i, \hat{\theta}).$$

M denotes the $n \times n$ matrix:

$$M = I_n - \frac{1}{n} \hat{G}_\theta \tilde{\gamma} \tilde{W}',$$

where, I_n is the $n \times n$ identity matrix,

$$\hat{G}_\theta \text{ is the } n \times d \text{ matrix : } \frac{\partial G(X, \theta)}{\partial \theta},$$

$$\tilde{W} \text{ is the } n \times c_\theta \text{ matrix : } H(W),$$

$$\tilde{\gamma} \text{ is the } d \times c_\theta \text{ matrix : } (\hat{D}' A \hat{D})^{-1} \hat{D}' A.$$

d is defined as the number of parameters in $\{\theta : \theta_1, \dots, \theta_d\}$. $H(\cdot)$ is a known vector valued function with independent components of dimension c_θ , again chosen arbitrarily for a higher precision scale. Applying such a vector valued function can be achieved with Legendre polynomials of order c_θ using the R function *legendre.polynomials* from the *orthopolynom* package. A is a $c_\theta \times c_\theta$ stochastic matrix, implementable with the R function *eigeninv* from the *eigeninv* package. \hat{D} is the $c_\theta \times d$ matrix:

$$\hat{D} = \frac{1}{n} \sum_{i=1}^n \tilde{w}_i' \hat{G}_{\theta i}.$$

D denotes the $L \times L$ matrix with (j, k) elements being:

$$\hat{d}_{jk} = \frac{1}{n^2} \sum_{i=1}^n \sum_{J=1}^n \hat{f}_{xw}(x_i, w_J) \phi_j(\mathcal{X}_i) \phi_k(\mathcal{W}_J),$$

where $\mathcal{X}_i = \frac{x_i - \min(X)}{\max(X) - \min(X)}$,

$$\mathcal{W}_i = \frac{w_i - \min(W)}{\max(W) - \min(W)}.$$

The set $\{\phi_j : j = 1, 2, \dots\}$ is a complete orthonormal basis for $L_2[0, 1]^{p+r}$, where $p \geq 1$ and $r \geq 0$. In application, such basis is satisfied by the Fourier basis series, and is implementable with the R function *bf* from the *ldr* package.

Φ denotes the $L \times n$ matrix with (j, i) elements being $\phi_j(w_i)$.

7 Estimation results

In this section, we outline the estimation results of the debt/GDP-growth relationship for the OLS, IV, semiparametric and semiparametric IV models, covering different country and time samples (OECD, OECD-excluding Japan, EU and PIIGS; 1971-2007/2010). We also report the results of the nonparametric test of exogeneity and the parametric against a nonparametric alternative test, with identification through instrumental variables.

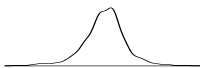


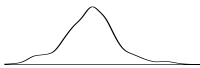
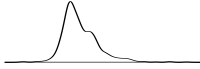
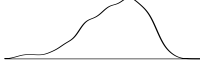
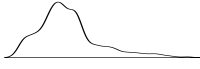
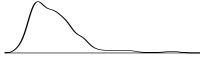
7.1 Linear and IV model estimates

Table 2 reports the linear (A) and IV (B) model estimates, as described in section 5.1, obtained by the standard OLS and 2SLS estimators respectively, for the 23 pooled OECD countries over the period 1971-2010. The variable summary statistics appear on the right section of table 2. The coefficient t-tests for both model estimations use HAC standard errors in order to correct for potential error heterogeneity and autocorrelation. The 2SLS estimator uses the two first lagged levels of the $debt/GDP_{i,t}$ variable as instruments in the first step regression, in addition to the other explanatory variables ($Z_{i,t} = \{debt/GDP_{i,t-1}, debt/GDP_{i,t-2}, X_{i,t}\}$). We restrict the number of lagged levels, as suggested by Roodman (2009), in order

to reduce the risk of weak instruments and keep more observations. The OLS estimate of the linear (A) model results in a significant (5% level) and negative debt/GDP-growth effect, such that an increase of 10 percentage points in the debt-to-GDP ratio is associated with a 7 basis point reduction in the succeeding medium/long term economic growth level. When we instrument the debt-to-GDP variable with its two first lagged levels, the effect of a 10 percentage point increase of the government debt ratio on growth rate changes to a 6 basis point reduction and is almost significant at the 5% level, with a t-test (HAC) p-value of 0.066. Accordingly, we do not observe a drastic change in the estimation of the debt/GDP-growth effect when instrumenting the debt-to-GDP variable in a linear parametric model framework. This result is in line with the findings of previous papers, in which the debt-to-GDP variable is instrumented with its lagged levels or first difference. Most control variables ($X_{i,t}$) in both estimated models have expected signs, with the exception of the average year of total schooling variable (*school*), which obtains a negative and significant coefficient, a contrasting result in comparison to the empirical literature. Intuitively, this difference can partly be explained by the distinction between the specification of our economic growth variable, which is based on an aggregated GDP measure, and that of Cecchetti, Mohanty, and Zampolli (2011) and Panizza and Presbitero (2014), which consists of a GDP per capita measure. One of the principal information the average year of total schooling variable (*school*) provides is the shared access to education across the population of a given country. What this measure reflects may be more closely related to income distribution than productivity. High levels of aggregated growth, sustained by strong productivity, do not necessarily imply an equal distribution of revenues across the population. Therefore, we partly justify our estimation result on the basis that the average year of total schooling variable is linked with a distinctive characteristic of our growth variable specification, in comparison to both articles. The estimated coefficients of the log of real GDP (*-RGDP*), the trade openness as % of GDP (*trade*) and the age dependency ratio (*dep*) are all significant at the 5% significance level, with coefficient signs that follow Cecchetti, Mohanty, and Zampolli (2011) and Panizza and Presbitero (2014). We find a negative effect of the annual population growth rate (*pop*) variable on long term growth, with varying significance levels according to the models. This coefficient

sign follows Cecchetti, Mohanty, and Zampolli (2011)'s results, which the authors justify as being in accordance with the growth theory. On the contrary, Panizza and Presbitero (2014) find a positive effect of the population growth variable. The different estimation results from the empirical literature suggest that the effect of the population growth variable, through this specific model specification, is sensitive to the selected data samples. The estimated coefficients of the countries and time specific dummy variables are not reported. Figure 2 reports the bivariate partial residual plots of both estimated models, with the variable representing the partial residuals ($growth - X\hat{\beta}$) on the y-axis and the debt-to-GDP variable ($debt/GDP$) on the x-axis. The black line represents the fitted linear effect of the debt-to-GDP variable on medium/long-term growth, such that its slope equals the estimated coefficient value of the $debt/GDP$ variable.

Table 2 – Linear and IV model estimates

| | Coefficient estimates | | Summary statistics | | |
|------------------------|-----------------------|-----------------------|--------------------|-------|---|
| | Linear (A) | IV (B) | Mean | SD | Density |
| Dependent variable: | | | | | |
| growth | | | 2.71 | 2.68 |  |
| Independent variables: | | | | | |
| debt/GDP | -0.0071** (0.003) | -0.0058* (0.003) | 45.85 | 29.77 |  |
| -RGDP | 5.6420*** (0.551) | 5.8026*** (0.593) | 12.67 | 1.51 |  |
| ngs | 0.0037 (0.019) | -0.0010 (0.020) | 23.35 | 5.25 |  |
| pop | -0.2545** (0.119) | -0.2388* (0.135) | 0.69 | 0.63 |  |
| school | -0.2980*** (0.067) | -0.3349*** (0.070) | 9.04 | 2.07 |  |
| openc | 0.0350*** (0.006) | 0.0361*** (0.007) | 62.93 | 30.68 |  |
| dep | -0.0637*** (0.015) | -0.0746*** (0.016) | 52.95 | 6.57 |  |
| crisis=1 | -0.1399 (0.121) | -0.1533 (0.121) | | | |

Notes: See section 4 for variable description. The left side of the table reports the linear (A) and IV (B) model regression coefficients and their HAC standard errors in brackets, computed with the R function *vcovHAC* from the *sandwich* package. On the right side, the sample average, standard deviation and kernel density distribution statistics of the variables are reported.

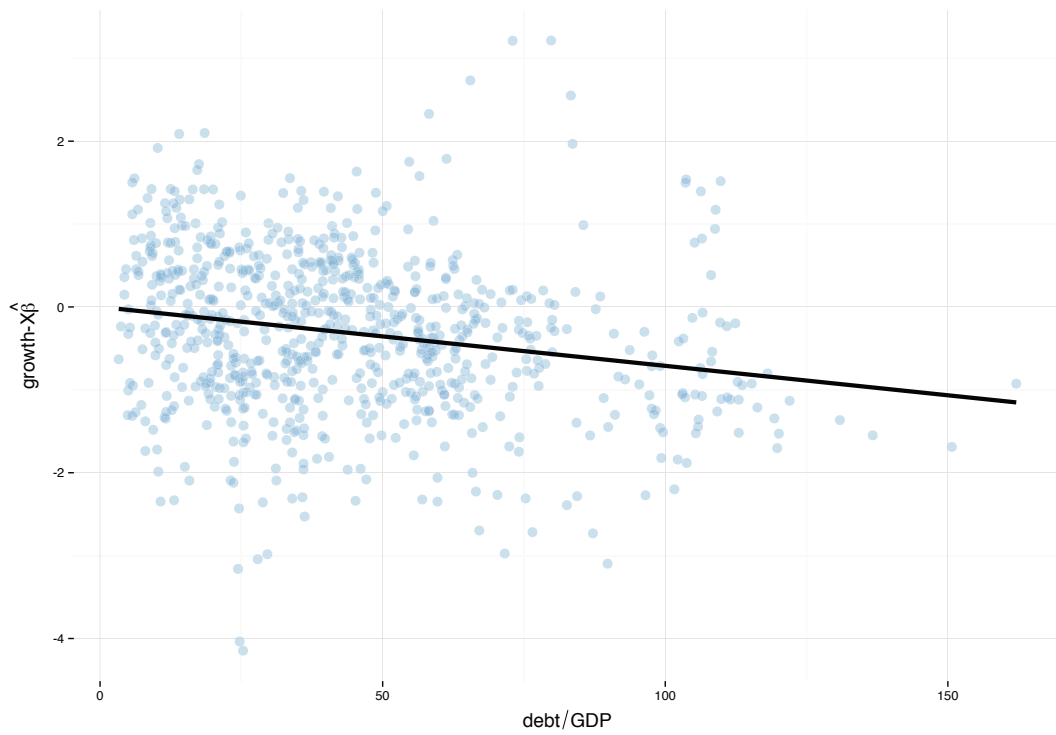
* significant at 10%.

** significant at 5%.

*** significant at 1%.

Figure 2 – Debt/GDP-growth relationship for 23 OECD countries, 1971-2010

(a) Linear model



(b) IV model



7.2 Semiparametric and Semiparametric IV model estimates

Figure 3 and 4 report the semiparametric and semiparametric IV estimates of the debt-growth relationship for the complete time sample (1971-2010) and a shorter sample that excludes the great recession (1971-2007). The estimated model specifications are based on section 5.1. In the semiparametric IV model, the $debt/GDP_{i,t}$ variable is instrumented semiparametrically, such that its two first lagged levels are specified nonparametrically ($Z_{1i,t} = \{debt/GDP_{i,t-1}, debt/GDP_{i,t-2}\}$) and the other explanatory variables linearly ($Z_{2i,t} = X_{i,t}$). Each figure contains six graphics: (a)-(b) Semiparametric - Semiparametric IV model for OECD countries, excluding Japan; (c) SP IV model for EU countries; (d) SP IV model for PIIGS countries; (e) SP IV model for OECD countries, excluding PIIGS; (f) SP IV model for EU countries, excluding PIIGS. We exclude Japan from our country sample because its debt-to-GDP observations consist of outliers and are highly sparsed, resulting in overfitted nonparametric estimates. The selection of the EU sub-group of countries is based on the methodological approach of Checherita-Westphal and Rother (2012). The PIIGS sub-group represents a cluster of European countries that experienced particular economic hardship and sovereign-debt puzzles during the last two decades. Additionally, its members are identified to have limited fiscal space according to Ghosh et al. (2013)'s findings, two elements suggesting the PIIGS countries potentially have higher debt intolerance levels.

In figure 3, the SP (a) and SP IV (b) model estimates of the debt/GDP-growth relationship for the OECD countries present similar characteristics. Both suggest an almost linear and slightly upward trend in growth levels for debt-to-GDP ratios between 0-65%, with a small hump shaped effect in the lower 0-20% range. Past the 65% level of government indebtedness, the debt/GDP-growth relationship estimation is characterized by complex non-linearities, expressed through two wave like effects. The two model estimations show no evidence of a threshold effect in the relationship. High debt-to-GDP ratios are associated with similar or slightly lower growth rates than moderate indebtedness levels. Instrumenting the debt-to-GDP variable with its lagged levels reduces the spread of the non-linear effect and confirms the absence of threshold or inverted-U effect, with a minimum-maximum

growth rate spread of almost 1 percentage point.

Figure 3 (c) reports the SP IV debt/GDP-growth estimation for the 15 EU countries sub-group and suggests stronger non-linearities in the relationship for debt-to-GDP levels above the 60% level. In fact, the non-linear growth spread is more than 3.5 percentage points inside the 78-120% debt-to-GDP range, providing some evidence of a post-78% threshold effect. However, the tipping point of this non-linear pattern (78%) is associated with higher growth levels of more than 1 percentage point than moderate debt-to-GDP ratios. Therefore, this threshold effect rather suggests an inverted U-shaped functional form, instead of a unique fall in growth rates for highly indebted countries. In fact, the observations at right tail of the threshold effect (high debt-to-GDP ratios) are only linked with a growth rate reduction of around 1 percentage point in comparison to low or average debt levels.

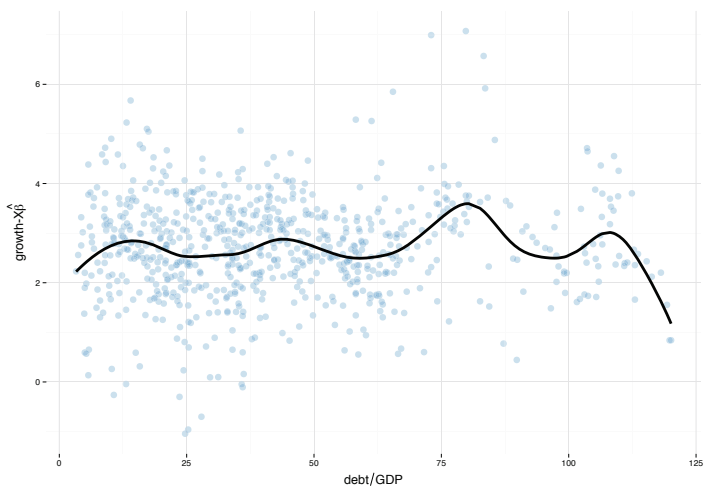
The analysis becomes truly interesting in figure 3 (d), where the SP IV estimation of the debt/GDP-growth relationship for the PIIGS subgroup of countries provides strong and surprisingly clear evidence of an inverted-U shaped effect. The minimum-maximum growth spread of the relationship is a little more than 3 percentage points, with an upward and almost linear trend in the debt-to-GDP range of 0-78%, followed by a clearly negative debt/GDP-growth effect in the post-78% region. The underlying non-linearities that characterize the estimated debt/GDP-growth functional form, specific to the PIIGS sub-group of countries, describes surprisingly well the non-linear patterns of the SP IV (b)-(c) estimates for the 22 OECD and 15 EU country samples. In fact, the components of both non-linear characterizations consist of a first hump effect, with a tipping point around the 78% debt level, and a second lower wave like effect around the 107% debt-to-GDP level. Based on this analysis, we propose that most of the non-linear effect of the debt/GDP-growth relationship for the 22 OECD country sample could be attributed to the PIIGS sub-group of countries.

In an attempt to test this last assumption, figures 3 (e) and (f) report the SP IV model estimations of the OECD and EU country samples that exclude the PIIGS sub-group. Convincingly, most of the non-linear pattern is absent from both estimated functional forms. The model estimation of the OECD sample, PIIGS excluded, suggests a very slight U-shaped and almost constant relationship, with

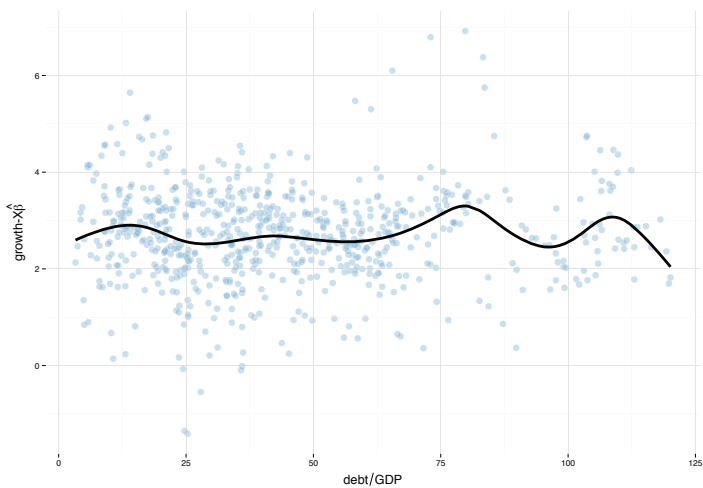
a growth spread of less than 0.5 percentage point for the entire debt-to-GDP domain. The estimate for the EU sample, PIIGS excluded, finds a linear and slightly positive trend, such that the non-linear pattern is now completely absent. The exclusion of the PIIGS sub-group of countries from the OECD and EU samples confirms our assumption that most of the non-linear effect in the debt/GDP-growth relationship can be explained by a specific sub-group of countries that share common characteristics, in this case, a potentially higher debt intolerance level. Our results are robust to the exclusion of the great recession in the time sample (see figure 4) and to the inclusion of Iceland in the PIIGS sub-group (PIIIGS). The estimated semiparametric models for the EU, PIIGS, OECD (PIIGS excluded) and EU (PIIGS excluded) are reported in figures 5 and 6 of the Annex section, for the 1971-2007/2010 periods respectively.

Figure 3 – Debt/GDP-growth relationship : OECD-excluding Japan / EU / PIIGS countries (1971-2010)

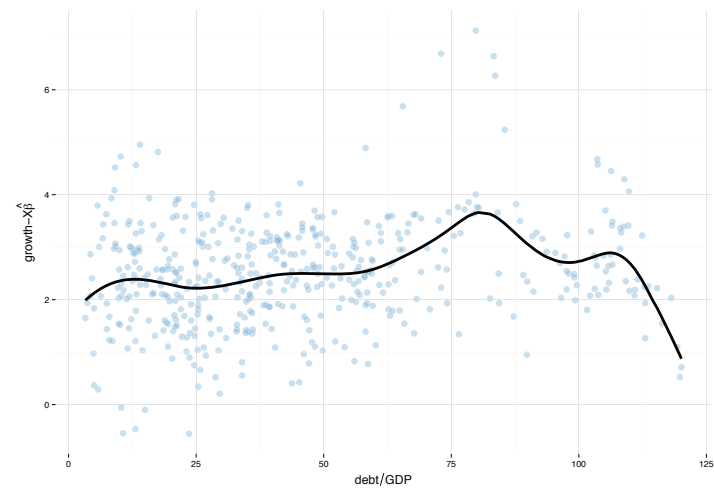
(a) Semiparametric OECD - excluding Japan



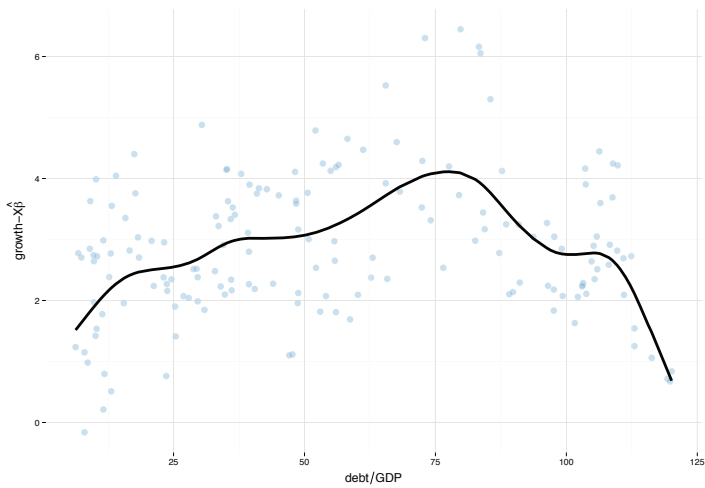
(b) Semiparametric IV OECD - excluding Japan



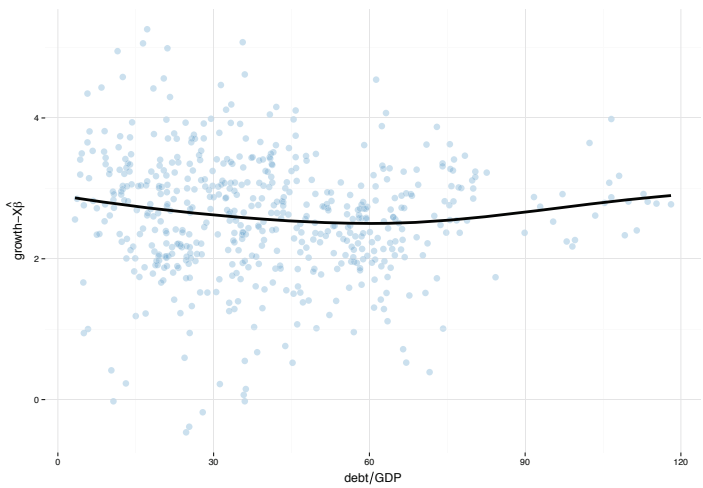
(c) Semiparametric IV EU



(d) Semiparametric IV PIIGS



(e) Semiparametric IV OECD - excluding Japan and PIIGS



(f) Semiparametric IV EU - excluding PIIGS

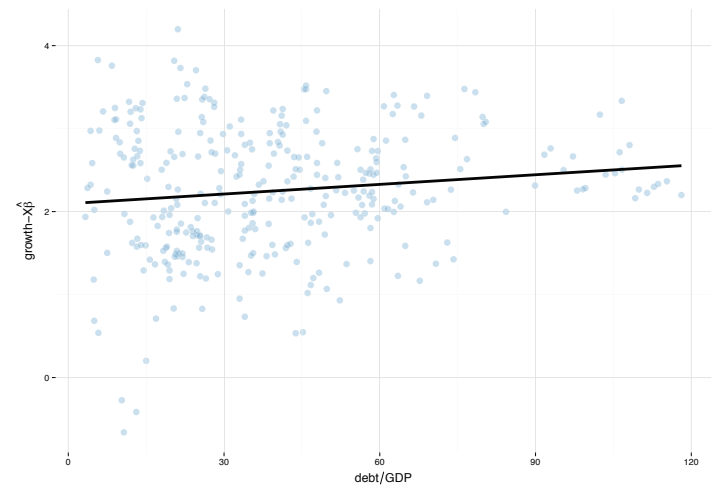
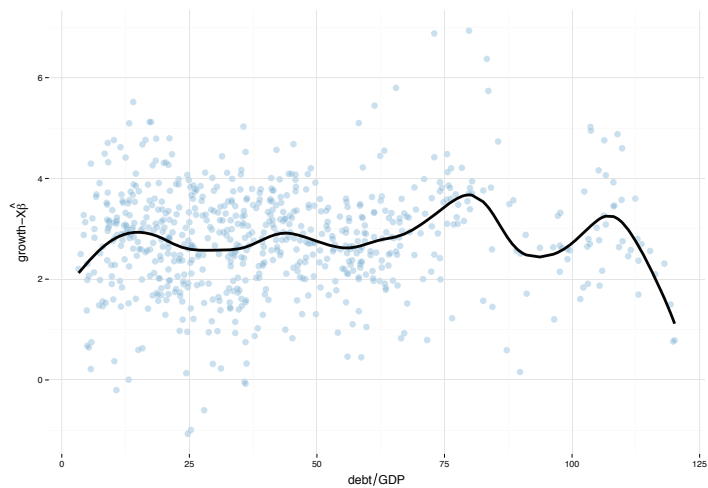
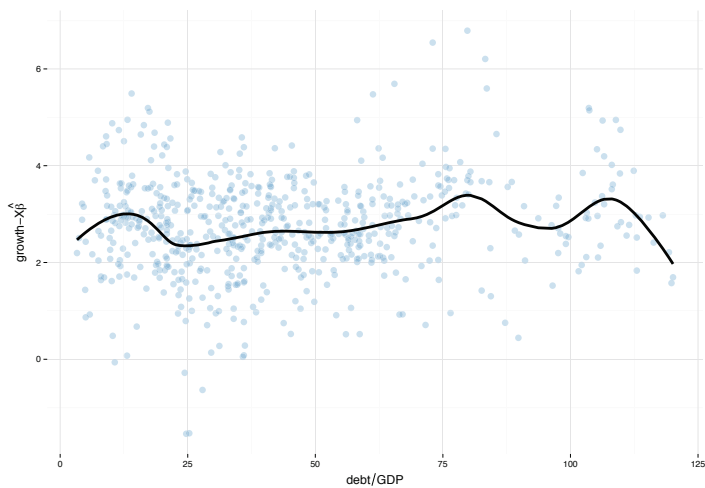


Figure 4 – Debt/GDP-growth relationship : OECD-excluding Japan / EU / PIIGS countries (1971-2007)

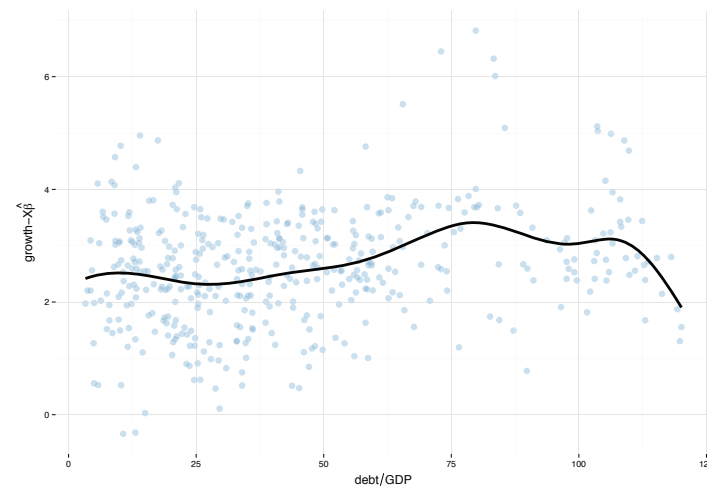
(a) Semiparametric OECD - excluding Japan



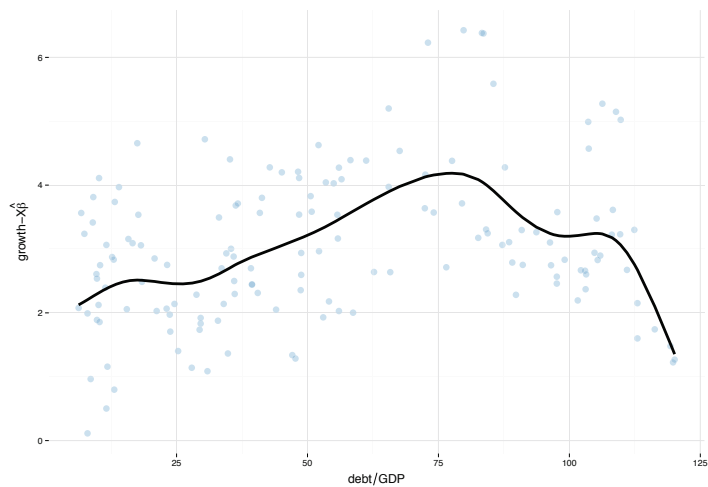
(b) Semiparametric IV OECD - excluding Japan



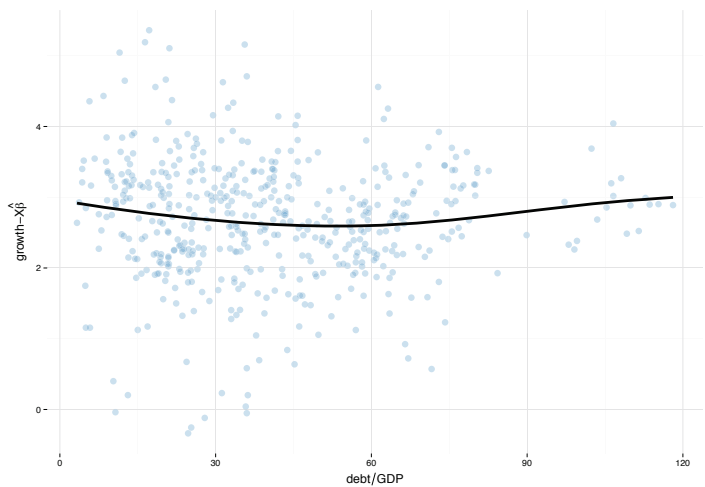
(c) Semiparametric IV EU



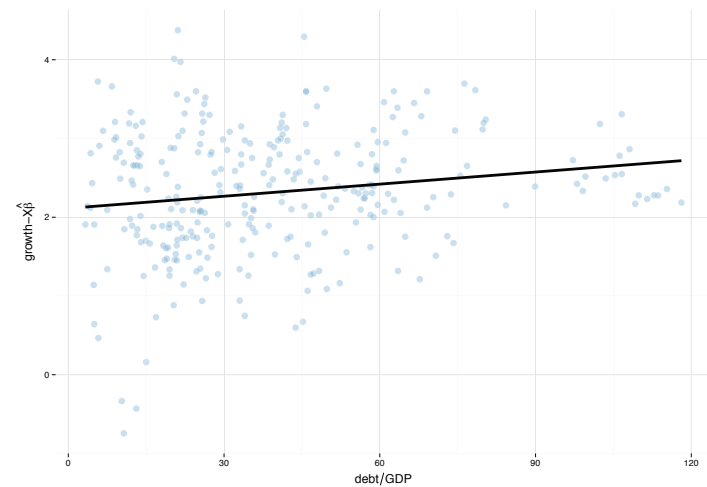
(d) Semiparametric IV PIIGS



(e) Semiparametric IV OECD - excluding Japan and PIIGS



(f) Semiparametric IV EU - excluding PIIGS



7.3 Specification test results

Table 3 reports the results of the nonparametric test of exogeneity and the parametric model against a nonparametric alternative test, as described in section 6. We specify the variables included in the testing procedure as:

$$\begin{aligned} y &= \overline{growth}_{i,t+1,t+6} - X_{i,t}\hat{\beta}_{OLS}, \\ x &= \text{debt}/GDP_{i,t}, \\ z &= \text{debt}/GDP_{i,t-1}, \end{aligned}$$

where $\hat{\beta}_{OLS}$ is the vector of OLS estimated coefficients of the set of control variables $X_{i,t}$ from the linear model (A). The dependent variable used throughout the test is generated by subtracting the estimated linear effect of $X_{i,t}\hat{\beta}_{OLS}$ from $\overline{growth}_{i,t+1,t+6}$, in order to reduce the potential for omitted variable bias and minimize potential curse of dimensionality problems associated with multivariate nonparametric estimations. The principal drawback of this methodological approach consist of linearly specifying the debt-to-GDP variable in the estimation of model (A), a necessary step for the generation of y . The precision parameter of the critical value (L), used by both tests, and the number of Legendre polynomials, required by the nonparametric alternative test (c_θ), were arbitrarily set to 20. Different precision parameters produced similar results. The two columns of table 3 report the test results for the complete 23 OECD countries and a second sample that excludes Japan.

The first line of table 3 describes the results of the nonparametric test of exogeneity. We recall the hypotheses of this test are:

$$\begin{aligned} H_0 &: E(y - G(x)|w) = 0, \\ H_1 &: E(y - G(x)|w) \neq 0, \end{aligned}$$

where $G(x) = E[y|x]$. The exogeneity of x requires that $G(x) = g(x)$, where $g(x)$ is any nonparametric function identified with the following condition:

$$E(y - g(x)|w) = 0.$$

Following section 6.1 and using the first lagged level of the debt-to-GDP variable as instrument, the test for both country samples rejects the null hypothesis representing the exogeneity of x , at a 5% significance level, and suggests an IV specification in the context of a nonparametric estimation of the debt/GDP-growth relationship.

The second and third line of table 3 report the results of the parametric against a nonparametric alternative test, with identification through instrumental variables. The test hypotheses are:

$$H_0 : E(y - G(x, \theta)|z) = 0,$$

$$H_1 : E(y - G(x, \theta)|z) \neq 0.$$

On the second line of table 3, the test assumes that $G(x, \theta) = x\theta$, such that H_0 assumes a linear parametric model specification. On the third line of table 3, H_0 defines $G(x, \theta) = x\xi_1 + x^2\xi_2$, in order to test a quadratic specification in the regressors against a nonparametric alternative. We recall that choosing a parametric specification against a nonparametric alternative requires that $G(x, \theta) = g(x)$, where $g(x)$ is any nonparametric function identified through:

$$E(y - g(x)|w) = 0.$$

Following section 6.2 and using the first lagged level of the debt-to-GDP variable as instrument, the test rejects a linear specification or quadratic effect in the regressors, at a 5% significance level. It suggests the alternative of a nonparametric specification of the debt/GDP-growth relationship, with identification through instrumental variables. This result is robust to both country samples.

Table 3 – Specification test results

| Specification tests | OECD | | OECD - excluding Japan | |
|----------------------------|----------|-------------|------------------------|-------------|
| | Stat. | Crit. (95%) | Stat. | Crit. (95%) |
| Exogeneity | 0.000041 | 0.0000004 | 0.000035 | 0.0000002 |
| Linear vs Nonparametric | 0.00052 | 0.00031 | 0.00053 | 0.00019 |
| Quadratic vs Nonparametric | 0.00052 | 0.00031 | 0.00054 | 0.00018 |

8 Discussion

The semiparametric IV estimations of the debt/GDP-growth relationship for OECD countries provide results that bind the contrasting conclusions from the theoretical and empirical literatures. First of all, like most of the reviewed empirical works, our results provide strong evidence regarding the presence of non-linearities in the functional form of the debt/GDP-growth relationship for the pooled sample of advanced countries. Following the conclusion of Minea and Parent (2012), our results show that this effect is characterized by a complex non-linear pattern and do not suggest a unique debt/GDP-threshold turning point common to all advanced countries. Secondly, in regard to C. Reinhart and Rogoff (2010b)'s findings, we confirm that high debt-to-GDP levels ($>78\%$) are associated with rapidly declining growth outcomes, a conclusion that holds exclusively for the PIIGS sub-group of countries. However, even for the PIIGS countries, this relationship still cannot be characterized by a unique fall of growth rates when the debt-to-GDP level reaches a given threshold. In fact, it is also accompanied by a positive growth effect for government debt ratios inside the 0-78% range, such that the absence of a debt/GDP-growth link for low to moderate indebtedness levels, as reported by C. Reinhart and Rogoff (2010b), is not confirmed. Instead, the debt/GDP-growth functional form for the PIIGS countries is best described as inverted U-shaped, closer to Checherita-Westphal and Rother (2012)'s conclusion.

Thirdly, our findings also grasp the theoretical views of Cochrane (2011), who supports the absence of a debt/GDP-growth link for countries in which agents are convinced that the government has the ability to pay off its debt in the future. Accordingly, the impact of higher debt-to-GDP levels on economic growth is in-

significant for countries identified to have low debt intolerance measures, a notion adapted from C. Reinhart and Rogoff (2003). Moreover, the *Institutional Investor ratings* of the PIIGS sub-group of countries are lower than most covered OECD countries (with the exception of Turkey and New-Zeland), suggesting relatively higher debt intolerance levels and lower confidence in the government fiscal management capacities (see table 5 of the Annex section). In an attempt to empirically test the theoretical view of Cochrane (2011), our relationship estimates show that if we exclude the PIIGS sub-group of countries from the OECD and EU sample, the functional form of the estimated debt/GDP-growth relationship changes drastically. The functional form of the advanced country sample, excluding the PIIGS sub-group, is rather described as slightly U-shaped or even linear and positive. The complex non-linear pattern identified for the completed sample of advanced countries vanishes almost completely. This suggests a constant or even positive debt/GDP-growth functional form for advanced countries with lower debt intolerance levels, what links Panizza and Presbitero (2014)'s study, which also identifies a positive and linear relationship when the public debt variable is instrumented. Additionally, we observe that the complex non-linear pattern contained in the relationship estimates for the 22 pooled OECD countries is characterized almost entirely by the functional form specific to the PIIGS sample.

In all, our results provide strong evidence for cross-country heterogeneity in the functional form of the debt/GDP-growth relationship. Our estimates for the different country samples provide divergent results, each corroborating with a specific theoretical or empirical proposal of the covered literature. We conclude that the effect of debt-to-GDP levels on medium/long-term growth tends to differ wildly across sub-groups of advanced countries with divergent debt intolerance measures. The identification of a strong heterogenous debt/GDP-growth relationship follows several studies of the literature. Principally, our conclusions are conceptually in accordance with Kourtellos, Stengos, and Tan (2013)'s study, which identifies a contrasting debt/GDP-growth effect when comparing low and high democracy countries, and Panizza and Presbitero (2013)'s extensive survey.

We are confident that the semiparametric IV estimator is an appropriate statistical tool for estimating and describing the debt/GDP-growth functional form. First, as mentioned in previous sections, the empirical investigation of this rela-

tionship requires particular focus on the potential for reverse causality and simultaneity bias, as well as for the control of omitted variables. This justifies the IV aspect of our estimator, which is further confirmed by the rejection of the nonparametric test of exogeneity. According to this test, in the context of a nonparametric specification, the IV approach is advisable even if the model specification already reduces reverse causality and simultaneity bias with growth predetermined variables. Secondly, the theoretical justifications and empirical evidences of a threshold or inverted U-shaped relationship advise an estimation strategy that is flexible enough to depict potential non-linear patterns. The rejection of a linear parametric specification or a quadratic effect in the regressors against a nonparametric alternative test, with identification through instrumental variables, confirms that the semiparametric IV estimator is suitable. The contrasting estimates from the linear/IV and the semiparametric IV models reasserts how the choice of a given estimator or statistical tool can affect the diagnostics of any empirical work.

Finally, we think our results and conclusions bring some clarification in understanding how some countries easily manage high debt-to-GDP levels, while comparable or even lower debt ratios imply harsh economic outcomes for others. Yet, we were not able to identify the specific functional forms that given fiscal management strategies imply. Based on our analysis, we cannot conclude that rolling the debt over, or other fiscal channel leading to higher debt-to-GDP ratios, necessarily imply worst economic outcomes for the PIIGS sub-group compared to other OECD countries. However, our estimation results can be linked with Ghosh et al. (2013)'s identification of a group of countries with limited fiscal space, which is almost identical to the PIIGS/PIIIGS sub-group. This suggests that countries with lower fiscal space experience rapidly decreasing economic growth in consequence of high debt-to-GDP ratios.

We provide evidence supporting strong heterogeneity in the debt/GDP-growth relationship across OECD countries, which invalidates the view that a unique debt threshold can dictate the fiscal health of all advanced economies. Our opinion follows the view that fiscal decisions should result of case by case studies, or alternatively based on the analysis of homogenous sub-groups of advanced countries. We think that measures similar to Ghosh et al. (2013)'s fiscal space have superior potential in guiding fiscal policy than common thresholds based on pooled

analyses. We hope our findings provide a deeper understanding of the risk high debt-to-GDP levels imply on economic growth in OECD countries and help resolve the actual public debt crisis raging in Greece and in the eurozone.

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9 Annex (p.69-73)

Table 4 – Summary table of the covered literature (formal model or method required)

| | Measure | Type | Model or method | Growth effect |
|--------------------------|----------|-------------|------------------------------------|-------------------------------|
| Diamond (1965) | Debt | Theoretical | Neoclassical growth model | Positive (conditional) |
| Blanchard (1985) | Debt | Theoretical | Neoclassical growth model | Positive (conditional) |
| Saint-Paul (1992) | Debt | Theoretical | Endogenous growth model | Negative |
| R.R. (2010) | Debt/GDP | Empirical | Descriptive method | Threshold (90%) |
| Woo and Kumar (2010) | Debt/GDP | Empirical | System GMM method | Negative |
| Cecchetti et al. (2011) | Debt/GDP | Empirical | OLS method | Negative |
| Minea and Parent (2012) | Debt/GDP | Empirical | Panel smooth threshold regression | Complex non-linear |
| Égert (2012) | Debt/GDP | Empirical | Non-dynamic threshold regression | Thresholds (20.38-55.35%) |
| Checherita et al. (2012) | Debt/GDP | Empirical | 2SLS and GMM methods | Inverted U-shaped |
| Checherita et al. (2012) | Debt/GDP | Theo./Emp. | Endogenous growth model | Inverted U-shaped |
| Greiner (2012) | Debt/GDP | Theo./Emp. | Endogenous growth model | Negative monotonic |
| Afonso et al. (2013) | Debt/GDP | Empirical | Non-dynamic threshold regression | Threshold (59%) |
| Baum et al. (2013) | Debt/GDP | Empirical | Dynamic threshold regression (IV) | Thresholds (67% and 95%) |
| Herndon et al. (2013) | Debt/GDP | Empirical | Descriptive and NP method | Negative and non-linear |
| Ghosh et al. (2013) | Debt/GDP | Theo./Emp. | Stochastic model sovereign default | According to fiscal space |
| Kourtellos et al. (2013) | Debt/GDP | Empirical | Structural threshold regression | According to democracy levels |
| Teles et al. (2014) | Debt/GDP | Theo./Emp. | Endogenous growth model | Positive (conditional) |
| Panizza et al. (2014) | Debt/GDP | Empirical | OLS and 2SLS methods | Positive (IV) |

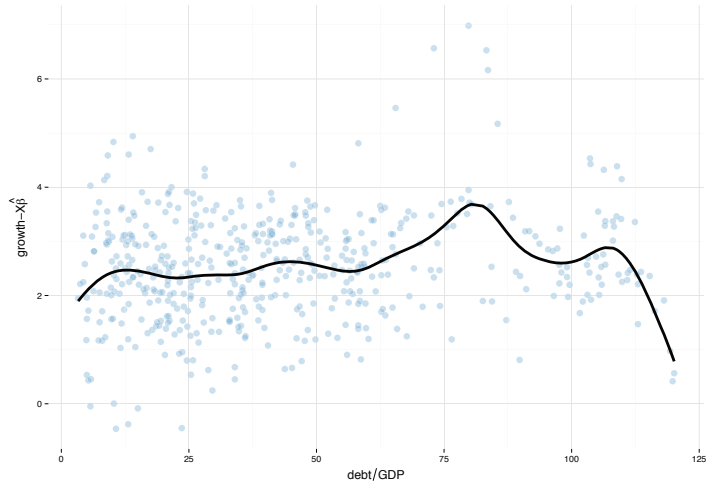
Table 5 – Average Institutional Investor rating (1979-2002) for 15 OECD countries

| Country | Average Institutional Investor rating (1979-2002) |
|---------------|---|
| Unites States | 92.8 |
| Japan | 92.5 |
| Canada | 86.0 |
| Norway | 84.3 |
| Sweden | 79.7 |
| Australia | 77.3 |
| Finland | 77.2 |
| Denmark | 76.9 |
| Italy | 76.4 |
| Spain | 73.8 |
| Ireland | 71.4 |
| New Zealand | 70.7 |
| Portugal | 63.3 |
| Greece | 54.5 |
| Turkey | 34.9 |

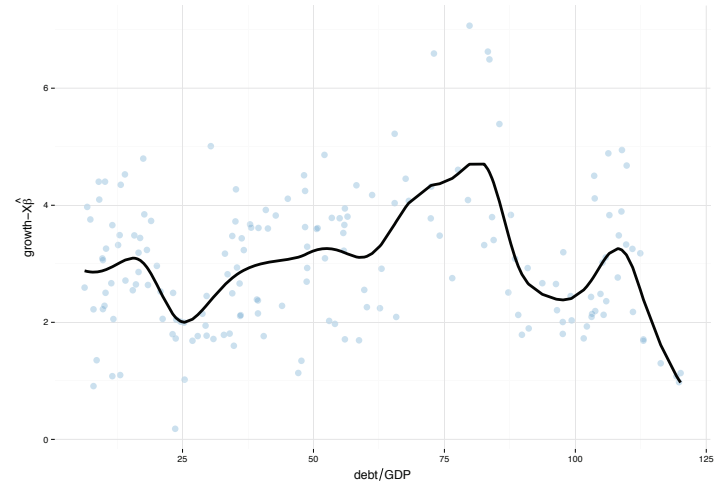
Source: *Institutional Investor*, various issues. Replicated from C. Reinhart and Rogoff (2003).

Figure 5 – Debt-growth relationship : EU / PIIGS countries (1971-2010)

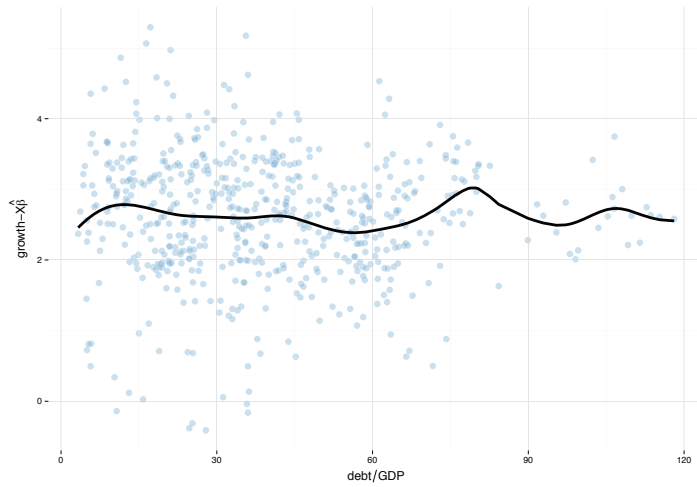
(a) Semiparametric EU



(b) Semiparametric PIIGS



(c) Semiparametric OECD - excluding Japan and PIIGS



(d) Semiparametric EU - excluding PIIGS

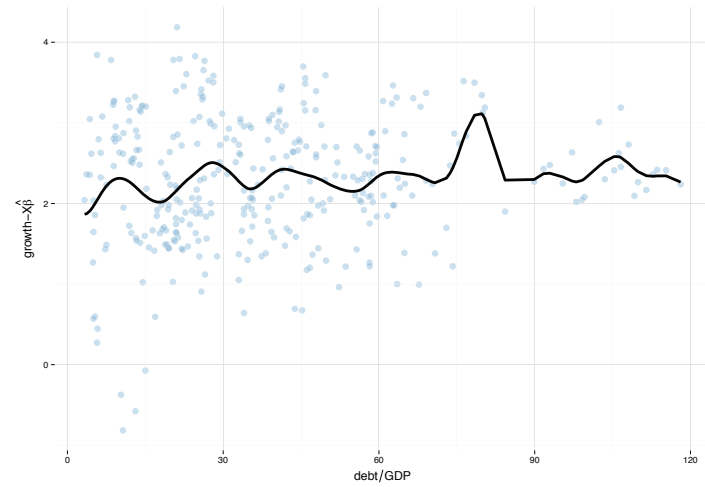
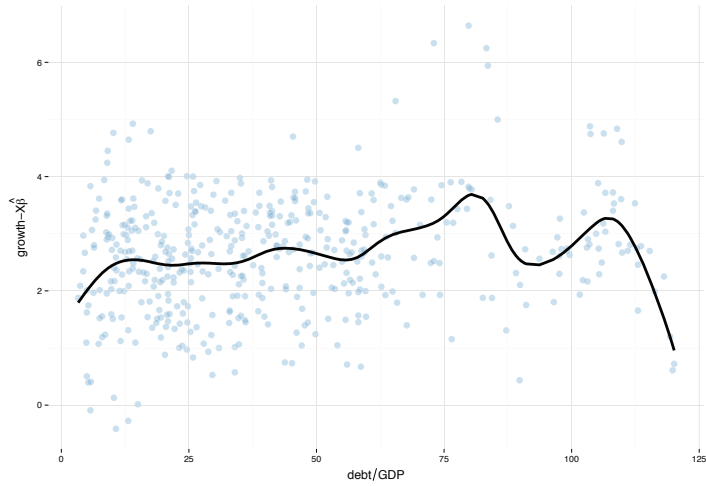
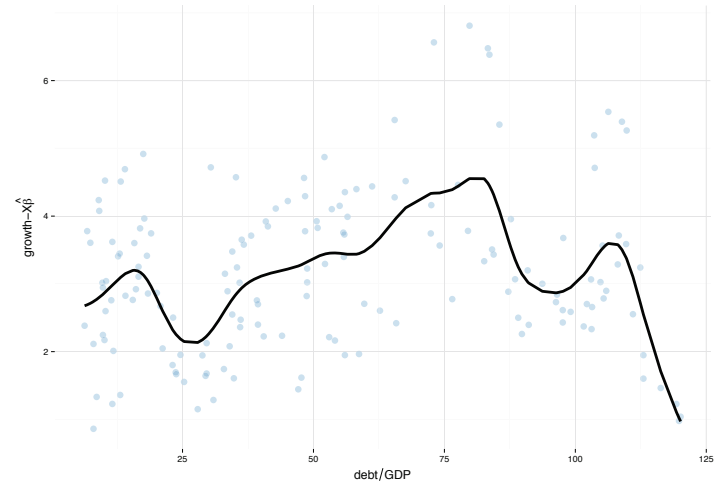


Figure 6 – Debt-growth relationship : EU / PIIGS countries (1971-2007)

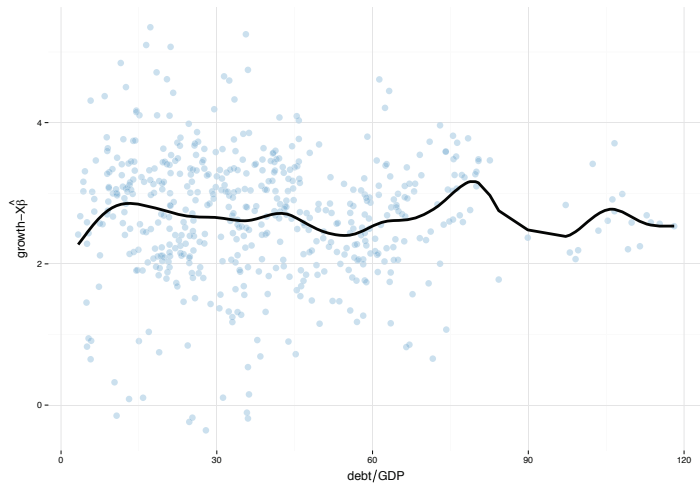
(a) Semiparametric EU



(b) Semiparametric PIIGS



(c) Semiparametric OECD - excluding Japan and PIIGS



(d) Semiparametric EU - excluding PIIGS

