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ABSTRACT

A common mining practice involves leaving rock "pillars" between mine openings to maintain the stability of the openings. A great deal of work has been done on the stability of coal pillars however, relatively little work has been done on the stability of hard-rock pillars. The work that has been done on hard-rock pillars is based on empirical studies, and most of the pillars in these empirical studies are rib pillars. The problem with empirical studies is that they are generally site specific and pillar failures must occur before an empirical strength curve can be calibrated to a particular mine site. This thesis presents a theoretical framework for assessing the stability of hard-rock mine pillars using rock mass strength and numerical modelling. The framework is based on the concept that the strength of hard brittle rocks is fundamentally controlled by a "cohesion loss" process.

This thesis shows, through the use of elastic numerical models, that by using traditional $m$ and $s$ parameters, derived from the Geological Strength Index (GSI), in the Hoek-Brown equation, pillar stability curves can be generated. The pillar stability curves generated using this procedure do not resemble the empirical curves derived by other authors. Further modelling was conducted using an elastic-brittle-plastic analysis to determine the far-field stresses that cause pillar failure. The corresponding elastic stresses were then calculated using the far-field stresses from the elastic-brittle-plastic analysis and pillar stability curves were generated using these elastic stresses. The curves for GSI values of 40 and 60, generated using this two step approach, followed the trend of the empirical data for the boundary between, stable and unstable pillars, or in other words, the onset of stress induced progressive failure for pillar width-to-height ratios between 0.5 and 1.2.
When the input parameters are based on the concept of “cohesion loss”, in which the Hoek-Brown parameters $m$ and $s$ are set to 0 and 0.11 respectively, the pillar stability curves generated are in good agreement with the empirical curves developed by others (Hedley and Grant, 1972; Lunder and Pakalnis, 1997; etc.). This is especially evident at pillar width-to-height ratios less than 0.7 where cohesion loss across the entire width of the pillar is more or less instantaneous and corresponds with complete pillar failure. One important note however, is that some of the empirical curves tend to have an asymptotic trend where as the theoretical curves do not. Stress induced failure of the skin of the pillar will tend to an asymptotic initiation stress value as the pillar width-to-height ratio increases, however, the strength of the entire pillar will continue to increase indefinitely as the pillar width-to-height ratio increases.
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1 INTRODUCTION-PROBLEM STATEMENT

In mining, the goal is to safely extract as much of the orebody as possible; 100% ideally. However, this is often not practical as the size of the mining stopes is constrained by the quality of the rockmass in which the mining is being carried out. Therefore, a common mining practice involves leaving rock "pillars" between openings to maintain the stability of the openings. Hence, a pillar can be defined as "the in situ rock between two or more underground openings" (Coates, 1981).

Mine pillars can have simple to complex geometries depending on the nature of the orebody, the mining method, and the purpose of the pillars. Tabular ore bodies use barrier pillars to give regional support and divide the orebody into panels. Rib pillars are used to separate individual mine openings and give local support. Post pillars are used for local support in some cut and fill operations. When the orebody is oriented vertically (steeply dipping) sill pillars are used to divide the orebody into multiple mining horizons and crown pillars are used to prevent the mining activity from affecting the ground surface above the mine (Figure 1.1).
The design of mine pillars can significantly affect the success of a mining operation. Pillars that are under-designed can lead to failure of individual pillars, or to domino-type failure of many pillars and hence to collapse of large areas of a mine, as was experienced at Rio Algom's Quirke Mine in Elliot Lake (Hedley, 1992). Zipf (1996) has reported other large domino type pillar failures. On the other hand if pillars are over-designed and left unnecessarily large it may be uneconomical to recover the ore. The design of mine pillars to date has largely been based on empirical studies (Gale, 1999). Particularly in the coal industry, a number of empirically derived formulas have been developed for determining the strength of coal pillars, e.g. Salamon and Munro (1967).

In the hard rock mining industry the strength of pillars has had relatively little study. Hedley and Grant (1972) modified the empirical formulae derived for coal pillars to fit the pillar failures observed in tabular uranium mines in the Elliot Lake region of
northern Ontario. More recently, Lunder and Pakalnis (1997) developed an empirical formula based on case studies of Canadian hard rock mines. These pillar design methods have relied on observed and measured behaviour of full scale pillars, both stable and failed. Since the empirical formulas are based on site specific case studies, the use of these empirical methods to design pillars a priori is limited.

While significant advances have been made in determining the strength of rock masses (Hoek and Brown, 1998), these methods have not been used effectively to determine the strength of pillars. This thesis examines the application of the Hoek-Brown strength criterion to hard rock pillar design. In particular this thesis examines the strength of hard rock pillars using the hypothesis that the strength of hard rocks is fundamentally controlled by a "cohesion loss" process. This hypothesis was introduced by Martin (1997) and used by Martin et al (1999) to establish the depth of failure around underground openings. Using this approach, this thesis provides a framework for determining a priori the strength of pillars using simple laboratory tests and conventional numerical models.

A sound theoretical framework for determining hard rock pillar dimensions will provide the mine engineer with better tools to evaluate various designs. As with all mining projects, field observations must be collected to refine the general design approach to suit the specific mining environment.

1.1 THESIS STRUCTURE

Chapter 1 of this thesis gives a brief introduction to the problem and outlines the thesis. The second chapter identifies the modes of pillar failure that are commonly
encountered. This chapter also looks at the failure of laboratory specimens, and the effect of local mine stiffness on the failure of a pillar.

The stress distribution within a pillar is an important consideration when trying to determine its stability. Chapter 3 looks at the stresses in pillars and compares the various methods used for obtaining these stresses. Chapter 4 reviews the empirical pillar strength formulae developed by a number of different authors, with a bias towards those developed specifically for hard rock mines.

In Chapter 5, the approach developed by Martin, Kaiser and McCreath (1999) is used to develop theoretical pillar stability charts, and these are compared both with the traditional strength parameters used for numerical modelling, and with the empirical curves generated by other authors. Two-dimensional elastic and elastic-plastic analysis are considered.

Chapter 6 concludes the thesis by summarizing the findings of the research.
2 PILLAR GEOMETRY AND PILLAR FAILURE

Pillars are usually designed to be rectangular or square shapes in both plan and section. The design of pillars relates the strength to pillar shape. Figure 2.1 illustrates the pillar dimension nomenclature used throughout this thesis. It is important to note that the pillar height is defined relative to the direction of the maximum pillar stress. For example, for a sill pillar in a steeply dipping orebody, the pillar height is actually in the horizontal direction as the maximum pillar stress will be in the horizontal direction (Figure 2.1).

![Figure 2.1: Definition of pillar dimensions used throughout thesis.](image)

2.1 LAB

Knowledge of the complete stress-strain curve for pillars is important for understanding the modes of failure that might be experienced. The testing of rock
specimens in the lab, used as model pillars, gives a good indication of how the various geometries of pillars might react in the field. Hudson et al. (1972) studied the shape of the complete stress-strain curve for various sizes and shapes of rock cores. For their tests they used cores of 20 mm (¾ inch), 50 mm (2 inch) and 100 mm (4 inch) diameter with length to diameter ratios of 3, 2, 1, ½ and 1/3. They found that the strength of samples with the same length to diameter ratio but with varying diameters was approximately the same. They termed this the "size effect". Hudson et al. (1972) also examined the "shape effect" which considers the effects of varying the length to diameter ratio of rock cores with a constant diameter (varying the length). They found that varying the length to diameter ratio had a significant effect on the compressive strength of the rock as well as the shape of the stress-strain curve in the post-peak region. Figure 2.2 presents the results of the tests conducted by Hudson et al (1972). The smaller the length to diameter ratio the higher the compressive strength of the sample and the flatter the post-peak region of the stress strain curve. Das (1986) made similar observations regarding the effect of sample width to height ratio (which is the inverse of length to diameter ratio) on stress-strain curves when studying 6 Indian coal seams. Hudson et al. (1972) explained why the post-peak region of the stress-strain curve is steeper for slender (large L/D ratio) samples. As failure propagates, the actual cross-sectional area of the sample decreases to almost nothing, however, the stress is still calculated using the original cross-sectional area of the sample. For squat (small L/D ratio) samples, the actual cross-sectional area of the sample during testing does not vary significantly from the original.
Figure 2.2: Influence of specimen size and shape on the complete stress-strain curve for marble loaded in uniaxial compression (Hudson et al., 1972).
2.2 FIELD

There are three modes of pillar failure which are commonly observed underground; (1) structurally controlled failure; (2) stress-induced progressive failure; and (3) pillar bursts.

Structurally Controlled Failure

Most rock masses contain pre-existing failure planes (discontinuities) known as joints, faults, etc. Structurally controlled failure occurs when the pillars are oriented unfavorably with respect to the discontinuities present within the rock mass. Failure of these planes is usually in the form of shear movement along the plane. This type of failure is often observed as corners of pillars coming off along well defined planes.

Progressive Failure

The second mode of pillar failure is termed stress-induced progressive failure. This is observed as slabs spalling off the walls of the pillars. The progressive spalling mode of failure, otherwise known as hour-glassing, is generally observed in squat pillars where the skin of the pillar which has little confinement and high tangential stresses causes cracking and slab formation parallel to the direction of the major principal stress in the pillars. Initially the core of the pillar remains intact, because it is still confined and, hence, the pillar still retains most of its load carrying capacity. As spalling occurs, the stresses flowing through the pillar are redistributed to the intact pillar rock. The loss of the slabs relaxes the confinement on the adjacent intact core rock in the pillar, and further
damage then occurs to the newly exposed pillar wall surfaces. If this type of progressive failure is allowed to propagate too far, then the intact core of the pillar can reach a critical cross-sectional area, and fail. Pritchard and Hedley (1993) classified the pillars at the Denison Mine into five categories depending on how far failure had propagated. Figure 2.3 illustrates the condition of the pillars in each of these five categories.
Figure 2.3: Pillar classification categories as defined by Pritchard and Hedley (1993).
If the loads around an opening were sufficient to cause additional stress-induced failure (Figure 2.6), Kaiser et al (1996) using the approach developed by Martin (1996), proposed that the depth of the failure could be approximated by the linear relationship given by:

\[
\frac{d_f}{a} = 1.34 \frac{\sigma_{\text{max}}}{\sigma_c} - 0.57(\pm0.05)
\]  

(1)

It is important to note that from the above equation the stress-induced failure initiates when the maximum tangential stress exceeds approximately \(1/3 \sigma_c\) (Figure 2.4).

Kaiser et al (1996) suggested that the first stage of stress-induced failure was the “hour-glass” effect commonly observed in hard rock pillar failure (Figure 2.5). They suggested that the failed material should be termed “baggage” because if unsupported, it simply forms detached slabs. They also proposed that the extent of this baggage could be predicted by:

\[
\Delta_{\text{hoo}} = \frac{h(w)}{2\sin\alpha} \left(1 - \frac{\sin\alpha}{\tan\alpha}\right)
\]  

(2)
Figure 2.4: Plot of equation used by Kaiser et al (1996) to define the depth of stress-induced failure.

Figure 2.5: Definition of baggage (after Kaiser, McCreath and Tannant, 1996).
Figure 2.6: Depth of stress induced failure (after Kaiser, McCreath and Tannant, 1996).

Pillar Bursting

The third mode of failure encountered in pillars is pillar bursts. This mode of failure is usually encountered when the following two constraints are satisfied: (1) The stress in the pillar must exceed the strength; and (2) the local mine stiffness must be less than that of the pillar. Based on the work by Martin (1996) and Kaiser et al (1996), when the pillar stress exceeds $1/3$ of the uniaxial compressive strength of the rock the first constraint is generally satisfied. Figure 2.7 uses data from Mah et al. (1995) to illustrate this point.

Care should be taken however when dealing with rockburst damage to pillars. Pillars can undergo both pillar bursts, which completely fail the entire pillar, or they can undergo bursts that merely eject rock from the outer highly stressed skin of the pillar while the core of the pillar remains intact (Figure 2.8).
Figure 2.7: Rockburst potential in pillars. Data from Mah et al. (1995). (after Martin, Kaiser, Maybee, 1998)

Figure 2.8: Schematic illustrating the difference between pillar skin bursts and pillar bursts.
2.3 **LOCAL MINE STIFFNESS**

Once the strength of the pillar is exceeded, the pillar will fail according to the shape of the post-peak portion of its stress-strain curve. The violence of the failure is governed by the stiffness of the surrounding mine environment (Zipf, 1999). If the local mine stiffness ($K_{LMS}$) is high compared to the post-peak stiffness of the pillar ($K_p$), then the failure will be nonviolent as the available energy is fully absorbed in the fracturing process (Figure 2.9). However, if the local mine stiffness is low, less than that of the pillar, then the failure will be violent as more energy is put into the failing pillar than is absorbed by the fracturing process (Figure 2.10). Thus, in this case there is excess energy that promotes violent failure. This concept is analogous to that presented by Hudson et al. (1972) with respect to the stiffness of the testing machine used in laboratory tests. A soft testing machine will cause samples to fail violently, whereas a stiff test frame will allow the sample to fail in a stable manner.

![Diagram](image)

**Figure 2.9: Stiff local mine stiffness versus pillar load-convergence.**
Figure 2.10: Soft local mine stiffness versus pillar load-convergence curve illustrating available excess energy to drive failure.

Hoek and Brown (1980) state that as the pillar width-to-height ratio increases, the post-peak region of the load-displacement curve has a more ductile behaviour. Hence, in order for squat pillars to fail violently, the local mine stiffness would have to be extremely low. Ormonde et al (1993) carried out laboratory tests on samples of dolerite rock that simulated pillars with width-to-height ratios of 0.5, 0.67, 1, 1.25, 1.5, 2, 3 and 4. The results from these laboratory tests supported the findings of Hoek and Brown (1980).

Unfortunately while the complete stress-strain curve of the pillar can be approximated by laboratory tests, the local mine stiffness is more difficult to assess. A common approach is to use 3-dimensional elastic models in a 2-step approach: first to calculate the load and displacements with the pillar in place, and second to do these calculations with the pillar removed. This simple approach is illustrated in Figure 2.11. However, Aglawe (1999) pointed out the difficulties of assessing the mine stiffness for complex mine geometries and stress conditions.
Figure 2.11: Schematic illustrating approach for evaluating local mine stiffness, $K_{LMS}$.

2.4 SUMMARY

The testing of laboratory samples can give a good indication of the type of stress-strain curve to be expected from various pillar geometries. Three modes of pillar failure are commonly encountered underground: (1) Structurally-controlled failure; (2) Stress-induced progressive failure; and (3) Pillar bursts. When dealing with a pillar failure in which the strength of the rock has been exceeded, the violence of the failure is largely dependant on the relationship between the local mine stiffness and the stiffness of the pillar in the post-peak region. If the local mine stiffness is high relative to the post-peak response of the pillar, then the failure will be nonviolent. However, if the local mine stiffness is low relative to the post-peak behaviour of the pillar then excess energy is available to drive unstable failure in a violent manner (i.e. rockburst).
3 STRESSES IN PILLARS

The elastic stress distribution in pillars is a function of the pillar geometry and the in situ stress state.

3.1 IN SITU STRESSES

The in situ state of stress is a difficult parameter to evaluate. Determining the stresses in pillars requires knowledge of the in situ stresses in the vicinity of the pillars. The vertical in situ stress magnitude is generally taken as the unit weight of the overlying rock (γ) times the depth (z) (Hoek, Kaiser and Bawden, 1997). This is not always the case, for example, adjacent mining or faults may alter this assumption, but it is a good starting point. Figure 3.1 indicates a number of vertical stress measurements for excavations at various depths and shows that the average vertical stress gradient is 0.027MPa/m. Table 3.1 summarizes some of the vertical stress gradients that have been measured around the world.
Figure 3.1: Vertical stress versus depth (Hoek, Kaiser and Bawden, 1997).
Table 3.1: Summary of measured vertical stress gradients in various rock types.

<table>
<thead>
<tr>
<th>Vertical Stress Gradient (MPa/m)</th>
<th>Location (rock type)</th>
<th>Depth (m)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0249</td>
<td>Elliot Lake (quartzites)</td>
<td>900</td>
<td>Hedley and Grant (1972)</td>
</tr>
<tr>
<td>0.0266±0.0028</td>
<td>World data</td>
<td>0-2400</td>
<td>Herget (1974)</td>
</tr>
<tr>
<td>0.0270</td>
<td>World data</td>
<td>0-3000</td>
<td>Brown &amp; Hoek (1978)</td>
</tr>
<tr>
<td>0.0265</td>
<td>World data</td>
<td>100-3000</td>
<td>McGarr (1978)</td>
</tr>
<tr>
<td>0.0266±0.0324</td>
<td>Canadian Shield</td>
<td>0-2200</td>
<td>Herget (1987)</td>
</tr>
<tr>
<td>0.0266±0.0008</td>
<td>Canadian Shield</td>
<td>0-2200</td>
<td>Arjang (1989)</td>
</tr>
<tr>
<td>0.027</td>
<td>URL, Granite</td>
<td>440</td>
<td>Martin (1990)</td>
</tr>
<tr>
<td>0.0285</td>
<td>Canadian Shield</td>
<td>0-2300</td>
<td>Herget (1993)</td>
</tr>
<tr>
<td>0.0260</td>
<td>Canadian Shield</td>
<td>0-2200</td>
<td>Arjang &amp; Herget (1997)</td>
</tr>
<tr>
<td>0.0264</td>
<td>HRL, Gneissic Granite Sandstones/Volcanics</td>
<td>150-420</td>
<td>Andersson &amp; Ljunggren (1997)</td>
</tr>
<tr>
<td>0.0249±0.00025</td>
<td>Sellafield, UK Sandstones/Volcanics</td>
<td>140-1830</td>
<td>Batchelor et al (1997)</td>
</tr>
</tbody>
</table>

CANMET (Herget and Arjang, 1990) compiled a database of in situ stress measurements at various depths within the Canadian shield (Figure 3.2). The linear best-fit line of this data gives a vertical stress gradient of 0.0236 MPa/m. Due to the consistency of the data from the various sources the vertical in situ stress can be estimated with a fair degree of confidence. In Canadian hard rock mines the vertical stress is normally the in situ minor principal stress ($\sigma_3$).
Figure 3.2: Minor principal stress versus depth using data from the CANMET (Herget and Arjang, 1990) database.

The horizontal in situ stress magnitudes are related to the vertical stress by the ratio of horizontal principal stress to vertical principal stress ($k$). Figure 3.3 compares the $k$ value to the depth below surface for a number of sites around the world and shows that $k$ values range from 0.5 to 3.5. Figure 3.4 illustrates the range of $k$ values recorded in the CANMET database (Some of the data with very large $k$ values is not shown on the graph). Because of this wide range of $k$ values, it is important to establish the $k$ at a particular site in order to properly determine the horizontal stress magnitudes.
Figure 3.3: $k$ ratio versus depth (Hoek and Brown, 1981).
Figure 3.4: Ratio of horizontal stress to vertical stress versus depth for the CANMET (Herget and Arjang, 1990) database.

3.2 TRIBUTARY AREA THEORY

Salamon and Munro (1967), Hedley and Grant (1972), and Krauland and Soder (1987) used the tributary area theory to determine average pillar stresses. Tributary area theory implies that the load on each pillar is a function of the vertical column of rock immediately above each pillar as well as that above the area between an individual pillar and any of its adjacent pillars (Figure 3.5). Provided pillars have regular geometry it is possible to express the average pillar stress as a function of the extraction ratio.
Figure 3.5: Plan view of geometry for tributary area analysis of pillars in uniaxial loading (Brady and Brown, 1992).

The average stress in a pillar found by the tributary area theory is expressed as a function of the extraction ratio \( e \) by:

\[
\sigma_p = \frac{\gamma z}{(1 - e)}
\]  

(3)

where \( \gamma z \) represents the in situ stress acting normal to the pillar axis. According to Brady and Brown (1992) the extraction ratio \( e \) can be expressed in terms of the dimensions given in Figure 3.5 as:

\[
e = \frac{((a + c)(b + c) - ab)}{(a + c)(b + c)}
\]

(4)
Figure 3.6: Tributary area theory for square pillars (Hoek and Brown, 1980).

The average stress ($\sigma_p$) on a square post pillar, using tributary area theory, is given by:

$$\sigma_p = \gamma z (1 + \frac{W_o}{W_p})^2$$

(5)

where $\gamma$ is the unit weight of the overlying rock, $z$ is the depth below surface, $W_o$ is the excavation width and $W_p$ is the pillar width (Figure 3.6).

3.3 NUMERICAL MODELS

While the tributary area theory gives a good approximation of the pillar stresses for simple uniform geometries, in reality pillar geometries are often quite complex and hence not amenable to tributary area theory. Since the mid 1980's numerical modelling has
been used extensively to establish the stress distribution in pillars. Boundary element programs such as NFOLD\(^1\) have been used to establish both the peak and post-peak response of pillars occurring in tabular orebodies (Krauland and Soder, 1987; Von Kimmelmann, 1984). Sjoberg (1992) used MINSIM, a similar boundary element program to NFOLD, to estimate the stresses in the pillars at the Zinkgruvan Mine.

### 3.3.1 TWO-DIMENSIONAL MODELLING

For the purposes of this thesis, a number of numerical models were used to determine the stress distribution in pillars of various geometries. Two-dimensional modelling results from Examine 2D\(^2\) (boundary element) and Phase2\(^3\) (finite element) were compared with the tributary area theory and three-dimensional modelling results from Map3D\(^4\) (boundary element). These results were used to determine the minimum number of pillars required in a two-dimensional analysis to give an accurate representation of the stresses found in a room and pillar operation.

#### 3.3.1.1 PILLAR STRESSES FROM EXAMINE 2D

A number of models were built and tested in the two-dimensional boundary element program Examine 2D. These initial models used pillars with a constant width-to-height ratio of 1 and excavations with a constant w/h ratio of 1. The number of pillars in the models was varied, within a constant in situ stress field with \(k = 1\), and the major

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\(^1\) Available from Golder Associates, Mississauga, Ontario, Canada.

\(^2\) Available from RocScience Inc. 31 Balsam Ave., Toronto, Ontario, Canada M4E 3B5; Internet: www.rocscience.com.

\(^3\) Available from Mine Modelling Pty. Limited, P.O. Box 637, 22 Attunga Way, Mt. Eliza, Victoria 3930, Australia.
principal stress at the core of the central pillar in the array was recorded for each run. This procedure was then repeated for $k$ values that ranged from 0.5 to 3. A 50 MPa in situ stress field was used for the $k = 1$ curve, and a 50 MPa major principal in situ stress and a 25 MPa minor principal in situ stress were used for the $k = 0.5$ and $k = 2$ curves. For the $k = 3$ curve, a major principal in situ stress of 75 MPa and a minor principal stress of 25 MPa were used. The maximum number of pillars used in the models was 13, and it was considered that these results were “correct” in that the addition of more pillars showed no significant change in the computed stresses in the central pillar.

The results of these model runs are summarized in Figure 3.7. Since the stresses required to cause failure of a pillar are relatively high a 10% error in the stresses obtained by numerical modelling will not cause significant errors in the stability assessment of the pillars. It can be seen from Figure 3.7 that the error for the value of pillar stress is acceptable with only 5 pillars in the array. Hence, an array of 5 pillars was used for the analyses in this section to balance the computer run time, with an acceptable level of accuracy.
Figure 3.7: Number of pillars in the array versus % change of stress from value with 13 pillars in the array for various values of $k$.

Figure 3.8 and Figure 3.9 are examples of the types of two-dimensional pillar array models used to generate the pillar stability charts. The pillars in any one array are all of the same dimensions. The excavations are also the same dimensions in any one model. Figure 3.8 (a) is an array with a pillar width-to-height (w/h) ratio of 0.5, Figure 3.8 (b) has a pillar w/h ratio of 1 and Figure 3.8 (c) has a pillar w/h ratio of 2. In all three cases the excavation w/h ratio is 1. Figure 3.9 is an example of an array with a pillar w/h ratio of 2 and an excavation w/h ratio of 2.
Figure 3.8: Pillar models for various pillar w/h ratios for excavations with a w/h ratio of 1.

Figure 3.9: Pillar model with pillar w/h=2 and excavation w/h=2.

For this modelling, the stress at the core of the pillar was used as the key or critical value of interest rather than the average stress across the pillar, because the pillar core stress is obtained with greater ease than the average pillar stress from numerical models. The core of the pillar was taken as the horizontal centre at mid-height of the pillar. Four models were run in Examine 2D to see if the stress at the centre of the pillar was an accurate representation of the average stress across the entire pillar at mid-height. The average stress at mid-height of the pillar was obtained by determining the stress at a number of grid points across the pillar and taking the average of all these stresses. The
four models were run in a 27 MPa hydrostatic ($k = 1$) in situ stress field and had pillar w/h ratios of 0.5, 1, 1.5 and 2. The results are summarized in Table 3.2.

**Table 3.2: Average pillar stress versus pillar core stress.**

<table>
<thead>
<tr>
<th>Pillar width to height ratio</th>
<th>Average pillar stress (MPa)</th>
<th>Pillar core stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>69.3</td>
<td>71.2</td>
</tr>
<tr>
<td>1</td>
<td>50.4</td>
<td>52.1</td>
</tr>
<tr>
<td>1.5</td>
<td>43.4</td>
<td>43.1</td>
</tr>
<tr>
<td>2</td>
<td>39.6</td>
<td>38.1</td>
</tr>
</tbody>
</table>

From Table 3.2 it is apparent that the core stress is a good approximation of the average stress across the entire pillar. At low width-to-height ratios the core stress slightly overestimates the average stress and, at high pillar w/h ratios the core stress slightly underestimates the average stress. Figure 3.10 is the plot of the major and minor principal stress distributions across the mid-height of the pillar with a width-to-height ratio of 0.5, which would be considered as a slender pillar. The major principal stress distribution, obtained from numerical modelling, is fairly uniform across a slender pillar. The arch shape of the major principal stress distribution apparent in Figure 3.10 can be attributed to the shadowing effect of the sharp corners of the excavations. This shadowing effect is also visible in the principal stress distributions for the pillars with width-to-height ratios of 1, 1.5 and 2 shown in Figures 3.8 to 3.11 respectively. The figures also illustrate that for more squat pillars, the confining stress at the centre of the
pillars increases. For all cases the minor principal stress (confining stress) at the excavation boundary is zero.

Figure 3.10: Principal stress distributions across a pillar with w/h = 0.5, k = 1, and far-field stress of 27 MPa.
Figure 3.11: Principal stress distributions across a pillar with w/h = 1, k = 1, and far-field stress of 27 MPa.

Figure 3.12: Principal stress distributions across a pillar with w/h = 1.5, k = 1, and far-field stress of 27 MPa.
Figure 3.13: Principal stress distributions across a pillar with w/h = 2, k = 1, and far-field stress of 27 MPa.

3.3.1.2 PILLAR STRESSES FROM PHASE2

While boundary elements are an efficient means to determine the elastic stress distribution in pillars, finite element programs offer the added functionality of being able to determine stress redistribution during pillar failure provided that the post-peak behaviour of the rock is known. Phase2 is a windows-based, user friendly program with automatic finite element mesh generation capabilities (Figure 3.14). An elastic analysis was carried out using Phase2 to evaluate if the automatic mesh generation was adequate to determine the stresses in pillars. Figure 3.15 shows the elastic stresses for a pillar width-to-height ratio of 1, k = 1 and the far-field stress equal to 27 MPa. Comparing Figure 3.15 to Figure 3.11, there is a 2.7 MPa difference in the value of the major principal stress, and a 0.5 MPa difference in the minor principal stress, at the core of the
pillar obtained using the two different numerical modelling programs. Therefore, the automatic mesh generation scheme in Phase2 is adequate for determining the stress distribution in pillars.

Figure 3.14: Mesh generated using finite element program Phase2.

Figure 3.15: Principal stress distributions, from Phase2 modelling, across a pillar with w/h = 1.
The next step was to compare the results of the two-dimensional elastic model with those from a three-dimensional elastic model. The three-dimensional boundary element program Map3D was used for this modelling. The two-dimensional model assumes that the pillars are actually ribs, whose out of plane length is significantly longer than their width (Hoek, Kaiser, Bawden, 1997). For the purpose of three-dimensional modelling the length of the ribs was taken as 10 times their width. Figure 3.16 compares the core stress in pillars of various lengths, all with a constant width-to-height ratio of 1 (width = 10m). There is a 5 MPa difference in the pillar core stress between, a pillar that has a length 4 times its width and a pillar with a length 6 times its width. The difference in pillar core stress between a pillar whose length is 6 times its width, and one whose length is 8 times its width is approximately 0 MPa. Therefore, once the length of the pillar exceeds the width by about 6 times the pillar core stress is fairly constant. Therefore using a pillar whose length is 10 times its width is a more than adequate representation of a rib pillar. In order to compare the results of the different modelling techniques the same 27 MPa hydrostatic stress field used in the previous section was employed. A model was constructed in which the width-to-height ratio of the pillars is 1. Figure 3.17 presents the major principal stress contours on a grid through the centre of the middle pillar in the three-dimensional model.
Figure 3.16: Three-dimensional pillar core stress versus length of pillar for pillars with constant width and height of 10 m.
Figure 3.17: Map3d model output of major principal stress contours.

Figure 3.18: Map3d output of minor principal stress contours.
Figure 3.18 shows the minor principal stress contours in the pillars of the three-dimensional elastic model. The principal stress distributions at mid-height of the centre of the central pillar are plotted in Figure 3.19. Comparing Figure 3.19 to Figure 3.11 generated using Examine 2D, shows that there is a 4.2 MPa difference in the value of the major principal stress, and a 0.7 MPa difference in the value of the minor principal stress, obtained at the core of the pillars. The results of the Phase2 analysis (Figure 3.15) show a 6.8 MPa difference in the value of the major principal stress and a 1.1 MPa difference in the value of the minor principal stress at the core of the pillars. Therefore, a two-dimensional analysis gives approximately the same results as a three-dimensional one as long as the third dimension of the pillar is significantly long to be a plain strain condition.

![Principal stress distributions across a pillar with w/h = 1, obtained from three-dimensional elastic modelling using Map3D.](image)

Figure 3.19: Principal stress distributions across a pillar with w/h = 1, obtained from three-dimensional elastic modelling using Map3D.
3.4 CONFINEMENT IN PILLARS

Knowledge of the confinement within a pillar is important when trying to determine the strength of the pillar because, rock strength itself is a function of confining stress, as indicated by the Hoek-Brown strength criterion (Hoek and Brown, 1980). In order to examine the confinement present in rib pillars of varying width-to-height ratios the two-dimensional finite element program Phase2 was used. When using this two-dimensional numerical modelling program, the out-of-plane in situ stress is required. This is the stress that acts normal to the plane modelled in two-dimensional space. In most instances this is the stress which acts parallel to the long axis of the excavation. The in situ vertical stress and out-of-plane stress were kept constant at 30 MPa for all of the model runs. The horizontal in situ stress was varied, in other words the $k$ value was altered. The minor principal stress at the core of the central pillar in the model was recorded and taken to represent the confining stress in the pillar. The results of the modelling are presented in Figures 3.20 and 3.21. Figure 3.20 illustrates that for pillars with width-to-height ratios greater than approximately 1 the horizontal in situ stress affects the confining stress in the pillars. The larger the pillar width-to-height ratio, the more effect the in situ horizontal stress has. For pillars with a width-to-height ratio of less than approximately 1, the confining stress in the pillars remains relatively constant regardless of the in situ horizontal stress. Figure 3.21 illustrates that the $k$ ratio affects the confinement in pillars with width-to-height ratios greater than approximately 1.
Figure 3.20: Confining stress in pillars of varying width-to-height ratios versus the horizontal in situ stress. The vertical and out-of-plane in situ stresses are held constant.

Figure 3.21: Increase in confinement as a function of $k$, the ratio of the far-field $\sigma_1$ and $\sigma_3$. 
Lunder and Pakalnis (1997) use the following equation to relate the pillar width-to-height ratio to the average confinement in a pillar. The average pillar confinement ($C_{pav}$) is defined as the ratio of the average minor to the average major principal stress at the mid-height of a pillar.

$$C_{pav} = 0.46 \times \left[ \log \left( \frac{W_p}{h} + 0.75 \right) \right]^{1.4} \left( \frac{W_p}{h} \right)$$

where: $W_p$ = the pillar width and, $h$ = the pillar height.

This equation shows that the average pillar confinement is a function of the geometry of the pillar. Figure 3.22 is a plot of the average pillar confinement for various pillar width-to-height ratios, and indicates that the average pillar confinement increases as the pillar width-to-height ratio increases.
Figure 3.22: Average pillar confinement as defined by Lunder and Pakalnis (1997) versus pillar width-to-height ratio.

Hoek also used the ratio of minor principal stress to major principal stress to determine the length of a crack that could form from an elliptical flaw as seen in Figure 3.23. The figure illustrates that the lower the $\sigma_2/\sigma_1$ ratio the longer the crack that can form and that a small amount of confining stress effectively reduces the ability of the crack to propagate. Figure 3.23 illustrates that for a minor principal stress to major principal stress ratio less than approximately 0.1 crack extension is a risk. Figure 3.22 illustrates that pillars with a width-to-height ratio less than approximately 1 have a minor principal stress to major principal stress ratio ($C_{pav}$) of less than 0.1 and hence, are at risk of crack extension, even in the core of the pillar. This can be related to pillars in that
slender pillars and the skin of squat pillars have very little confining stress and hence are susceptible to cracks that grow parallel to the major principal stress in the pillar.

![Diagram of stress distribution in pillars]

Figure 3.23: Relationship of fracture growth as a function of confining stress expressed as the ratio of $\sigma_3/\sigma_1$, (Hoek, 1968).

### 3.5 SUMMARY

Tables 3.3 to 3.5 give a comparison of the results obtained from the different methods of stress analysis presented in this chapter. For the sake of comparison all pillars were assumed to be rib pillars with lengths 10 times their width. Width-to-height ratios of 0.5, 1.0 and 2.0 were used along with a hydrostatic ($k = 1$) in situ stress field of 27 MPa. The results show that for the case of rib pillars, where length is significantly greater...
than the width, a two-dimensional analysis can be used to estimate the real world three-dimensional stresses in the pillars. Table 3.3 shows that at low pillar width-to-height ratios \((w/h = 0.5)\) the two-dimensional models used tend to slightly underestimate the pillar stresses for a given loading. For a pillar width-to-height ratio equal to 1.0 the pillar stresses from the two-dimensional analysis are in much better agreement with the pillar stress obtained using Tributary Area Theory.

Table 3.3: Comparison of rib pillar results from different stress analysis techniques for pillars with width-to-height ratio of 0.5.

<table>
<thead>
<tr>
<th>Stress Analysis Technique</th>
<th>Average major principal stress (MPa)</th>
<th>Variation in principal stress (\sigma_1) (%)</th>
<th>Average minor principal stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tributary Area Theory</td>
<td>81</td>
<td>-</td>
<td>N/A</td>
</tr>
<tr>
<td>Examine 2D</td>
<td>69.3</td>
<td>-14.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>Phase2</td>
<td>66.0</td>
<td>-18.5</td>
<td>-0.19</td>
</tr>
<tr>
<td>Map3D</td>
<td>78.9</td>
<td>-2.6</td>
<td>-0.34</td>
</tr>
</tbody>
</table>
### Table 3.4: Comparison of rib pillar results from different stress analysis techniques for pillars with width-to-height ratio of 1.

<table>
<thead>
<tr>
<th>Stress Analysis</th>
<th>Average major principal stress (MPa)</th>
<th>Variation in principal stress ( \sigma_1 ) (%)</th>
<th>Average minor principal stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tributary Area Theory</td>
<td>54</td>
<td>-</td>
<td>N/A</td>
</tr>
<tr>
<td>Examine 2D</td>
<td>50.4</td>
<td>-6.7</td>
<td>3.1</td>
</tr>
<tr>
<td>Phase2</td>
<td>47.8</td>
<td>-11.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Map3D</td>
<td>54.6</td>
<td>+1.1</td>
<td>3.8</td>
</tr>
</tbody>
</table>

### Table 3.5: Comparison of rib pillar results from different stress analysis techniques for pillars with width-to-height ratio of 2.

<table>
<thead>
<tr>
<th>Stress Analysis</th>
<th>Average major principal stress (MPa)</th>
<th>Variation in principal stress ( \sigma_1 ) (%)</th>
<th>Average minor principal stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tributary Area Theory</td>
<td>40.5</td>
<td>-</td>
<td>N/A</td>
</tr>
<tr>
<td>Examine 2D</td>
<td>39.6</td>
<td>-2.2</td>
<td>10.6</td>
</tr>
<tr>
<td>Phase2</td>
<td>37.7</td>
<td>-6.9</td>
<td>9.2</td>
</tr>
<tr>
<td>Map3D</td>
<td>41.9</td>
<td>+3.5</td>
<td>10.8</td>
</tr>
</tbody>
</table>
4 EMPIRICAL PILLAR DESIGN

One of the main methods used to date for determining the strength of pillars is back analysis (Gale, 1999). This empirical method requires that data on observed pillar failures is collected and strength envelopes for pillars are fitted to this data. A great deal of work, using this approach, has been carried out on the stability of coal pillars (Salamon and Munro, 1967; Mark, 1999); however, limited studies have been completed on hard rock pillars. Most of the hard-rock pillars that have been studied are rib pillars.

4.1 SALAMON AND MUNRO

Salamon and Munro (1967) studied 125 pillars in the South African coalfields after the collapse at the Coalbrook North Colliery (Salamon, 1999). They used the following equation for the strength of coal pillars:

\[ \sigma_{ps} = Kh^aW_p^b \] (7)

where: \( K \) = the strength of a unit cube of coal,
\( h \) = the height of a pillar,
\( W_p \) = the pillar width and,
\( a, b \) = empirical constants.

Salamon and Munro determined values of \(-0.66\) and \(0.46\) for the constants \( a \) and \( b \), respectively. A summary of values for these constants as determined by other authors is given in Table 4.1.
Table 4.1: Constants suggested by various authors for empirical pillar design (after Hoek and Brown, 1980).

<table>
<thead>
<tr>
<th>Author</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salamon and Munro (1967) – analysis of collapsed pillar areas in South Africa</td>
<td>-0.66</td>
<td>0.46</td>
</tr>
<tr>
<td>Greenwald, Howarth and Hartman (1939) – in situ tests on small pillars in USA</td>
<td>-0.85</td>
<td>0.5</td>
</tr>
<tr>
<td>Holland and Gaddy (1957) – extrapolation of small scale laboratory tests</td>
<td>-1.00</td>
<td>0.5</td>
</tr>
<tr>
<td>Bieniawski (1968) – interpretation of tests on in situ coal specimens in S. Africa</td>
<td>-0.55</td>
<td>0.16</td>
</tr>
</tbody>
</table>

4.2 **HEDLEY AND GRANT**

Hedley and Grant (1972) investigated the stability of hard rock mine pillars, based on pillars in the Elliot Lake Uranium Mines. The uranium is found in conglomerate reefs separated by massive quartzite beds. Their database consisted of 28 pillars, ranging in depth from 150 to 1040 metres. Three of the pillars were described as being crushed or totally failed. Two pillars were partially failed and the remaining 23 pillars were described as stable. They determined the following equation for pillar strength:

$$\sigma_{ps} = 133 \frac{W_p^a}{h^b}, \text{MPa}$$  \hspace{1cm} (8)$$

This equation has the same form as Salamon and Munro’s equation, with values of 0.5 and 0.75 for the constants $a$ and $b$, respectively as presented in Equation 8. They used a value of 133 MPa to represent the strength of a unit cube of rock. The laboratory uniaxial
compressive strength of the intact pillar rock was determined to be 230 MPa. This equation was meant for square pillars however, Hedley and Grant felt that it would be applicable to the rectangular pillars they were studying. These rectangular pillars had lengths that were in the order of 10 times their width, and are considered as rib pillars.

It is common practice within the literature to present pillar data and pillar strength equations in the form of a pillar stability graph. Potvin et al. (1989) describe pillar stability graphs in the following manner:

"The y-axis of the graph has been chosen to represent a relative index of pillar loading. It is calculated as the ratio of average induced pillar load versus the compressive strength of the intact rock. The x-axis of the chart takes into account pillar shape by plotting the ratio of the pillar width to pillar height. This will account for the increased strength provided by the shape and confinement of the pillar."

Figure 4.1 is a pillar stability graph showing the equation used by Hedley and Grant (1972) and Pritchard and Hedley (1993) and the data used to derive the equation.
Hedley and Grant (1972).

Pritchard and Hedley (1993):

4.3 POTVIN, HUDYMA AND MILLER

Potvin et al. (1989) published a paper that presented an empirical curve derived from rib pillars from open stope mines in Canada. When plotted in the form of a pillar stability graph, their line has the following form (Figure 4.2):

\[
\frac{\sigma_{ps}}{UCS} = 0.4162 \frac{W_p}{h}
\]

Potvin et al. (1989) consider that their pillar strength line was less conservative than Hedley and Grant's curve (Figure 4.1), because Hedley and Grant's curve had been derived from the response of smaller pillars.
4.4 VONN KIMMELMANN, HYDE AND MADGWICK

The Selebi and Phikwe mines in Botswana provided an excellent case study for Von Kimmelmann et al. (1984) to determine an empirical pillar stability graph. These mines provided a database of 57 massive sulfide pillars consisting of 47 square pillars and 10 long or rib pillars. The uniaxial compressive strength of the massive sulfides was determined to be 94.1 MPa. Von Kimmelmann et al. (1984) used the same pillar strength equation as Salamon and Munro (1967) with the exception that they used a value of 65 MPa for the strength of a unit cube of rock ($K$). They indicated that the $K = 65$ MPa value gave a curve with a factor of safety between 1.2 and 1.3. The equation has the following form (Figure 4.3):

Figure 4.2: Pillar stability graph, Potvin et al. (1989).
\[ \sigma_{ps} = 65 \frac{W_p^{0.46}}{h^{0.66}}, \text{MPa} \]  

(10)

Figure 4.3: Pillar stability graph, Von Kimmelmann et al. (1984).

4.5 **SJOBERG and KRAULAND AND SODER**

Sjoberg (1992) studied the pillars in the Zinkgruvan Mine in Sweden and Krauland and Soder (1987) studied the pillars in the Black Angel Mine in Greenland. These authors used the following pillar strength formula, proposed by Obert and Duvall in 1967:

\[ \sigma_{ps} = \sigma_{pl} (0.778 + 0.222 \frac{W_p}{h}) \]  

(11)
where $\sigma_{pl}$ is defined as the strength of a pillar with a width to height ratio of 1. Sjoberg was looking at sill pillars in a rockmass with a uniaxial compressive strength ($\sigma_c$) of 240 MPa and found that a value of 74 MPa for $\sigma_{pl}$ fit his data well (Figure 4.4). Krauland and Soder found that a value of 35.4 MPa for $\sigma_{pl}$ fit their data well in a rockmass with a $\sigma_c$ of 100 MPa (Figure 4.5).

![Pillar stability graph, Sjoberg (1992).](image)

Figure 4.4: Pillar stability graph, Sjoberg (1992).
Figure 4.5: Pillar stability graph, Krauland and Soder (1987).

4.6 LUNDER AND PAKALNIS

Lunder and Pakalnis (1997) compiled a database of essentially rib pillar histories from a number of hard rock mines. The database includes pillar case histories from Lunder (1994), Hudyma (1988), Hedley and Grant (1972), Von Kimmelman et al. (1984), Krauland and Soder (1987), Sjoberg (1992), and Brady (1977). Using this combined rib pillar database Lunder and Pakalnis proposed “The Confinement Formula” for determining the strength of hard rock mine pillars given as:

$$\sigma_{ps} = (K \ast UCS) \cdot (C_1 + C_2 \ast \kappa)$$

where: $K$ = a rock mass strength size factor which Lunder and Pakalnis average at 44%,

$UCS$ = the unconfined compressive strength of intact pillar material (MPa),
$C_1, C_2 = \text{empirically derived constants which were determined to be } 0.68 \text{ and } 0.52 \text{ respectively and,}$

$kappa = \text{the mine pillar friction term.}$

In its most basic form "The Confinement Formula" consists of two parts. One part takes into account the effect on strength between a pillar and a laboratory sample of the same rock, and the other term takes into account the shape of the pillar.

The mine pillar friction term kappa is defined as follows:

$$kappa = \tan^{-1}\left(\frac{1 - Cpav}{1 + Cpav}\right)$$ \hspace{1cm} (13)

$Cpav$ is the average pillar confinement, which is defined as the ratio of the average minor to the average major principal stress at the mid-height of a pillar. The average pillar confinement can be found using the following equation (presented earlier in section 3.4-see Figure 3.22):

$$Cpav = 0.46 \left[ \log\left(\frac{W_p}{h} + 0.75\right) \right]^{1.4}$$ \hspace{1cm} (14)

Lunder and Pakalnis (1997) used "The Confinement Formula" to generate pillar stability charts, as shown in Figure 4.6. The "average pillar confinement" is presented on the horizontal axis and the average pillar stress divided by the $UCS$ is on the vertical axis. The Factor of Safety equal to 1 line (upper curve on Figure 4.6) gives a good approximation to the point of failure of the pillars in the database. However, as with all problems involving excavations in rock, a Factor of Safety greater than 1 should be used because of the inherent heterogeneity of the rock. Lunder and Pakalnis (1997) also plot
the Factor of Safety equal to 1.4 curve (lower curve on Figure 4.6). This curve puts almost all of the unstable and failed pillars on the correct side of the line. This figure illustrates the point that as the confinement in the pillar increases so does the strength of the pillar.

Figure 4.6: Pillar Stability Graph (after Lunder and Pakalnis, 1997) using “average pillar confinement”.

Figure 4.7 is a pillar stability graph from Lunder and Pakalnis (1997) with the same vertical axis as Figure 4.6 but with the pillar width-to-height ratio on the horizontal axis. Once again the Factor of Safety equal to 1.4 curve (lower curve on Figure 4.7) appears to place most of the unstable and failed pillars on the correct side of the curve.
4.7 SUMMARY

The authors mentioned in this section have all proposed equations that can be plotted in the form of curves on pillar stability graphs. The equations of the author's curves are summarized in Table 4.2. Figure 4.8 presents the pillar strength curves of the previously mentioned authors combined on a single pillar stability graph for easy comparison. The general trend of all of these lines, based on empirical case studies, is very similar with the exception of that by Potvin et al. (1989), if extrapolated past a pillar width to height ratio of 1.25. The scatter in the lines can partly be attributed to the fact that the different authors had different classifications for "failure" of a pillar.
The empirical curves give consistent results. However, they rely on failures to occur. Ideally the design engineer would like to have a tool available which would allow him/her to predict the performance of pillars in the preliminary design stages. This tool would have to correspond well with the empirical curves presented in this chapter. The next chapter of this thesis presents a theoretical framework for assessing the strength of the hard rock pillars a priori and compares it to the empirical curves presented in this chapter.
Table 4.2: Summary of Empirical Pillar Strength Equations for Hard Rock Mines.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Pillar Strength Equation</th>
<th>UCS (MPa)</th>
<th># of Pillars in Database</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedley and Grant, 1972, Pritchard and Hedley, 1993</td>
<td>$\sigma_{ps} = 133 \frac{W_p^{0.5}}{h^{0.75}}$</td>
<td>230</td>
<td>28</td>
</tr>
<tr>
<td>Potvin et al., 1989</td>
<td>$\frac{\sigma_{ps}}{UCS} = 0.4162 \frac{W_p}{h}$</td>
<td>-</td>
<td>47</td>
</tr>
<tr>
<td>Von Kimmelmann et al., 1984</td>
<td>$\sigma_{ps} = 65 \frac{W_p^{0.46}}{h^{0.66}}$</td>
<td>94.1</td>
<td>57</td>
</tr>
<tr>
<td>Krauland and Soder, 1987</td>
<td>$\sigma_{ps} = 35.4(0.778 + 0.222 \frac{W_p}{h})$</td>
<td>100</td>
<td>287</td>
</tr>
<tr>
<td>Sjoberg, 1992</td>
<td>$\sigma_{ps} = 74(0.778 + 0.222 \frac{W_p}{h})$</td>
<td>240</td>
<td>9</td>
</tr>
<tr>
<td>Lunder and Pakalnis, 1997</td>
<td>$\sigma_{ps} = (0.44 \times UCS) \times (0.68 + 0.52 \times \kappa)$</td>
<td>-</td>
<td>178</td>
</tr>
</tbody>
</table>
Figure 4.8: Combined pillar stability graphs of authors cited in this section.
5 ROCK MASS STRENGTH AND PILLAR STABILITY GRAPHS

This chapter presents a theoretical framework for assessing the strength of hard rock mine pillars. The advantage of this theoretical approach over the empirical strength curves presented in Chapter 4 is that no failures are required and hence, this pillar strength criterion can be used in the preliminary design process.

The strength of a rock mass is usually expressed in the form of a power law (Brady and Brown, 1992). The following equation proposed by Bieniawski in 1974 is a good example of a power law equation for the peak strength of rock:

\[
\frac{\sigma_\perp}{\sigma_c} = 1 + A \left( \frac{\sigma_\perp}{\sigma_c} \right)^k
\]

where: \( k = 0.75 \) and

\( A \) - ranges from 3 to 5 depending on the rock type (Brady and Brown, 1992).

This equation suggests that the strength of a rock mass is very dependent on the confining stress, \( \sigma_3 \).

Section 3.4 showed that the average pillar confinement (\( C_{pav} \)) is less than 0.07 for pillars with width-to-height ratios less than 1. The sketch in Figure 5.1 taken from a report by Hoek (1982) gives a good illustration of the effect of pillar width-to-height ratio on the strength of pillars. Slender pillars have little or no confining stress across their entire width and are hence relatively weak as compared to squat pillars, which have a fair amount of confining stress at their core.
Figure 5.1 Influence of pillar shape on confining stress and strength of a pillar (Hoek, 1982).

5.1 ELASTIC ANALYSIS

Elastic modelling was carried out using the 2-dimensional boundary element program Examine 2D developed at the University of Toronto. The goal of the elastic models was to create theoretical pillar stability graphs. Failure of the pillars in the models was achieved by increasing the far-field stresses at a constant $k$ value. Failure of a pillar was considered to have occurred when the factor of safety at mid-height of the pillar dropped below 1.0 across the entire cross-section of the pillar (Figure 5.3). The elastic model allowed for a user defined “factor of safety” to be used rather than the “strength factor” provided by the program. Figure 5.2 (a) illustrates how the “strength factor” is
calculated and Figure 5.2 (b) illustrates how the "factor of safety" is calculated. At a value of unity the "strength factor" and Factor of Safety are the same. However, above and below this value, they differ.

\[
\text{Strength Factor} = \frac{A_{\text{max}}}{A}
\]

\[
\text{Factor of Safety} = \frac{B_{\text{max}}}{B}
\]

Figure 5.2: Strength Factor versus Factor of Safety (McCreath and Diederichs, 1994).
Figure 5.3: (a) Examine2D strength factor contours for model of 5 pillars with $k = 1$. 
(b) Enlarged view of central pillar.

The contours in Figure 5.3 represent the strength factor calculated at the various points. The figure illustrates the strength factor contours in the pillar immediately prior to failure. A further increase in the far-field stresses would cause the strength factor across the entire mid-height of the pillar to drop below unity. It is the pillar core stress at the condition represented by Figure 5.3 that was taken to represent the stress at failure of the pillar ($\sigma_p$).

5.1.1 PILLAR STRENGTH USING $m$ AND $s$

The empirical Hoek-Brown failure criterion (Hoek and Brown, 1998) is currently in widespread use:

$$\sigma_1 = \sigma_3 + \sqrt{m \sigma_c \sigma_3 + s \sigma_c^2} \tag{16}$$

where: $\sigma_1, \sigma_3$ = the principal effective stresses at failure,
\[ \sigma_c = \text{the unconfined compressive strength of the intact rock and} \]

\[ m, s = \text{parameters defined by Hoek and Brown (1998).} \]

The \( m \) and \( s \) parameters can be derived from the Geological Strength Index (GSI) and \( m_i \), which is the \( m \) value for an intact specimen of the rock. The GSI is related to Bieniawski’s 1989 Rock Mass Rating by the following equation (Hoek and Brown, 1998):

\[ \text{GSI} = \text{RMR}^{89} - 5 \quad (17) \]

The Geological Strength Index (GSI) can be obtained from the rock mass quality \( Q' \) by the following equation:

\[ \text{GSI} = 9 \ln Q' + 44 \quad (18) \]

Potvin et al. (1989) found that the rock mass quality \( Q' \) in Canadian hard-rock mines, with what is termed as “blocky” ground, had the distribution shown in Figure 5.4. Using equation 18 and the database presented in Figure 5.4 the GSI value at most Canadian hard-rock mines ranges from 65 to 77. For the following analyses, a GSI value of 70 and a \( m_i \) value of 25 were assumed for the pillars. The \( m_i \) value of 25 relates to a fairly competent rock such as a norite or gneiss (Hoek and Brown, 1988), which are very common in hard rock mines. The values of \( m \) and \( s \) for the rockmass used in the model were determined using the following equations for an undisturbed rock mass:

\[ s = \exp(\text{GSI}-100/9) = 0.036 \quad (19) \]

\[ m = m_i \exp(\text{GSI}-100/28) = 8.56 \quad (20) \]

The \( m \) value is related to the frictional strength of the rock and the \( s \) value is related to the cohesional strength of the rock. The Hoek-Brown strength criterion assumes that both the cohesional and frictional strength of the rock are mobilized at the same time. The \( m \) and \( s \)
parameters determined above are substituted in the Hoek-Brown equation along with the uniaxial compressive strength of the rock which, for the purpose of this analysis was taken as 240 MPa. The curve obtained is plotted in principal stress space in Figure 5.5.

Figure 5.4: Distribution of rock mass quality in Canadian hard-rock mines with "blocky" ground, data from Potvin et al., 1989.
Figure 5.5: Traditional Hoek-Brown strength curve in principal stress space.

This same approach is used to generate theoretical pillar stability charts where the vertical axis is normalized by the uniaxial compressive strength ($\sigma_c$) of the rock and the horizontal axis is the pillar width-to-height ratio.
Figure 5.6: Empirical pillar stability graphs discussed in Section 4 compared to the predicted pillar stability using Hoek-Brown failure criterion with $m = 8.56$, $s = 0.036$ and $\sigma_c = 240$ MPa.

The pillar stability curve generated by the conventional Hoek-Brown approach in a hydrostatic stress field ($k = 1$) for a Factor of Safety equal to 1 is shown in Figure 5.6. This curve does not correspond at all with the empirical curves of the other authors (Figure 5.6), particularly for pillar width-to-height ratios greater than 0.75. The Hoek-Brown failure criterion comprises two components; a cohesive strength component and a frictional component. For low confining stress conditions (pillar w/h < 0.5), the strength of the pillars is underestimated. The $m$ and $s$ parameters obtained from the Geological Strength Index tend to give more importance to the frictional $m$ parameter which requires confining stress to be mobilized. The contribution of the confining stress present in
pillars with width-to-height ratios greater than 0.5 causes the frictional strength component to increase significantly. The \( m \) and \( s \) Hoek-Brown parameters presented above are based on the notion that friction and cohesion are mobilized at the same time. Figure 5.6 suggests that this assumption might not be correct for pillars. The next section explores the concept behind brittle Hoek-Brown parameters which suggest that failure is controlled by a cohesion loss process and that the frictional strength is not initially mobilized.

### 5.1.2 Brittle Hoek-Brown Parameters

The Hoek-Brown failure criterion was originally developed for application to rock around underground openings under confined conditions (Hoek and Brown, 1980). Section 5.1.1 showed that \( m \) and \( s \) Hoek-Brown parameters derived from the Geological Strength Index do not generate pillar stability charts that correspond to the empirical database presented in Chapter 4.

Recently, Martin, Kaiser and McCreath (1999) and Martin, Kaiser and Maybee (1998) showed that the traditional Hoek-Brown failure criterion significantly underestimated the depth of stress-induced brittle failure around underground openings. They suggested that the strength of brittle rocks was controlled by the cohesive strength and that the normal-stress dependent frictional strength component could be ignored. Martin and Chandler (1994) have shown that different amounts of plastic strain were required to mobilize the cohesive and frictional strength components, respectively. Thus, these strength components were not mobilized simultaneously.
Initial fracturing in brittle rocks is a function of principal stress difference, ie. shear stress, and is equivalent to a loss of cohesion. Loss of cohesion initiates when the principal stress difference \((\sigma_1 - \sigma_3)\) is approximately \(1/3 \sigma_c\). This criterion is equivalent to Mohr-Coulomb with \(\Phi\) equal to 0, of form:

\[
\tau = \frac{1}{6} \sigma_c + 0
\]  
(21)

This criterion can also be written in Hoek-Brown terms as:

\[
\sigma_1 = \sigma_3 + \sqrt{0 + s \sigma_c^2}
\]  
(22)

Substituting \(s = 0.11\) into Equation 22 gives:

\[
\sigma_1 = \sigma_3 + \frac{1}{3} \sigma_c
\]  
(23)

Equation 23 is a rearranged form of the principal stress difference \((\sigma_1 - \sigma_3)\) given earlier. Therefore use the Hoek-Brown failure criterion with \(m = 0\) and \(s = 0.11\). Martin et al. (1999) termed these parameter values "brittle Hoek-Brown parameters". The brittle parameters imply that strength is dependant only on principal stress difference, and is otherwise independent of confining stress, where as, conventional Hoek-Brown implies that strength is strongly dependant on confining stress.

Figure 5.7 compares the conventional Hoek-Brown failure envelope for a typical brittle hard rock mass with a Geological Strength Index (GSI) of 70 \((m = 8.56\) and \(s = 0.036\)) with the Hoek-Brown brittle parameters suggested by Martin et al. (1999). Figure 5.7 shows that fracturing occurs at much lower stress levels under high confinement, than predicted by conventional Hoek-Brown parameters.
Figure 5.7: Hoek-Brown strength envelopes.

The brittle parameters can be used in the program Examine2D to generate theoretical pillar stability graphs in the same manner and with the same model geometry as with the frictional parameters. One minor problem arises when trying to do this in that
Examine2D will not allow a 0 value to be entered for the $m$ parameter. A value of 0.000001 was used instead. Figure 5.8 compares the theoretical pillar stability chart generated using the brittle parameters with the one generated using the traditional $m$ and $s$ parameters based on the Rock Mass Rating system. The curves shown were generated in a hydrostatic stress field ($k = 1$).

![Graph showing pillar stability chart](image)

**Figure 5.8:** Pillar stability chart: Conventional versus brittle Hoek-Brown parameters.

Figure 5.9 compares the curve generated using the brittle Hoek-Brown parameters with the empirically based pillar stability curves reviewed in chapter 4. The brittle parameter curve corresponds well with the empirical curves, particularly for $w/h < 2$, although the curvature is curved upward.
Figure 5.9: Brittle parameter elastic pillar stability chart along with the empirical curves of the authors mentioned in Chapter 4.

To further illustrate the difference in the numerical model results using the brittle Hoek-Brown parameters and the traditional \( m \) and \( s \) parameters obtained by using the Geological Strength Index, the strength factor contours obtained from two Examine 2D models are presented. In both instances the pillar width-to-height ratio is 1 and the in situ stresses are hydrostatic \( (k = 1) \), with the same magnitude. Figure 5.10 presents the strength factor contours obtained using the brittle parameters \( (m = 0, s = 0.11) \) and Figure 5.11 presents the strength factor contours obtained using conventional Hoek-Brown parameters \( (m = 8.56, s = 0.036) \). In Figure 5.10 the pillar is failed (The strength factor contours are below one across the centre of the pillar). However, the strength factor
contours in Figure 5.11 are above one across most of the pillar (This pillar is a long way from the condition described as failure at the start of section 5.1).

Figure 5.10: Strength factor contours of a pillar using the brittle Hoek-Brown parameters ($m = 0, s = 0.11$ and $\sigma_c = 240$ MPa).

Figure 5.11: Strength factor contours of a pillar using $m = 8.56, s = 0.036$ and $\sigma_c = 240$ MPa obtained using GSI = 70.
5.1.3 EFFECTS OF STRESS FIELD VARIATION ON PILLAR STABILITY

GRAPHS GENERATED BY ELASTIC MODELS

Very rarely when designing underground excavations do we encounter a case where the horizontal and vertical stresses are of the same magnitude (i.e. hydrostatic stress field). As mentioned in Section 3 the weight of the overlying rock is often a good estimate for the vertical stress, and the horizontal stress is related to this by the $k$ ratio. With this in mind further pillar stability charts were generated for various $k$ ratios using Hoek-Brown brittle parameters. Figure 5.12 presents pillar stability charts for $k$ values of 0.5, 1 and 1.5. The elastic model also allows a second line with a Factor of Safety of 1.4 to be generated. In Figure 5.12 the lower line for each $k$ value represents a Factor of Safety of 1.4. The ability to generate this second line is beneficial because when designing openings there is a gray area around the factor of safety equal to one line in which some pillars fail and some do not and this line is able to represent one of the bounds of this area. The value of 1.4 was chosen because Lunder and Pakalnis (1997) had used that value and so it was an attempt to compare the results generated by this work to that of their database.

Figure 5.12 illustrates that for pillars with width-to-height ratios less than one $k$ has little or no effect on Factor of Safety. This is because for slender pillars, the horizontal in situ stresses flow around the array of pillars, and do not contribute to the confining stresses in the pillars. In Section 3.4, it was shown that the higher the relative value of the horizontal in situ stress (the larger the $k$ value), the higher the confining
stresses in the pillars (the lower the principal stress difference in the pillars) and hence, the stronger the pillars.

![Graph showing the effect of pillar loading](image)

**Figure 5.12: Effect of $k$ on pillar strength.**

5.1.4 **EFFECT OF INCLINED PILLAR LOADING**

Another problem that is often encountered in underground excavations is that the in situ principal stresses are not oriented in the same direction as the excavation axis. This can be caused by natural phenomena such as faults and dykes, by adjacent mining or by the orientation of the mining in space. This rotation of the in situ stresses relative to the pillars can impose shear stresses on the pillars. Hedley (1992) showed that at Quirke Mine in Elliot Lake, pillars which were oriented along the dip of the formation had symmetrical safety factor contours about the centre of the pillar whereas pillars which were oriented along strike had unsymmetrical safety factor contours, and were less stable.
Hedley (1992) also showed that rib pillars oriented along strike were weaker than those oriented along dip. The orebody dips at approximately 20 degrees to the South. The minor principal stress in Elliot Lake is vertical and the major principal stress is horizontal. The pillars are thus loaded by a normal stress perpendicular to the orebody and shear stress acting parallel to the dip of the orebody. In the case of dip rib pillars this shear stress acts parallel to the pillar's long axis and hence does not have a major effect on the stability of the pillars. For rib pillars oriented along strike the shear stress acts on the narrow cross-section of the pillar and hence affects its stability. If one looks at the seismic activity associated with the cascading pillar failure at Quirke Mine (Figure 5.14), it is apparent that most of the seismic activity is located near the haulage drifts were the pillars are oriented along strike. Pritchard and Hedley (1993) showed that the failure in pillars at the Denison Mine was oriented such that it corresponded with the angle of the principal stress relative to the pillars.

Figure 5.13: Stability of rib pillars aligned on dip and strike (Hedley, 1992).
Figure 5.14: Location of rockbursts associated with cascading pillar failure at Quirke Mine (Hedley, 1992).

Figure 5.15 presents the major principal stress distributions in a dip rib pillar and a strike rib pillar. The in situ minor principal stress is vertical with a magnitude of 15 MPa and the in situ major principal stress is horizontal with a magnitude of 20 MPa. The in situ intermediate principal stress was also assumed horizontal with a magnitude of 20 MPa for this modelling. The minor principal stress distributions in the same pillars as shown in Figure 5.15 are presented in Figure 5.16. These figures illustrate that the major principal stresses are higher in the strike rib pillar than in the dip rib pillar although the magnitude of the in situ stresses is the same. In both instances the minor principal stresses are very low at the core of the pillars. In terms of principal stress difference the
strike rib pillar has a larger difference than the dip rib pillar which illustrates that rotation of the in situ stress field alone, without a change in magnitude, can affect the stability of a pillar.

Figure 5.15: Major principal stress contours in a: (a) dip rib pillar, (b) strike rib pillar.

Figure 5.16: Minor principal stress contours in a: (a) dip rib pillar, (b) strike rib pillar.

Figure 5.17 illustrates that while the orientation of the far field principal stresses has relatively little effect on the strength of slender pillars (w/h < 1.0), stress field
orientation has a significant effect on squat pillars. The figure presents five different stability graphs, generated using Hoek-Brown brittle parameters. The ratio of in situ major to minor principal stress was kept constant at 1.5 for each case. The five cases have the major principal stress oriented at angles of 0, 22.5, 45, 67.5 and 90 degrees from horizontal. The pillar axis is parallel to the global vertical axis. The case where the major principal stress is at an angle of 90 degrees from the horizontal is equivalent to a $k$ of 0.67. It can be seen in Figure 5.17 that the worst case is when the far-field stresses are oriented at an angle of 45 degrees from the horizontal (45 degrees to the pillar vertical axis). These results imply that for squat pillars it is essential to know the correct orientation of the in situ stress field (ie. the effects of adjacent excavations).

Figure 5.17: Effect of in situ stress field rotation on predicted pillar stability graphs using Hoek-Brown brittle parameters ($m = 0$ and $s = 0.11, k = 1.5$).
5.1.5 **EFFECT OF VARYING EXCAVATION SPAN**

Models were run to look at the effects of varying the span of the excavations on the theoretical pillar stability charts developed using the brittle parameters. The results are presented in Figure 5.18. The curve labeled "room width / room height = 1" is the $m = 0$, $s = 0.11$ curve presented earlier in Figure 5.8. The other curve was generated assuming that the excavation spans were doubled while maintaining the same height.

![Figure 5.18: Effect of varying excavation span on the pillar stability charts.](image)

The curves show that varying the excavation span does not have much effect on the strength of the pillars. Although the pillar strength is not significantly affected, increasing the room width will increase the pillar stresses under the same in situ stress conditions. This can easily be explained by looking back at the tributary area theory.
presented earlier. Increasing the excavation span but maintaining the pillar dimensions increases the extraction ratio and hence the stress in the pillars.

5.2 ELASTIC-PLASTIC ANALYSIS

Boundary element programs are adequate for modelling the performance of rock in the elastic region of the stress-strain curve and for determining the stress condition at which failure initiates. However, once failure initiates, the problem becomes non-linear (plastic region) and hence a finite element analysis program with a plasticity model is better suited for modelling the post-peak, non-elastic behaviour. In the case of rock two types of post-peak behaviour are generally considered: (1) elastic-perfectly plastic, for which the peak and residual strength parameters are the same (Figure 5.19 (a)); and (2) elastic-brittle-plastic, for which the post-peak residual strength parameters are reduced (Figure 5.19 (b)).

![Figure 5.19](image-url)  
Figure 5.19: Two commonly used constitutive models for the simulation of rock mass behaviour: (a) elastic-perfectly plastic, (b) elastic-brittle-plastic.

The elastic-brittle-plastic behaviour is more representative of a brittle rock mass behaviour as the residual strength is considerably less than the peak strength.
Phase2 is a finite-element program developed at the University of Toronto by Rocscience Inc. This windows based user-friendly program was used in much the same way as the boundary element program Examine 2D to generate theoretical pillar stability charts. As with the boundary element modelling an array of five pillars was used and the core stress at mid-height of the pillar was used to define the term "pillar stress", rather than the average stress.

When using the finite element program Phase2 an external boundary that denotes the extent of the finite element region is required in the model. This boundary can either be added manually to the model or it can be automatically located by defining an expansion factor and a shape. The expansion factor places the external boundary that multiple of the excavation width away from the excavation boundary. For the modelling performed in this chapter a rectangular external boundary with an expansion factor of 10 was used. The distance the external boundary is from the excavation boundary has an effect on the accuracy of the results obtained from the program (Table 5.1). To obtain the results presented in Table 5.1 all model parameters were kept constant as presented in section 3.3.1.2 with the exception that the expansion factor used to generate the external boundary was varied.

Table 5.1: Comparison of pillar core stress with variations of expansion factor in Phase2.

<table>
<thead>
<tr>
<th>Expansion Factor</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pillar Core Stress ($\sigma_p$)</td>
<td>42.6</td>
<td>46.5</td>
<td>49.8</td>
<td>51.1</td>
</tr>
</tbody>
</table>
The external boundary as well as the excavation surfaces must also be discretized. The whole model was discretized using the default value of 200 excavation nodes. Since the central pillar is the one that is of most interest for the purpose of generating the theoretical pillar stability charts, it was custom discretized by selecting the two lines which represent its walls. A custom discretization of 40 nodes for the two lines was used. The degree of discretization in a finite element model has a significant effect on the size of the finite elements that are generated by the automatic mesh routine of the program. Three-noded triangles were used for the finite elements. By discretizing the walls of the central pillar quite finely more finite elements are generated in the central pillar and hence the results produced by the model are more accurate in this region. Figure 5.20 illustrates the pillars and finite element mesh used for an array of pillars with width-to-height ratios of 1. If displacement values are desired from Phase2 modelling, special attention must be given to the value of the dilation parameter. The dilation parameter is related to the plastic volumetric strain of the rock mass. For the modelling presented in this section the dilation parameter was set at its default value of 0.
Output from the finite element program was interpreted differently than the output of the elastic boundary element program. Once the stresses in the non-linear models exceeded the peak strength, the stresses were redistributed, hence decreasing the stresses in the failed regions, and increasing the stresses in adjacent regions. The program allows the user to examine the finite elements that have yielded (exceeded peak strength). The program does this by marking the finite elements with either an X or an O depending on the type of failure. The X denotes that the element has failed in shear and an O denotes that the element has failed tension. Figures 5.19 to 5.21 present typical output from the program Phase2 for an elastic-brittle-plastic analysis. They are zoomed views of the central pillar in Figure 5.20. The contours represent the distribution of the major principal stresses. Figure 5.21 is an intact pillar. The pillar in Figure 5.22 displays some yielding of the pillar walls. Practical experience indicates that although there is some
localized failure this pillar still retains its full load carrying capacity. Figure 5.23 is a pillar with extensive yielding. In Figure 5.24 it can be seen that all of the finite elements at mid-height of the pillar have yielded, which is indicative of the pillar response described in Section 2.2 as a “failed pillar”. It is the far-field stress condition one load increment less than the one that caused this condition that was used to obtain the pillar core stress at failure of the pillar.
Figure 5.21: Intact pillar from Phase2 modelling.

Figure 5.22: Partially failed pillar from Phase2 modelling.
Figure 5.23: Failed pillar from Phase2 modelling.

Figure 5.24: Finite element mesh overlain on Figure 5.23.
5.2.1 ELASTIC-PERFECTLY PLASTIC ANALYSIS

Theoretical elastic-perfectly plastic pillar stability charts were first generated in a hydrostatic stress field using the conventional Hoek-Brown frictional parameters of $m$ and $s$ set to 8.56 and 0.036 respectively. As was mentioned earlier non-linear modelling requires that residual $m_r$ and $s_r$ parameters also be defined. For perfectly plastic analysis residual $m_r$ and $s_r$ parameters are the same as the elastic $m$ and $s$ parameters namely 8.56 for $m_r$ and 0.036 for $s_r$. The pillar stability chart produced using this approach is presented in Figure 5.25, compared with the empirical pillar formulae given in Table 4.2.

![Figure 5.25: Elastic-perfectly plastic pillar stability chart using conventional Hoek-Brown parameters ($m=8.56, s=0.036; m_r=8.56, s_r=0.036$).]
It is evident from Figure 5.25 that this model somewhat underestimates the strength of slender pillars (width-to-height ratios less than 0.5) and grossly overestimates the strength of pillars with width-to-height ratios greater than 0.5.

The brittle Hoek-Brown parameters cannot be used in an elastic-plastic analysis. The idea behind the brittle parameters is that it takes different amounts of plastic strain for cohesion and frictional strength to be mobilized. The cohesion is mobilized first, and then after the onset of failure the frictional strength of the rock mass becomes mobilized. This means that the brittle parameters \((m = 0, s = 0.11)\) give a good indication of the onset of failure but at a certain point the cohesive strength of the rock drops and the frictional strength is mobilized. Hence, residual \(m_r\) and \(s_r\) parameters require values which reflect this drop in cohesive strength and increase in frictional strength. Hoek, Kaiser and Bawden (1997) suggest \(m_r = 1.0\) and \(s_r = 0.11\) as values for the residual Hoek-Brown parameters. The problem then arises that past a critical normal stress level \((\sigma_n^*)\), the residual strength becomes greater than the peak strength. Figure 5.26 illustrates this problem by presenting the peak and residual strength curves in Mohr stress space.
5.23 ELASTIC-BRITTLE-PLASTIC ANALYSIS

Theoretical pillar stability charts were generated assuming a strain softening response of the rock mass where, the residual Hoek-Brown parameters are given different values from the peak strength. In the case of rock, the residual parameters are lower than the peak strength parameters to represent a weakening of the rock mass after some failure has occurred. The elastic region $m$ and $s$ parameters were set to 8.56 and 0.036 respectively. The residual $m_r$ and $s_r$ parameters were taken as suggested by Hoek, Kaiser and Bawden (1997). A value of 1.0 was assigned to $m_r$ and a value of 0.01 was assigned to $s_r$. The results of this modelling in a hydrostatic stress field are plotted in Figure 5.27 along with the empirical pillar formulae given in Table 4.2.

Figure 5.26: Peak and residual strength curves in Mohr stress space.
Figure 5.27: Elastic-brittle-plastic pillar stability chart using $m=8.56$, $s=0.036$ and $m_r=1$, $s_r=0.01$.

While this approach gives slightly better results than the elastic-perfectly plastic approach using conventional Hoek-Brown parameters, it still underestimates the strength of narrow pillars, and grossly overestimates the strength of squat pillars when compared to the empirical data.

5.3 FAILURE AND ELASTIC ANALYSIS

In an attempt to relate the elastic-brittle failure process to the combined database of Lunder and Pakalnis (1997) and Hedley and Grant (1972) the following modelling methodology was used.
GSI values of 80, 60 and 40 which are representative of a Very Good, a Good and a Fair Quality rock mass, respectively, were chosen. The following Hoek-Brown parameters were derived for these GSI classes using the equations in Section 5.1.1, a uniaxial compressive strength of 230 MPa, and a $m_i$ value of 22, which were considered to be representative values for a brittle rock mass:

Table 5.2: Peak strength Hoek-Brown $m$ and $s$ parameter values for various GSI values.

<table>
<thead>
<tr>
<th></th>
<th>80</th>
<th>60</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>10.77</td>
<td>5.27</td>
<td>2.58</td>
</tr>
<tr>
<td>$s$</td>
<td>0.108</td>
<td>0.0117</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

The residual Hoek-Brown parameters were taken to be; $m_r = 1$ and $s_r = 0.001$ (Hoek, Kaiser, Bawden, 1995). A $k_{max} (\sigma_1/\sigma_3)$ value of 2 and a $k_{min} (\sigma_2/\sigma_3)$ value of 1.66 were used. The far-field stresses were increased until the elastic-brittle-plastic model showed that the entire cross-section of the pillar had yielded. The elastic stress distribution in the pillar was determined by re-running the model under elastic conditions. The difference between the two models is that as the pillar yields in the elastic-brittle-plastic model the stresses are redistributed. Where as in the elastic model the stresses continue to increase in the pillar. This procedure was repeated for pillars with width-to-height ratios ranging from 0.5 to 3. The elastic stresses in the pillars were chosen in an attempt to directly relate this work to that of the empirical case studies presented in Chapter 4 as most of the
empirical analyses used elastic stress analysis techniques for back calculating the pillar stresses.

The elastic stresses were then used to generate the pillar stability curves labeled GSI 80, GSI 60 and GSI 40 in Figure 5.28. The predicted pillar strength for GSI 80 exceed all of the empirical failure criteria. The rest of the curves presented in Figure 5.28 do relate to the empirical data in one sense or another. There is very little empirical data for pillar width-to-height ratios greater than 1.5, hence it is difficult to assess the validity of any of the curves in this region. However, the asymptotic nature of the empirical curves assumes that past some pillar width-to-height ratio the strength of pillars becomes constant. In reality pillars continue to increase in strength as their width-to-height ratio increases however, the stress induced failure of the skin of the pillar will at some point initiate at a constant pillar stress regardless of the pillar width-to-height ratio. The GSI 60, GSI 40 and brittle parameter \( m = 0, s = 0.11 \) curves all show the trend of continuously increasing pillar strength. In the pillar width-to-height ratio range of 0.5 to 1.2 the shape of the GSI 60 and 40 curves agree with the onset of stress induced failure (stable to unstable transition). In this same range of pillar width-to-height values the brittle parameter \( m = 0, s = 0.11 \) curve from Section 5.1.2 reflects the strain/cohesion loss controlled failure (unstable to failed transition).
Figure 5.28: Pillar stability graph showing Lunder and Pakalnis (1997) curves and database, Pritchard and Hedley (1993) curve, brittle parameter \( (m = 0, s = 0.11) \) curve and the results of the Phase2 modelling using GSI 40, 60 and 80.

### 5.4 SUMMARY

Pillar stability charts generated using conventional Hoek-Brown parameters derived from Geological Strength Index (GSI) values which are representative of the rock masses found in most Canadian hard-rock mines, do not correspond well with the empirical strength equations summarized in Table 4.2. The conventional Hoek-Brown parameter curves show a large increase in pillar strength as pillar width-to-height ratio and hence confinement increases. This tendency is not apparent in the empirical data. The Hoek-Brown brittle parameter \( (m = 0, s = 0.11) \) curves are in much better agreement with the empirical data. Therefore, pillar stability assessment is a matter of determining when
failure initiation ($d_f$ from Section 2.2) is equal to half the pillar width. At this point progressive failure by cohesion loss will lead to pillar collapse.

Some other important conclusions that came out of this chapter were that:

1. The ratio of the horizontal stress to vertical stress only affects the strength of pillars with width-to-height ratios greater than 1.0.

2. Rotation of the in situ stress field relative to pillars can impart shear stresses on the pillars; hence, a pillar that was stable when loaded axially can become unstable if the stresses are rotated without an increase in the magnitude of the stresses.
6 CONCLUSIONS

The goal of this thesis was to provide the mine design engineer with a theoretical framework for determining, a priori, the strength of hard rock pillars. In order to accomplish this, various modes of pillar failure and the stresses present in the pillars were examined. The various methods of determining the stresses present in the pillars were compared and it was found that tributary area theory gave acceptable results for simple geometries but it only gave the average principal stress in the pillar. Numerical models are better suited to determining the stresses in pillars. The modelling showed that an array of rib pillars could be modelled by a two dimensional model as long as the out of plane dimensions are adequate to make the case one of plain strain. Short rib or post-pillars require three-dimensional modelling.

The next step was to examine the work of other authors who had also analyzed hard rock pillars. For the most part the work of the other authors was based on empirical studies of known failures. The strength curves produced by these authors were relatively consistent, with minor differences most likely being attributable to limited data (in some instances), and the use of varying definitions for pillar failure.

The thesis then examined the hypothesis introduced by Martin (1997), that the strength of hard rocks is fundamentally controlled by a "cohesion loss" process. Through the use of elastic numerical models it was shown that by using conventional $m$ and $s$ parameters, derived from the Geological Strength Index (GSI), in the Hoek-Brown equation, pillar stability curves were generated which did not resemble the empirical
curves presented by the other authors. Non-elastic numerical models using conventional plasticity theory were also run using the well known Hoek-Brown \( m \) and \( s \) parameters based on the Geological Strength Index. Some of the curves generated using this approach agree with the onset of progressive stress induced failure in the empirical database for pillars with width-to-height ratios less than 1.2. Theoretical curves generated with an elastic model using the \( m = 0, s = 0.11 \) parameters set forth by Martin et al (1999) for a cohesion only strength criteria fit well with the empirical curves generated by all but one of the authors (i.e., Potvin et al., 1989). Pillars with width-to-height ratios less than approximately 0.7 tend to loose cohesion across their entire width at once and the brittle parameters reflect this.

The ratio of in situ horizontal stress to vertical stress \((k)\) and the orientation of the principal stress tensor with respect to the pillars affect the strength of pillars, especially pillars with width-to-height ratios greater than 1.0. Below a pillar width-to-height ratio of approximately 1.0 the confinement in a pillar is relatively unaffected by varying the far field stress tensor.

The theoretical framework presented in this thesis gives the mine engineer a sound basis for determining a priori the strength of mine pillars based on available data. As with all projects as the design is implemented field observations will have to be collected and the design refined to suit the specific application. This is particularly important for the design of pillars in ore, as the brittle parameters \((m = 0, s = 0.11)\) were established for pillars in hard brittle rocks and may change significantly for less brittle rocks.
7 REFERENCES


