Compressive Strength and Modulus of Elasticity of Masonry Prisms

by


A thesis submitted
to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirement for the degree of
Master of Engineering

Department of Civil and Environmental Engineering

Carleton University
Ottawa, Ontario
August 18, 1999
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0-612-48451-3
Abstract

Two important elements in structural engineering are the strength and the stiffness of the structure. In masonry, the primary components are, first of all, strong and stiff masonry units, and secondly, the relative weak and “soft” mortar.

Masonry units typically have compressive strengths ranging from 30 MPa to over 100 MPa in North America, and the standard compressive strength of the mortar is in the range of 10 to 25 MPa. Their respective moduli of elasticity when tested independently are significantly different as well, although it is generally accepted that the units are much stiffer than the mortars.

When these two components are put together to form structural elements, the overall strength and modulus are two pieces of important data that are needed by engineers to design for the strength and deformation of the structure properly.

The research program for this thesis is composed of computer simulation studies and laboratory testing. The computer simulation was designed to study the behaviour of the masonry model with respect to the strength and modulus of elasticity using data obtained from the Phase I testing program. Parametric analysis of the model behaviour was also conducted to examine the effects of different input parameters for the simulation model.
The second part of the research program consisted of laboratory testing of combinations of different strengths masonry units and mortar mixes. The testing program was designed to study high and low strength masonry units in combination with two strength extremes of mortar mixes. This thesis studied the envelope of this range of high strength unit/high strength mortar, high strength unit/low strength mortar, low strength unit/high strength mortar and low strength unit/low strength mortar combinations.

The test results were compared with other recent published test results and Canadian Standards.

The findings of this testing program clearly demonstrate the need for a better understanding of the mortar/masonry unit interface, and a better measuring technique for the deformation of mortar joints within a masonry prism assemblage. The properties of the mortar are changed due to the moisture absorption by the unit, and this change alone is significant enough to alter all the properties of the mortar when compared with the non-absorptive specimens prepared for standard mortar testing. Measuring methods using strain gauges across the mortar joints are unreliable due to the physical size of the strain gauges in comparison to the joint itself.
Acknowledgements

The author wishes to express his deepest gratitude and appreciation to his supervisor Professor G. T. Suter for his guidance and teaching throughout the entire research and reporting program. The author also wishes to extend his gratitude to Professor S. Kennedy for his guidance and cooperation in reviewing and helping to select an appropriate finite element program for the research.

The author also wishes to express his sincere appreciation towards Mr. K. McMartin and his staff for their assistance in the laboratory testing phase of the program.

The author further expresses his appreciation to the Department of Civil and Environmental Engineering of Carleton University for the crucial financial support for the duration of this Master’s Program, and to the National Research Council for the generous support at the early stage of the laboratory testing program.

The author especially wishes to express his heartfelt appreciation to his loving wife, and his children for their unconditional love, encouragement, and support throughout the years.
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\{F^a\} \quad \text{Vector of applied loads.}

\[ K \] \quad \text{Coefficient matrix.}

\[ K^r \] \quad \text{Tangent matrix.}

\{F^w_i\} \quad \text{Vector of restoring loads corresponding to the element internal.}

\{u\} \quad \text{Vector of unknown DOF (degree of freedom) values.}

i \quad \text{Subscript representing the current equilibrium iteration.}

E_m \quad \text{Modulus of elasticity of prism.}

f_{br} \quad \text{Compressive strength of brick.}

f_m \quad \text{Compressive strength of mortar.}

f'_m \quad \text{Compressive strength of prism.}

\gamma_{wa} \quad \text{Ratio of moduli of elasticity between brick and mortar.}

W_a \quad \text{Water absorption percentage (\%).}

\text{MPa} \quad \text{MegaPascal (N/mm^2).}

\text{GPa} \quad \text{GigaPascal (10^3 N/mm^2).}
1 Introduction

Two important pieces of data in any structural element are the strength and the modulus of elasticity of the material. In Engineered Masonry, these two components are the compressive strength, \( f'_{m} \), and modulus of elasticity, \( E_{m} \), of the element. The compressive strength is important because it determines the bearing capacity of the element; the modulus of elasticity is important because it provides the estimate of deformation of the element under loading.

Full size testing can be done to obtain these data, however, full size testing programs are typically too expensive to be conducted, and in some cases, impractical because of other limitations. Prisms testing in the past has been proven to be cost-effective and to provide some measure of correlation with full size testing. The Canadian Standards Association\(^1\) also recognises this test method to reflect the compressive strength and the modulus of elasticity of the structural element.

1.1 The Basic Challenge

Typical compressive strengths of masonry units produced in Canada are relatively high. They are usually of the order of 60 MPa and up, but the strength of the mortar, on the other hand, is very low. Typical compressive strengths of Type "S" or Type "N" mortars are about 12.5 MPa and 9.5 MPa respectively according to Canadian standards. The modulus of elasticity of masonry units is also relatively high.
when compared with the modulus of elasticity of typical mortar that is used in the structure. When a typical masonry element is built, the masons combine the masonry units with mortar to form that structural element. When these two different materials with vastly different properties are combined to form the structural element, the properties of the combined element are not readily identifiable.

Fig. 1 shows a schematic diagram of the relationships of the stress-strain curves of masonry unit, mortar and the prism assemblage made of these two components as expected from past experience. A prism assemblage is a term used to describe the small element constructed with several masonry units and mortar in stack.

![Schematic diagram of stress-strain curves of masonry unit, mortar and prism assemblage.](image-url)

Fig. 1 Schematic diagram of stress-strain curves of masonry unit, mortar and prism assemblage.
bond so that testing can be done on a smaller scale. The initial tangent of each stress-strain curve is defined as the initial modulus of elasticity of that particular material or prism. When these two materials are combined together, one would normally expect the stress-strain curve of the prism to lie somewhere between the two boundary curves. Drysdale explained this relationship in his book *Masonry Structures- Behaviour and Design* in detail. The question is where would the stress-strain curve of the prism assemblage lie for any given set of masonry unit and mortar combination.

To add to the complexity, while the properties of the masonry unit can be easily measured and tested, the properties of the mortar within the structural element are quite different from the results obtained by testing the standard mortar cube specimens.

This testing program is an attempt to correlate the relationship of the compressive strength and the modulus of elasticity of the prism assemblages using different combinations of high and low strength of masonry units and relatively high and low strength mortars.

In actual structures, many more aspects are just as important for the durability performance of the structure. Considerations for rainwater penetration, freeze-thaw cycles in the Canadian climate, bond strength of the mortars, creep and so on are also critical to the overall performance of the structure. However, they are beyond the scope of this testing program.
1.2 Research Test Program

Due to a limited amount of research results available, (Chapter 2 of this thesis outlines some of the recent published results by other researchers), there are many variables that need to be pre-determined before actual testing could take place. Therefore the research program was split into the following three different stages:

1. Preliminary test program to determine the testing criteria;

2. Computer simulation of the prisms using a finite element analysis program;

3. Final laboratory testing to determine the strength and the modulus of elasticity of masonry prisms.

1.3 Preliminary Test Program

Unlike ready-mixed concrete which is plant mixed and therefore usually quite reliable and consistent in terms of its slump, air content, compressive strength, and other properties throughout a project, mortar for masonry work is usually site mixed, and in many small projects even hand-mixed in a wheelbarrow; its properties are therefore highly variable.

The water/cement ratio, or more often in the masonry industry the water/binder ratio when both Portland cement and hydrated lime are used as the binding materials, is only one of the variables that would affect the strength and modulus of elasticity of the mortar. The water/binder ratio is defined as the ratio between the weight of water to the total weight of cement and lime. The labourer
hand-mixes the mortar and adds variable amounts of water to the mortar mix based on his experience and the amount of moisture present in the sand. The mason adds additional water to the mortar as required by the construction conditions and retemper it as necessary; this creates another variable. The contractor uses different brands of cement or lime on the same job, or different batches of bulk sand with different moisture content; all this contributes to the variability of the mortar strength and its modulus of elasticity. Even storing the bulk sand on site without proper protection against the weather elements would seriously affect the moisture content of the sand, and therefore, ultimately affect the water/binder ratio.

In general, however, it can be concluded that the compressive strength of mortar increases with an increase in cement content and decreases with an increase of sand, water or air content. Retempering is usually associated with a decrease in mortar compressive strength. The amount of the reduction in strength depends on the amount of water added and the time between mixing and retempering. It is this variability of mortar that may affect the overall performance of the masonry wall or structural elements. Although this is widely accepted as the norm, there are very few quantitative research reports available for reference.

Therefore, a first preliminary test program was carried out to prepare different mortar mixes and test their strengths to determine which mixes would be better suited for the final test program.
With co-operation from the National Research Council of Canada (NRC), the early stage of testing which consisted of mixing different batches, storage of specimens, and the subsequent compression testing of those mortar specimens, were all carried out at the Montreal Road campus of the National Research Council in Ottawa, Ontario.

1.4 Computer Simulation Program

A computer finite element analysis program was used to simulate different scenarios or properties of the materials, and examine the changes in strength and modulus of the prism. With the advance of computer technologies, it is both viable and cost-effective to simulate different scenarios to check against laboratory testing.

Unfortunately, there are so many commercial "finite element analysis" programs available with vastly different capacities and functions, that the task of selecting a suitable program was not easy. Some programs can only handle elastic analysis; some programs can only handle two-dimensional simulations; some limit the material properties to metals; others have a tedious user interface; and others are just not capable of computing large triangular matrices.

After evaluating different programs in the computer laboratory of the Department of Civil and Environmental Engineering of Carleton University and discussions with faculty members, the final decision was to use the program designed by ANSYS. This computer program can handle three-dimensional problems, non-
linear analysis, and most importantly, has a "concrete" element type that can simulate cracking and crushing of the element.

The simulation, first of all, consisted of the simple circular area and square area to confirm the stress flow and displacement pattern of split-tensile testing of a circular mortar cylinder and a cubic specimen of masonry unit. This was done to confirm that the simulation inputs are representative for preparation of the final simulation process.

The final process was to simulate the behaviour of the masonry prism assemblages with different masonry unit and mortar properties.

### 1.5 Final Test Program

The final test program was to test suitable prism assemblages with different combinations of masonry units and mortars. The task included the selection of suitable low and high strength masonry units to match with the appropriate soft and strong mortars. These different combinations would give good upper and lower boundaries of the possible spectrum of strengths and moduli of elasticity of the prism assemblages.

The final testing program itself was further divided into two stages. Stage I included testing different prisms using very high and low strength mortars with the same masonry units, and mounting sufficient strain gauges for strain measurement during compressive testing. The properties of the masonry units and mortars were
also separately tested. The data recorded were used to determine which mortar and masonry units were to be used in the second stage of the final test program.

Stage II of the final test program included building and testing five prisms for each combination of different masonry units and mortars. These combinations were high strength mortar with high strength unit, high strength mortar with low strength unit, low strength mortar with high strength unit, and finally low strength mortar with low strength unit. Different masonry units were considered to arrive at the final selection of two units. Over 20 different mortar mixes were tested to finally choose two mixes for the final program.
2 Recent Research Results

With shrinking financial resources and ever-advancing computing power, computer simulation is becoming more and more useful to researchers. J.G. Rots and many others started to use “finite element analysis” programs to simulate the interaction of mortar/masonry unit interface of prisms. The approach by Rots was to compare the effect on masonry prisms using the modulus of elasticity of mortar and a very low modulus neoprene material in-between the masonry units. The author proposed an estimating formula for the relationship between the vertical stress of the prism to the horizontal stress in the masonry unit. Unfortunately the author appeared to have used the basic compressive strength of the mortar as obtained from traditional non-absorptive mould specimens without taking into account the change in properties of mortar after moisture was absorbed by the masonry units in the prisms. Although the author concluded “that the edge effects and non-homogeneous deformation are important factors that cannot be ignored in masonry materials characterization,” the author did not quantify such edge effect and switched to reporting other aspects of the masonry structures.

S. Stöckl, H. Bierwirth and H. Kupfer first reported in 1994 at the Tenth International Brick and Masonry Association Conference (IBMAC) that the testing
of the same mortar using different testing methods could yield drastically different results. They utilised a unique steel loading brush to virtually eliminate the frictional effect of regular rigid loading platens. They reported that the compressive strength of mortars obtained from the traditional testing method must be used carefully as the non-absorptive specimens and the aspect ratio of the specimens always underestimated the actual compressive strength of the mortar joints. In certain conditions the strength of the mortar joints could be as much as four times the strength obtained from typical traditional cube specimens tested according to the German testing standards. This result was obtained by testing mortar joint specimens cut out from prism assemblages utilizing the steel wire loading platens to eliminate the frictional effect of the traditional loading platens. They were one of the very few who actually reported quantitatively the increase in mortar joint strength compared with the traditional cube specimen strengths.

S. Stöckl, K. Beckhaus and TH. Fritsche were again among the first to report the characteristics of actual bed joints in test prisms. They concluded that the modulus of elasticity of the same mortar could vary as much as ten times depending on the test method used. The type of test specimen, the production method of the test specimen, the size of test specimen, and the measurement system were just several factors affecting the measured results of the modulus of elasticity of the same mortar. They further concluded that “anyone analyzing masonry behaviour for example by means of finite elements” should take the differences into account to avoid “getting unrealistic calculation results using wrong material laws.”
Australian researchers A. W. Page, and D. S. Brooks expressed a similar common philosophy of underestimating the strength of masonry structures when prism testing results were compared against the Australian code. They revealed that when using the Australian code method to estimate the masonry structure's strength based on the correlation between the compressive strength of masonry unit and mortar type would typically result in a much lower strength than actual field tested results. Since the code method of estimation does not reflect the workmanship, site conditions and other factors, it generally tends to be much more conservative. These researchers recommended the use of prism testing to accurately reflect the actual workmanship and the actual materials used at a particular project site. They did not propose, however, any estimating formula that could be used to estimate the strength or modulus of elasticity in the event that prism testing is not feasible for that particular project.

G. T. Suter and E. M. F. Naguib suggested Poisson’s ratio of brick masonry is between 0.16 to 0.20 between bedding planes. They used computer 3-D modelling together with reviewing other previous research results in reaching their conclusion. They concluded the modulus of elasticity parallel to its bed planes was smaller than that perpendicular to its bed planes. This anisotropic nature of masonry structures makes any estimating process much more complex than a simple empirical formula could cover.
A. W. Hendry also confirmed the seemingly straightforward compression test on a masonry unit was not simple at all. Depending on the rate of loading, capping material used, and aspect ratio of the testing specimen, the test results would vary. He suggested a loading rate between 7 MPa/min to 40 MPa/min would be appropriate for consistent "a statistically significant but not large variation in apparent strength." The capping material and its thickness also affected the strength of the masonry units. Grinding the testing specimen flat was discussed as an option, however not recommended due to the inherent nature of the workmanship difficulties and machine limitations.

Hendry also noted the tensile strength of a masonry unit was best based on a split-tensile test because of its uniform tensile stress distribution throughout the entire depth of specimen. He went on to suggest other researchers use the split-tensile test result to calculate the compressive strength of the same unit.

He further pointed out that the workmanship factor was the most important factor among any others in masonry structures. Incomplete filling of joints would inevitably give rise to inaccurate results. To ensure proper workmanship, professional masons were employed to construct the prism assemblages at different phases of the final test program of this thesis. A purpose-made timber jig was employed in the final test program to ensure a uniform joint thickness of prisms was achieved.
The testing method could also significantly affect the test results. R. G. Drysdale & H. E. Wong suggested that the test prisms should be at least four units high to avoid the frictional influence of loading platen restraint. The final test prisms of this thesis were all eight units high to allow for the Linear Variable Displacement Transducers (LVDT) to be mounted on specially made mounting frames.

L. Binda et. al. concluded that "the use of strain gauges for displacement detection in the case of small specimens and soft materials with high porosity may give unreliable results due to the influence of the glue stiffness." This is particularly true for mortar joints regardless of the strength of the mortar. She also concurred with Stöckl that loading platens consisting of steel brushes would be the most successful device to avoid platen confinement in compressive testing.

J.J. Brooks and B. H. Abu Baker recently published their findings after reviewing 184 sets of modulus/strength data. They proposed an empirical formula relating the modulus of brick prisms to the strength of their components, i.e. the mortar and unit. In their proposed formula, they included the water absorption of the unit as one of the variables, together with the strengths of the mortar and unit. With that many factors that could influence the modulus of the prisms, they claimed that with their proposed formula they could predict the modulus with better accuracy. They reported an error coefficient of about 26% when compared with the British Standards and the corresponding Euro-Code. This was, according to the authors, a drastic improvement from any other currently used estimation methods. Appendix B
highlights the formulae proposed in their report, and presents the corresponding estimation of the modulus of elasticity for the materials used in Phase II of the final testing program.
3 Preliminary Test Program

3.1 General

In order to understand the behaviour of mortar, different batches of mortar were prepared and tested in this preliminary testing phase of the program. This preliminary test program was critical in selecting suitable mixes for the final test program; mortars with different proportions of ingredients were made and tested at seven days after the specimens were prepared.

The mortars were prepared with assistance from the National Research Council (NRC); the mixes were prepared in the mechanical mixer at NRC using white Type 10 Portland cement, Type ‘S’ hydrated lime, and aggregate sand from a local quarry. While proprietary masonry cements are popular among masons in the construction industry because of their good workability, they contain different filler materials that are not listed in the manufacturer’s published material data. It is because of their proprietary secrecy that the true composition of masonry cements cannot be identified, and hence this type of cement was judged unsuitable for the research purposes in this thesis.
3.2 Materials

In order to control the variables for the entire testing program, the materials used were carefully selected and maintained. Throughout the testing program, Type 10 white Portland cement from the same manufacturer was used.

Since mortar is usually exposed to weather, in Canada freezing of moisture within the mortar is inevitable during winter months. For the durability of the mortar it is important to have a minimum of 8 to 10% air content within the hardened mortar to allow the moisture to have the space to freeze within the mortar without prematurely failing the mortar. Initially, air-entrainment agent originally developed for reinforced concrete was added to the trial mixes in an attempt to produce the desired air content.

Different mixes using different proportions of Portland cement, hydrated lime and sand aggregate with air-entrainment agent were tested for air content, among other plastic properties. The measurement of the air content of the mortar was done using the portable laboratory air content measuring instrument available at NRC. The instrument is based on the theory that air volume expands with a reduction of pressure; the measurement of volume expansion was indirectly measured by the linear potentiometer at different pressure settings. Unfortunately, the measurements were not reliable, as different readings were recorded for mortars with exactly the same proportion of ingredients. The inconsistency of air content in the mortar was first attributed to the fact that the air-entrainment agent was originally developed for
reinforced concrete mixtures, and the dosage needed for a mortar batch was very small and therefore very difficult to administer accurately. The second factor that led to the inconsistent measurement was that the equipment was later found to be malfunctioning by NRC laboratory staff.

After testing different batches of mortar mixes in the laboratory at NRC and at one of the repointing projects in Ottawa, the final air-entrainment agent chosen for this research program was Type ‘SA’ hydrated lime manufactured by Bondcrete. This product designation was to signify that the hydrated lime satisfies the Canadian Standards Association’s requirement for Type ‘S’ hydrated lime with air entrainment already pre-mixed during the manufacturing process of the hydrated lime. This type ‘SA’ hydrated lime was consistently producing an air content of between 10-11% throughout the repointing project. The field measurement was done with the more reliable air content measuring instrument made of steel construction for checking air content in concrete.

Recently, A. H. P. Maurenbrecher and K. Trischuk\textsuperscript{12} conducted an extensive study on the properties of pointing mortar. In their study, various air entrainment agents were used. They reported that using different brands of air entrainment agents in the mortar mixes or Type ‘SA’ lime were all successful in achieving over 10% air content consistently when the mixes were fluid enough to have a flow table reading of 145 to 150%. Although this report was only available after the current testing program for the thesis had been completed, it did confirm that the use of Type ‘SA’
lime was an effective method to ensure proper air content in all the mortar mixes used in the final test program.

The aggregate sand used throughout the entire testing program was river sand that was well graded with a gradation within the Canadian Standards Association’s upper and lower gradation limits.

3.3 Mortar Mixes

The initial trial mixes of mortar ranged from a pure cement/sand mix to a cement/lime/sand mix. Pure cement/sand mixes ranged from a ratio of 1:3 to 1:7.5 to obtain the high and low limits of compressive strength, whereas the cement/lime/sand mixes ranged from 1:0.25:3 to 1:3:10 to obtain high and low limits. Water/binder ratios were recorded after the mixes achieved bedding mortar consistency; they ranged from 0.9 for high cement content mixes for the 1:3 cement/sand mix or 1:0.25:3 cement/lime/sand mix, to 1.25 for high sand content mixes on the other end of the spectrum. Since the air content is important in regular site work in the Canadian climate, the pure cement/sand mixes were eliminated for the final testing program due to the inconsistency of air content measurement when using an air-entrainment agent. Type ‘SA’ lime was used throughout the final test program in order to achieve a consistent air content in the mortar. Therefore, all mortar types used in the final test program were Portland cement/lime/sand (PC/L/S) mixes.
Plastic properties of the mortar mixes were also recorded, although they were just recorded to ensure that the laboratory mixes were similar to the actual field mixes by masons. Each workable mix had a 135% to 160% flow table reading. Air contents were initially measured with the portable air content measuring instrument at NRC, but were later concluded to be not reliable as the instrument was not functioning properly at the time of testing. An air meter designed for measuring air-content for concrete was finally used to measure the air content; it consistently recorded 10 to 11% for all mortar mixes using Type ‘SA’ hydrated lime.

A Vicat cone penetration test was also performed for each of the preliminary trial mixes. The test is a measure of workability of the mortar by measuring the penetration depth of a conically shaped metal head that was released from the top surface of the wet mortar in a fixed size container. This test gave some indication of the workability of the mortar; any mortar mixes that had a cone penetration test result of less than 20 mm were considered as having poor workability and were therefore rejected. Mortar specimens obtained from these low workability mixes were also likely to have visible horizontal separation cracks at half height of the specimen showing the distinct different lifts during the moulding of the specimens. This clearly suggested that the standard tempering procedure during moulding was not adequate when the mortar was too dry due to low workability.
3.4 Compressive Test Specimens

Six 50 mm cube specimens were prepared for each mortar mix using non-absorptive moulds. All specimens were moist cured for six days using polyethylene wrapping which effectively achieved 100% humidity inside the wrapping; this was followed by one day of laboratory curing at about 50% relative humidity and 20° to 22° C before testing. All compressive testing of mortar cube specimens was done directly on the platens of a universal testing machine without capping materials placed on the specimen, and with all specimens rotated 90° to ensure the rough top surface of the specimens was not in contact with the loading platens.

The compressive strengths of the mortar ranged from 1.8 MPa for the 1/3/10(PC/L/S) mix to 18.0 MPa for the 1/0.25/3(PC/L/S) mix. The mixes selected for the final test program were a 1/3/10(PC/L/S) mix for the low strength mortar and a 1/0.5/4(PC/L/S) mix for the high strength mortar.
4 Computer Simulation

4.1 General

With the advance of computer technologies, the finite element analysis technologies are becoming increasingly more powerful and readily available. Many of the commercially available finite element programs can simulate the behaviour of many civil engineering structures so that a better understanding of the structures can be achieved without physically testing the full size, or even the reduced model, of the structures. However, a basic understanding of material properties must first be achieved prior to the use of computer programs in order to perform a realistic simulation.

4.2 Basic Simulation Approaches

There are many different approaches to simulate the behaviour of masonry structures. The most common approaches\textsuperscript{13,14,3} are:

1. Equivalent homogeneous material or macro-simulation approach – this approach assumes an equivalent homogeneous material of the masonry structure despite the well documented fact that both masonry units and mortar behave quite anisotropically;
2. Smear line element approach – this approach utilizes line elements representing horizontal and vertical joints of the masonry structure; and

3. Detailed definition or micro-simulation approach – this approach defines mortar and masonry units with clear boundaries and their individual properties.

These different simulation methods offer different degrees of accuracy and therefore should be used according to the requirements of individual situations. The first approach offers a general behaviour simulation of the structure and is better suited for studying large size structures. The second approach offers a better accuracy of the behaviour of a masonry structure and is suitable for simulation of an individual element of the structure such as a wall with openings to study the concentration of stress. The last approach studies the detailed interaction between mortar and masonry units and is most suitable for the current study as it provides the most detailed accuracy during simulation.

Fig. 2 shows the typical prism used in the final test program. It consists of eight masonry units, together with a compressive fibre material on top and bottom of the prism to reduce the frictional effect of the platen during loading. No strain gauges were mounted on the prisms during the final Phase II test program as it was clear from the Phase I testing results that:

a) The strain gauges mounted on the masonry units of the prisms yielded the same results as those mounted on the masonry units during individual
compressive testing. Therefore, the modulus of elasticity of the masonry units was measured with the strain gauges mounted on the individual units during compressive testing of the units only.

b) The smallest practical size of strain gauge used was 5 mm long with a plastic mounting base plate of 10 mm length. Therefore, the 10 mm thick standard mortar joints used in the final Phase II test program were just too small for strain gauges to be mounted on them without interference between the edges of the gauges and the masonry units. Smaller gauges of 2 mm length were available but not suitable for a porous material like mortar that might have significant localized voids.

For computer simulation work, it was not necessary to deal with the entire prism. Due to symmetry, the simulation only needed to model one-quarter of the masonry unit and one-eighth of the mortar joint. Fig. 3 shows the computer model used in the computer simulation.
Fig. 2  Typical arrangement of test prism.
Fig. 3  Basic computer model for simulation.
4.3 Material Properties

Using the ANSYS finite element program, the basic properties of the masonry unit and the mortar were entered into the program. The basic properties for the masonry unit for the purpose of computer simulation were defined as compressive strength, modulus of elasticity, modulus of rupture, and Poisson’s ratio. Several other properties might affect the actual site condition, but were not essential in the computer simulation; some of these are the rate of absorption, the moisture content, and size of pores within the units. Although these factors may affect the properties of the prism assemblages at actual job sites, they were not critical in the computer simulation simply because the computer program did not have the capability to actually simulate the process of mortar hardening and water absorption.

The compressive strength of masonry units was determined by the normal compressive testing of 50 mm cubes in the case of a stone unit, and half bricks of different units in the case of brick units. The moduli of elasticity of the units were determined by strain gauges mounted on the specimens during compressive testing, and also on some masonry prisms to confirm the results from the compressive testing in the Phase 1 of the final testing. A third set of strain gauges was mounted on the small beam specimen for the modulus of rupture test in Phase 1 of the final testing program to confirm the modulus of the masonry unit. Photo 1 shows a typical strain gauge mounting with epoxy compound on the surface of the beam specimen. Photo 2 shows the typical strain gauge arrangement on the prism assemblages.
Photo 1  Typical mounting of strain gauge using epoxy glue on the unit.
Photo 2  Arrangement of strain gauges on trial prism.
The modulus of rupture of masonry units was determined by using a combination of two different methods. The first method was using a two-point loading on simply supported beams of dimensions 50 mm x 50 mm x 300 mm, and the second method was split-tensile testing using a line load across the top of the specimen of a 50 mm cube specimen. Fig. 4 shows a schematic test arrangement of a simply supported beam specimen and Photo 3 shows the actual testing arrangement of the beam specimen on the testing machine. Two-point loading ensured a uniform bending moment between the two loading points.

Fig. 4 Typical arrangement for modulus of rupture test of masonry unit on simply supported beam.
Photo 4 shows the top view of the beam specimen indicating the location of the strain
gauge mounted at the top fibre of the specimen that lies between the two loading
points. Photo 5 shows the failure plane of the specimen after failure; a clean failure
plane was to be expected for all tensile stress failures of the beam specimen.
Photo 4  Location of strain gauge on simply supported beam specimen.
Fig. 5 shows the schematic test arrangement for the split-tensile test of a stone masonry unit and Photo 6 shows the actual failed specimen after testing. In order to verify the computer modeling procedures, a simple model of the split-tensile test was used in the finite element analysis program to check against the known theoretical concept. Fig. 6 is the finite element computer simulation of the split-tensile test model. The loading was in the Y-direction with a negative value as indicated by the downward arrow, and the model was restrained at the centre of the area at the bottom as indicated by the chevrons. The stress contour was plotted in Fig. 7 as the intensity of the stress in the X-direction; Fig. 8 plots the principal stresses of each

![Photo 5](image)

Photo 5 Typical failure plane on the specimen as expected in split-tensile test.
Fig. 5 Schematic diagram for split-tensile test of a stone unit.

Photo 6 Split specimen after failure.
Fig. 6 Computer model for split-tensile test.
Fig. 7 Contour of X-direction stress intensity.
Fig. 8 Principal stresses directions and magnitude.
Fig. 9 Principal stress plot to identify the directions of the stresses only.
element centroid. This plot shows the magnitude of the stresses in proportion to the intensity of the stress, that is the longer the arrow, the higher the stresses. The direction of the arrow pairs also indicates whether the element is in compression or tension. The arrows pointing towards the centroid are the compression stresses and those pointing away from the centroid are the tensile stresses experienced by that element. This plot demonstrates the tensile stresses experienced in the X-direction when the downward loading is applied in the middle of the area. For clarity, the same principal stress is re-plotted in Fig. 9 using the same length of arrows to show the directions of the stresses only.

The property that was not actually obtained by tests was the Poisson’s ratio; it was initially entered as 0.15 based on previously documented results\(^\text{15}\).

The basic properties of mortar are defined as compressive strength, modulus of elasticity and Poisson’s ratio. The modulus of rupture of the mortar was not considered essential as the failure mechanism of the prisms was always initiated by the splitting of the masonry units, whereas the mortar joints were still under a tri-axial compression state\(^2\) just prior to the failure of the prism. Fig. 10 shows the schematic relationship of this phenomenon of the equivalent tri-axial compression state of the mortar. As the prism was subjected to uni-axial vertical loading, the mortar would attempt to expand laterally because of the Poisson’s effect, however, the masonry unit did not expand at the same rate due to its much higher modulus of elasticity. Therefore, if good bonding was achieved at the bed joints between the masonry units
and mortar, the masonry units were restricting the lateral expansion of the mortar. This restriction results in a lateral bi-axial compression confinement pressure thus subjecting the mortar to tri-axial compression. Similar to reinforced concrete, the compressive strength would increase significantly when lateral-confining pressure was applied.

The reverse was true, however, for the masonry units; the lateral expansion of the mortar acted as the tensile forces on the masonry units, thus the masonry unit was undergoing a uni-axial compressive and bi-axial tensile stress state. The net result was the prism would fail by the tensile splitting of the masonry units once the units exceeded their modulus of rupture. This applied loading would then be considered as

![Diagram of stress states](image)

Fig. 10 Effective stress of masonry unit and mortar under vertical compression of prism.
the ultimate strength of the prism. This phenomenon is also referred to as the indirect tensile testing of prism assemblages by some researchers.

Similarly, the compressive strength of the mortar was determined by the compressive strength of standard 50 mm cube specimens from non-absorptive plastic moulds as shown in Photo 7. The modulus of elasticity of the mortar was also determined by strain gauges mounted both on cube specimens and on joints of prisms as shown in Photo 8, and also by a Linear Variable Displacement Transducer (LVDT) mounted on a 3” diameter x 6” high cylindrical specimen as shown in Photo 9. Poisson’s ratio was assumed as 0.15 from previous research data.

The compressive strengths of masonry units and mortars are two of the most tested properties for typical projects simply because the specimens are relatively easy and inexpensive to prepare when compared with testing for other properties. Most importantly, they are reasonably reliable strength indicators for commercial or research laboratories.

Other properties, however, are seldom tested, because they are either very expensive to carry out or often very unreliable. The preparation work for testing using strain gauges is both expensive and, at times, erroneous if the gauges are not prepared properly. Before the strain gauge can be mounted on the specimen, the surface of the base material must be sanded, thoroughly cleaned, and treated with epoxy resin to fill the voids on the surface. The strain gauge must then be carefully applied on the surface of the epoxy resin under specific manufacture’s instructions.
The data transmitting wiring is then soldered onto the conducting terminals before the strain gauge readings can be recorded with another remote recording machine or computer. The strain gauge will simply give erroneous result if any one, or any combination, of the preparation procedures is not followed correctly. Another possible area of concern is that the epoxy resin fills the voids of the base material and may have altered the properties of the base material somewhat.

Photo 7 50 mm cube mortar specimen being prepared in a non-absorptive plastic mould.
Photo 8 Strain gauge mounted on cube specimen to determine the stress-strain relationship during compressive testing.
Photo 9  LVDT mounting instrumentation to determine the modulus of elasticity of mortar on a cylindrical specimen.
4.4 Computer Simulation Modeling Procedure

Only a quarter of the masonry unit and one-eighth of the mortar joint were needed for simulation of a prism assemblage because of symmetry. Therefore, the co-ordinates of the corners for the masonry unit and the mortar were entered into the computer to form two connecting solid volumes. The volume representing the mortar was fitted with elements that had the properties of mortar. The volume representing the masonry unit was further divided into two regions. All these two regions were fitted with elements that had the same properties of the masonry unit. The volume closer to the mortar, however, was fitted with smaller elements, and the volume further away from the mortar was fitted with larger elements. This would provide better computation accuracy near the mortar/unit interface plane. Fig. 11 indicates the three regions of the computer model represented by the three different volumes. Fig. 12 shows the element configuration in the model.

Each type of elements was defined with its own properties in terms of its respective uni-axial compressive strength, cracking tensile stress or modulus of rupture, initial modulus of elasticity and stress-strain relationship. The stress-strain relationships were all multi-linear approximations. Fig. 13 shows one of the multi-linear stress-strain curves approximating the non-linear characteristic of masonry unit, and Fig. 14 shows a typical stress-strain relationship of mortar.
The three principal X-Y, X-Z, and Y-Z planes were assigned as planes of symmetry for the simulation. The applied loading was on the top surface of the model, in the negative Y-direction representing the compressive loading. Fig. 15 shows the direction of loading pressure indicated by the downward arrows and the planes of symmetry indicated by the chevrons.

Fig. 11 3-D model used in computer simulation.
Fig. 12  Computer model showing the sizes of finite elements.
Fig. 13  Stress-strain relationship of elements representing mortar.
Fig. 14  Stress-strain relationship of element representing masonry unit.
Fig. 15  Diagram showing the loading pattern and planes of symmetry.
The simulation was carried out in sub-steps of 1 MPa increments until the model failed due to non-convergence of differential equations. The final loading was then further sub-divided into 0.1 MPa increments to obtain the failure pressure to the nearest 0.1 MPa. This compressive loading would then be defined as the ultimate strength of the prism in the computer model. Since the elements defined in the simulation contained no steel reinforcement, as soon as they reached either the cracking stress (modulus of rupture) or the crushing stress (ultimate compressive stress), the matrix equations would fail to converge and this signaled the program to terminate further calculations.

Fig. 16 shows typical displacement vectors of the elements in the model under compressive loading from the Y-direction. The results are discussed in later chapters.

In order to study the effect on the failure strength of the model due to changes in properties of different individual components, the input variables were changed and the failure pressures recorded. These changes were carried out systematically one variable at a time.
Fig. 16  Typical displacement vector of edges of the elements under compression.
5 Finite Element Analysis Theory

The Finite Element Analysis uses a “discretization” process with the following set of simultaneous equations to equate the internal forces with the external applied loads:

\[ [K][u] = \{F^e\} \]

where \([K] = \) coefficient matrix

\[ \{u\} = \text{vector of unknown DOF (degree of freedom) values} \]

\[ \{F^e\} = \text{vector of applied loads} \]

This set of equations would be correct if the matrixes are all linear. However, if the simulation were to be non-linear, this simple matrix solution would give rise to incorrect accumulation of computational errors.

Fig. 17(a) shows the relationship of the assumed true response and the calculated responses. An iteration process is therefore required to overcome this shortcoming of straight simulation and Fig. 17(b) shows the general schematic of the iteration process.
Fig. 17  Schematic diagram of Newton-Raphson iteration process.
When the coefficient matrix \([K]\) is itself a function of unknown DOF values, then the original matrix equations become non-linear equations. The Newton-Raphson method is an iterative process of solving the non-linear equations and can be written as:

\[
\begin{align*}
\left[K_i^T\right]\{\Delta u_i\} &= \{F^s\} - \{F^r_i\} \\
\{u_{i-1}\} &= \{u_i\} + \{\Delta_i\}
\end{align*}
\]

where \([K_i^T]\) = Tangent matrix

\[i = \text{subscript representing the current equilibrium iteration}\]

\[\{F^r_i\} = \text{vector of restoring loads corresponding to the element internal loads.}\]

The matrixes \([K_i^T]\) and \(\{F^r_i\}\) are evaluated based on the values given by \(\{u_i\}\), and the term \(\{F^s\} - \{F^r_i\}\) represents the residual vector or sometimes referred to as the out-of-balance load vector. This is the amount of out-of-equilibrium from the system. Fig. 18 shows the graphical relationship of a single solution iteration of a single Degree of Freedom (DOF) model.

In this case, \([K_i^T]\) represents the tangent of the stiffness matrix, \(\{F^r_i\}\) is the restoring force matrix calculated from the element matrix, and \(\{u_i\}\) is the displacement matrix of the system. As seen from the graph, the tangent solution does not represent the actual response, and therefore an iteration process is necessary for
the solution to converge. The solution obtained at the end of each iteration process represents the load level \( \{F^u\} \). The final converged solution would be in equilibrium, and in such case, the restoring load vector \( \{F_i^n\} \) computed from the current stress state would equal the applied load vector \( \{F^a\} \) within the acceptable tolerance.

The ANSYS program implements this Newton-Raphson procedure and the program would go through all the intermediate steps until convergence is reached for each load step. Since masonry prisms typically behave in a non-linear manner, this method is necessary for accurate simulations. The convergence criterion was set at \( 10^{-6} \), a relative small value in order to ensure the computation error did not exceed the simulation errors.

![Graph](image)

**Fig. 18** Iteration process for Newton-Raphson Solution.
6 Computer Simulation Results

6.1 General Observation

The computer model was to simulate the behaviour of the masonry prism under vertical compressive loading until failure. Fig. 19 shows the displacement nodes of the elements in the model as simulated using the ANSYS finite element analysis program. It shows the displacement of the nodes of the elements is changing gradually from the vertical negative Y-direction near the top to the lateral direction near the horizontal X-Z plane. Fig. 20 shows the enlarged portion of the deformation of the model at the joint interface. The negative Y-direction (downward) pressure causes the vertical dimension to decrease, and this deformation is directly related to the strain measurement. At the same time, the Poisson’s effect causes the X- and the Z-direction dimensions to increase. However, the rate of increase depends on the modulus of elasticity of the elements; that is, the elements with a lower modulus of elasticity would expand laterally more than the those with a higher modulus of elasticity.

Fig. 21 shows the directions of the principal stresses of each element at its centroid. Each element has three pairs of arrows representing its three principal stresses. The arrows pointing away from the centroid of the element represent the tensile stresses, whereas those arrows pointing towards the centroid of the element
would indicate compressive stresses. Fig. 22 shows the same plot of principal stresses except the magnitude of the stresses are scaled, in other words, the higher the stresses, the longer the line between the arrows, and vice versa.

From these plots it is clear that those elements near the masonry unit/mortar interface but inside the mortar have all three principal stresses pointing to their respective centroid of the elements, indicating compressive stresses in all three principal directions. This matches exactly the theory of the equivalent tri-axial compression stress of the mortar.

The elements just above the unit/mortar interface representing the masonry units all show compressive stresses in the Y-direction parallel to the loading. However, for the ones near the symmetry planes of the masonry unit, arrows are pointing away from the centroid of the elements indicating tensile stresses are present in the two principal stresses perpendicular to the loading pressure. When the tensile stress in these elements exceeds the input criterion of the modulus of rupture, the elements in the simulation would “crack” and the simulation would stop due to non-convergence of the triangular matrix. This clearly explains the laboratory observation that the failure mode of prisms is always the splitting of the masonry units and not the compressive failure of the mortar.
Fig. 19  Computer simulation showing the direction of displacement vectors.
Fig. 20  Enlarged portion showing the deformation at mortar joint interface.
Fig. 21 Principal stresses of elements showing the tensile stress of the masonry unit near the joint interface.
Fig. 22  Principal stresses of elements showing the magnitude of stresses with proportional length of arrows.
The failure strength of the prism in the simulation is dependent on any of the following three conditions: it depends on the modulus of rupture of the masonry unit, the compressive strength of the masonry unit and of the mortar. The computer model would indicate prism failure when the integration points on the elements do not converge, and this non-convergence will occur when:

a) the compressive stress in the elements exceeds its respective pre-defined maximum strength of either the mortar or masonry unit, or,

b) the tensile stress of the elements exceeds the input value for the modulus of rupture of the masonry unit. This usually occurred just above the unit/mortar interface.

Using the basic compressive strength of the standard 50 mm cube non-absorbent specimens, the simulation result will produce a non-convergent result once the compressive stress of the mortar reaches the input compressive strength data. This occurs because the computer program interprets this compressive stress as the "crushing" stress of the element, thereby generating non-controllable displacements of the nodes. The program will indicate local failure of the mortar, and will halt the simulation. This will occur once the loading pressure exceeds the compressive strength of the mortar, even though laboratory testing clearly reports that a much higher strength can be achieved. As previously stated, earlier research work has observed that, the failure of a prism is initiated by the splitting of masonry units when they reach their modulus of rupture stress. The crushing strength of the mortar can be
much higher than that recorded by testing the non-absorptive specimens for two reasons. First, the mortar is experiencing an effective tri-axial compressive pressure; second, the moisture absorption during setting would increase the mortar strength considerably. Therefore, in order to allow the simulation to continue until the phenomenon of splitting of masonry unit could be achieved, with no other data being available for the effective lateral confining pressure of the mortar, the crushing stress of the mortar was eventually omitted in the simulation.

When the mortar tries to expand laterally under vertical loading, and with good bonding between mortar and unit, the net effect is a tensile stress being developed in the masonry unit at or very close to the joint interface. The computer simulation clearly demonstrates this phenomenon in agreement with actual laboratory testing and the previously reported failure mechanism when the crushing strength of the mortar is omitted. The result is that the simulation will stop only when the tensile stress of the elements representing the masonry unit exceeds its pre-determined modulus of rupture.

6.2 Parametric Analysis

In order to gain a better understanding of the different factors influencing the ultimate strengths of the prisms that are not normally tested in laboratory, different simulations using different constants were conducted and the results were analyzed. The possible parameters that can be changed are Poisson’s ratio of masonry unit,
Poisson’s ratio of mortar, the modulus of elasticity of masonry unit, and the modulus of elasticity of mortar.

6.2.1 Change in Poisson’s Ratio

Poisson’s ratio of the masonry unit and the mortar is a piece of data that is not normally tested for and reported in traditional compressive testing, but is important in computer simulation as it influences the lateral expansion of the masonry unit and the mortar under compressive loading. Based on previous research results, the initial Poisson’s ratios were chosen as 0.15 for both masonry units and mortar.

Fig. 23 shows the maximum tensile stress of the masonry elements near the interface was 1.9 MPa as indicated in the red zones of the plot, with Poisson’s ratio of masonry unit being set at 0.20. Fig. 24 shows the maximum tensile stress of the masonry elements was 2.7 MPa, with every other parameter being constant but Poisson’s ratio of the masonry unit changed to 0.10. These simulations indicated that when Poisson’s ratio of the masonry unit was reduced, the tensile stress of the elements in the masonry unit, in the horizontal directions, increased. When the prism is subjected to vertical loading, the masonry units and the mortar will expand laterally at different rates due to their different moduli of elasticity even if their Poisson’s ratios are the same. Therefore, if Poisson’s ratio of the masonry unit is reduced, the difference in the rate of lateral expansion is further increased causing an increase in tensile stress of the masonry unit.
Fig. 23  X-axis stress contour with Poisson's ratio of masonry unit being 0.20.
Fig. 24  X-axis stress contour with Poisson's ratio of masonry unit being 0.10.
This ultimately reduces the compressive strength of the prism as the tensile stresses in the masonry would reach the modulus of rupture at a much faster rate.

Fig. 25 shows the tensile stress near the mortar/masonry unit interface with Poisson’s ratio of the mortar being 0.10. Fig. 26 shows the same interface with Poisson’s ratio of the mortar increased to 0.20. The X-axis stress plots from the two different simulations revealed the tensile stress in the masonry unit increased from 1.0 MPa to 3.8 MPa just by changing the value in Poisson’s ratio of mortar. This is just the reverse of the previous parameter. As Poisson’s ratio of the mortar increases, so does the difference in the lateral expansions of the mortar and the masonry units. This increase of the difference causes the increase of the tensile stress in the masonry unit.

It can be concluded that the higher the difference of Poisson’s ratios between the masonry unit and the mortar, the faster the prism would fail. This result is important as this relationship would explain one aspect that contributes to the variability of masonry structures simply because traditional strength testing of the masonry unit or mortar simply does not reflect variations of Poisson’s ratio of the unit. Without the proper material data, the commonly obtained data of the compressive strengths of masonry unit and mortar are insufficient to understand the complete behaviour of the prisms.
FIG. 25  X-axis stress contour with Poisson's ratio of mortar being 0.10.
Fig. 26  X-axis stress contour with Poisson's ratio of mortar being 0.20.
6.2.2 Change in Modulus of Elasticity

The effect of changes in Young's modulus of the masonry unit was examined next. The result showed that the higher the Young’s modulus of unit, the higher the tensile stress in the unit along the interface. Fig. 27 shows the plot of the nodal stress in the X-axis for modulus of unit being set at 54 500 MPa, and Fig. 28 shows the corresponding plot when the modulus of unit was set at 100 000 MPa. With the lower modulus of elasticity, the tensile stress was only 0.17 MPa, while the higher modulus elasticity caused the tensile stress to increase to 2.4 MPa. The reverse is true for modulus of mortar. The simulation revealed the higher the Young’s modulus of mortar, the lower the tensile stress in the stone along the interface. It appears that the relative difference between the masonry unit and the mortar caused the increase or decrease in the tensile stress in the masonry unit. The higher the relative difference, the higher the tensile stress.
Fig. 27  X-axis stress contour with modulus of elasticity of masonry unit set at 54 500 MPa.
Fig. 28  X-axis stress contour with modulus of elasticity of masonry unit set at 100 000 MPa.
The results of the computer simulation of a masonry prism suggested that the interface of the mortar and the unit was the key investigation area. The principal stresses of the elements changed from tri-axial compression in the mortar, to bi-axial tension in the masonry just above the joint interface. A test program to investigate the relationship of the modulus of elasticity of the prism and across the mortar section was therefore developed.

Since there was an other research program being carried out with Ohio stone at the National Research Council (NRC), it was decided to use the same Ohio stone to compare with their findings. Ohio stone is a type of sandstone used quite frequently in the Ottawa area. This sandstone exhibits a fairly homogeneous behaviour within the stone units compared with limestone which is more layered and therefore behaves quite differently in orthogonal directions. The size of Ohio stone unit was chosen such that both strain gauges and DEMEC points could be mounted on the stone surface to obtain Young’s modulus of the stone. The recorded value can then be used to compare with values obtained from other different measuring methods, as well as with NRC’s value. Fig. 29 shows the arrangement of the prisms for the phase I of the final test program.
Fig. 29  Typical prism layout in Phase I test program.
7.1 Prism Construction and Testing

The size of the stone unit was 100 mm x 100 mm x 80 mm; 10-mm length strain gauges were mounted on opposite faces of the unit. Furthermore, two DEMEC discs were mounted at 50 mm spacing, also on opposite faces of the same unit, for direct strain measurements using a 50 mm DEMEC gauge. Strain gauge data were recorded during compressive testing along with its associated loading, with the on-line data recording computer connected to the compression testing machine. At the same time, the 50 mm DEMEC gauge was used to obtain strains at varies stress levels. These two sets of strain measurements were compared against each other, as well as against the results supplied by NRC.

The thickness of mortar joints was chosen as 20 mm to allow for the mounting of strain gauges across the mortar joints. This relatively thick joint was needed for the installation of suitable size strain gauges directly on the mortar joints without interference of the masonry units; 5-mm length gauges with 10 mm base plastic mounting plates were used throughout. This allowed the strain gauges to have a minimum 2 to 3 mm spacing between the ends of the strain gauge bases and the unit/mortar joint interfaces of the prisms. Another pair of strain gauges were mounted in the transverse direction at the centre of the mortar joint to record the strain in the orthogonal direction of the loading.

Two different strengths of mortar, one with extremely high strength of 25.5 MPa and the other with relatively low strength of 5.3 MPa, were selected in this
Phase I test program. The mortar mix proportions were obtained from earlier test results; the high strength mortar of 25.5 MPa was achieved with the mix of 1/0.25/3 (PC/L/S), representing 1 part of white Portland cement, 0.25 part of Typs ‘S’ hydrated lime and 3 parts of sand, while the low strength mortar of 5.3 MPa was achieved with a 1/2/8 (PC/L/S) mixture. The mortar mixes and the prisms were prepared and constructed by an experienced mason to ensure good workmanship. The prisms were then wrapped with plastic for moist curing for 7 days, and laboratory condition curing for another 21 days before the first compressive testing sequence. The laboratory conditions were at about 20 to 22° C and 35 to 50 % relative humidity.

Instead of loading the prisms to its ultimate strength and failure, these prisms were loaded to approximately 25 percent of the estimated ultimate capacity based on the preliminary data supplied by NRC. The prisms were unloaded prior to repeating another two cycles of loading and unloading.

The testing was carried out on a Tinius-Olsen 400-kip universal testing machine in the Structural Laboratory of Carleton University. The loading was applied at a rate of 0.05 kN per second but halted at every 1.5 kN interval for the 50 mm DEMEC measurements across the mortar joints and on the stone to be taken. A 200 mm DEMEC measurement for the overall strain of the prisms was also recorded at the same loading levels.
7.2 Test Procedures of Mortars

The properties of mortar were measured with the compressive testing of standard 50 mm cube specimens prepared in non-absorptive moulds, and 100 mm cube specimens prepared in purpose-made non-absorptive vinyl-lined plywood moulds to compare the sizing effect of the mortar. The compressive testing was carried out with two 10-mm strain gauges mounted on opposite sides of the cube specimens to obtain the stress-strain relationship of each mortar type. Fig. 30 shows a typical arrangement of the strain gauges on the mortar specimens for compressive strength and modulus of elasticity measurement, and Photo 10 shows the actual mortar specimens with strain gauges.
Fig. 30  Typical strain gauge locations on mortar specimen.

Photo 10  Different sizes of mortar specimen with strain gauges.
As mentioned earlier, 5 mm strain gauges were mounted on the mortar joints in vertical and horizontal directions on the prisms to obtain the stress-strain relationships, and the approximate value of Poisson's ratio of the mortar. Photo 11 shows the prism assemblage with all the strain gauges connected into the data-recording computer.

Photo 11 Prism assemblages with all strain gauges connected to data-recording computer.
7.3 Test Procedures for Ohio Stone

The properties of Ohio stone were measured using different testing methods and specimen types. The first test involved compressive tests on 50 mm cube specimens with two 10-mm strain gauges mounted on opposite sides to obtain the stress-strain relationship and the compressive strength of the stone. The second involved 50 mm x 50 mm x 200 mm simply supported beam specimens loaded at two points to obtain the modulus of rupture of the stone; 10 mm strain gauges were also mounted on the top compression fibre and bottom tension fibre to compare the stress-strain relationship of the stone obtained by other methods. The third test involved split-tensile tests on 50 mm cube specimens to compare the modulus of rupture obtained from the beam specimens. The fourth involved the mounting 10 mm strain gauges on the stone units of the prisms to measure the stress-strain relationship of the stone unit during the compression tests of the prisms.
8 Phase I Test Results

8.1 Properties of Mortars

The stress-strain relationships of the mortars are plotted in Figs. 31 to 34. The plots are the average value of the two strain gauges mounted on opposite sides of each specimen. Fig. 31 shows the stress-strain relationship of the 1/2/8 mortar on a 50 mm cube specimen and Fig. 32 is the relationship on a 100 mm cube specimen of the same mortar mix; Fig. 33 shows the stress-strain relationship of the 1/0.25/3 mortar on a 50 mm cube specimen and Fig. 34 shows the relationship on a 100 mm cube specimen.

For the 1/2/8 mortar, the maximum strength of the 50 mm cube specimen was 5.3 MPa and of the 100 mm cube was 4.6 MPa, representing 86% of the 50 mm cube. This is consistent with previous findings that the size of cube specimens affects the ultimate strength of the same mortar mix.
Fig. 31  Stress-strain relationship of 1/2/8 mortar for 50 mm cube specimen.
Fig. 32  Stress-strain relationship of 1/2/8 mortar for 100 mm cube specimen.
Fig. 33  Stress-strain relationship of 1/0.25/3 mortar for 50 mm cube specimen.
Fig. 34  Stress-strain relationship of 1/0.25/3 mortar for 100 mm cube specimen.
For the 1/0.25/3 mortar, the maximum strength of the 50 mm cube specimen was 25.5 MPa and of the 100 mm cube specimen 19.7 Mpa, representing 77% of the 50 mm cube strength.

### 8.2 Properties of Ohio Stone

The compressive strength of the 50 mm cube Ohio stone specimen was determined as 58.0 MPa. The modulus of rupture of the Ohio stone was calculated as 2.8 MPa for the two-point simply supported beam testing. This value was in good agreement with the results from the average value of 2.9 MPa obtained from the split-tensile tests for three 50 mm cube specimens. Fig. 35 shows the plot of the stress-

![Stress-Strain Relationship of Ohio stone in simply supported beam test](image)

**Fig. 35** Stress-strain curves of Ohio stone on top and bottom fibre of the simply supported beam specimen.
strain relationship of the stone, recorded by the strain gauges that were mounted on the beam specimen.

The initial modulus of elasticity for the Ohio stone was also obtained as 8.0 GPa. The modulus of elasticity was well within other measuring methods used in this thesis, as well as within the reported result from NRC.

8.3 Prism Test Result

Phase I of the test program recorded several different stress-strain relationships of Ohio stone and mortars, in addition to the overall relationship for the prisms. Fig. 36 shows the different stress-strain relationships of the prism constructed with the weak mortar and Fig. 37 shows the matching stress-strain relationships of the prism made with strong mortar. Each figure has three different sets of data representing DEMEC gauges at different locations of the prisms.

The first set, consisting of 50 mm DEMEC data, was used to determine the modulus of elasticity of the Ohio stone only; six measuring points were averaged to plot each data point on the curve. The second set, consisting of 50 mm DEMEC data, was used to determine the modulus of elasticity of the section across the mortar joints. Four data points were recorded for the average value used for each point of the plot. The third set of data was plotted using 200 mm DEMEC data to measure the overall strain deformation of the prisms; only two measuring points were available for the average value used to plot the graph.
Stress-strain curve for weak mortar with 200mm Demec & 50mm Demec gauges

Stress-strain relationships of Ohio stone prism with 1/2/8 mortar.

Fig. 36
Stress-strain curve for strong mortar with 200mm Demec & 50mm Demec gauges

Fig. 37 Stress-strain relationships of Ohio stone prism with 1/0.25/3 mortar.
The forth set of data points were the weighted average of all 50 mm DEMEC points as shown on the plots in Fig. 36 and Fig. 37, used to compare the modulus of elasticity of the prisms obtained from the 200 mm DEMEC points. Both curves confirmed a good match of the initial modulus of elasticity of the prisms between two different sets of DEMEC data points. It was decided, therefore, in the phase II of the final test program, only the measurement across the 200 mm length would be needed to obtain the overall modulus of elasticity for the prisms.

These two plots also showed the initial modulus of elasticity of both prisms were approximately the same regardless of the mortar strengths. Both test results showed the initial moduli of elasticity of the prisms were in the range of 7 800 MPa. When compared with the basic modulus of elasticity of the Ohio stone, the initial modulus of elasticity of the prisms did not appear to be affected by the strength, nor the modulus of elasticity, of the mortar.

Six strain gauges were mounted on each of the prisms to record the modulus of elasticity of Ohio stone. The average modulus of the stone of these six sets of data was 8.0 GPa. This was in good agreement with the 2-point load simply supported beam test, and was also in good agreement with the separate testing carried out by the National Research Council of Canada12. This good agreement of results led to the conclusion that strain gauges could be omitted in phase II of the test program. Therefore, in phase II of the final test program, the measurement of the modulus of elasticity for the masonry units would be done with the strain gauges mounted on the
compressive testing specimens only and no strain gauges would be used on the masonry units of the prisms.

Fig. 38 shows the plots of the average values of stress-strain relationship of Ohio stone as well as the vertical and the transverse directions of mortar, using the strain gauge data recorded during the compression testing of the prism made with 1/2/8 mortar. The modulus of elasticity of the mortar is interpreted as about 5.5 GPa, and together with the transverse direction strain data, the value of Poisson's ratio is calculated as about 0.14; this value is also in good agreement with the value of 0.15 used in the Finite Element Analysis simulations.

Fig. 39 shows similar plots of the strain gauge data of the Ohio stone prism made with the 1/0.25/3 mortar. The modulus of elasticity of Ohio stone was again consistent with other tests conducted. However, the modulus of elasticity of the mortar for this strong mortar was interpreted as 14.0 GPa with Poisson ratio recorded as approximately 0.14.

The moduli of elasticity of these two different mortars are very different. Furthermore, the values also differ quite significantly from the values obtained from the compressive testing of their respective non-absorptive cube specimens.
Fig. 38  Average stress-strain relationships of Ohio stone and 1/2/8 mortar in vertical and horizontal directions.
Stress-strain relationship of Ohio stone and 1/0.25/3 mortar

- - - Vertical strain gauges on Ohio Stone
- - - Vertical strain gauges on mortar
- - - Horizontal strain gauges on mortar

Fig. 39 Average stress-strain relationships of Ohio stone and 1/0.25/3 mortar in vertical and horizontal directions.
This is to be expected as it is well documented that the mortar strengths of absorptive specimens, and in the joints, are always higher than those of the non-absorptive specimens. This response is further explained in detail in the recent published papers authored by Stöckl, Bierwirth, and Kupfer⁴, and Stöckl, Beckhaus and Fritsche⁵. They reported that not only the compressive strength measured in the actual mortar joints can be four times that of the strength obtained from regular non-absorptive specimens, the modulus of elasticity of the same mortar can differ up to ten times depending on the testing method chosen.

They have suggested multiplying the standard test results obtained in accordance to the German Standard DIN 18555 by four to estimate the compressive strength of the mortar joint; in this they have accounted for the effect of the water absorption process of the brick unit, and of the friction on the rigid loading platen. However, they still have not proposed any formula for estimating the effective lateral confinement pressure experienced by the mortar.
9 Final Test Program – Phase II

9.1 Mortar Mixes and Masonry Units

After the results of the phase I test data had been reviewed, it was decided that the range of the mortar strengths should be 2.0 MPa for the low-end, and about 15 MPa for the high-end strength. These values are closely related to the Type “O” mortar for the low strength and Type “M” mortar for the high strength mortar. Type “K” mortar was not being considered as it would be a purely “lime/sand” mortar mix and not likely to be representative for modern masonry work.

Since the low-strength of the masonry unit was to have a low compressive strength and most sandstones have compressive strengths of 65 MPa and higher, the decision was made to select a solid restoration style clay brick. After searching several sources, it was decided to choose an American restoration brick with an average compressive strength of about 30 MPa. This type of brick was newly manufactured to the sizes and shapes of older bricks for the restoration of heritage buildings, hence the name of restoration brick. As a high strength masonry unit of about 100 MPa, a sandstone flagstone from a nearby quarry from the Province of Quebec was selected. These two strength levels provided good lower and upper strength limits for the masonry units.
9.2 Properties of Mortar Mixes

Further trial mixes of mortar with different compositions were prepared and tested prior to the final composition being decided. The final mix for the low strength mortar was 1/3/10 (PC/L/S), with a water/binder ratio of 1.2. This mix produced an average of 2.1 MPa compressive strength at 28 days and would be referred as the low strength mortar. The high strength mix was 1/0.5/4 (PC/L/S), with a water/binder ratio of 0.9. This mix produced an average of 14.4 MPa compressive strength at 28 days and would be referred to as the high strength mortar. As it was not the controlling factor of failure for the prisms, the modulus of rupture for the mortar was no longer tested in the phase II program. The mortars were mixed in a wheelbarrow by a professional mason, who measured the portions of each component into the wheelbarrow, dry mixed the mixture, and then added the pre-determined amount of water slowly while mixing the mortar.

9.3 Construction of Prisms

For compatibility reasons, the dimensions of the flagstone units were chosen as 90 mm x 90 mm x 30 mm to closely match the sizes of a quarter of a brick. To ensure good quality of cutting, both flagstone and restoration bricks were cut by a local commercial cutting facility in Ottawa. The final dimensions of the restoration bricks were 90 mm x 90 mm x 27 mm thick due to the thickness of the cutting blade.

The mason mixed one batch of mortar at a time, using the wheelbarrow available at the laboratory as shown in Photo 12. Both the flagstone prisms and the
restoration brick prisms were constructed using the same batch of mortar. The procedure was then repeated for the other mortar. Six prisms for each combination were constructed all in the same day.

The prisms were eight units high, with standard 10 mm joint thickness. In order to ensure good quality of workmanship, the same professional mason who prepared the mortar was retained to construct these prisms. A jig marked with the height of each unit was used throughout the construction of prisms to ensure uniformity of the joints. Fig. 40 shows the typical arrangement of prisms while Photos 13 and 14 show typical prism construction of flagstone and restoration bricks. Photo 15 is a close-up view of several joints of the flagstone prism to show the moisture being drawn from the mortar into the flagstone. This moisture absorption process enhances bonding between the mortar and the masonry units. However, this same process also alters the properties of the mortar in the prism assemblages.

The prisms were then stored in the laboratory wrapped in plastic for 7 days of moist curing, and then under laboratory conditions of about 20° C and 30 to 50 percent relative humidity for another 20 days.

One prism from each combination was trial tested on the 14th day after construction to record the approximate failure loading of each combination. This information was used to predict the final failure load of each combination of prisms. The final testing of prisms was carried out on the 28th or 29th day after construction. Since the properties of the masonry units do not change, but the mortar strength will
increase over time, the prisms with the same mortar mix were tested on the same day. That is, all five stone prisms with weak mortar and all five brick prisms with the same mortar were tested on the 28\textsuperscript{th} day, and all ten prisms with strong mortar were tested on the 29\textsuperscript{th} day.

### 9.4 Test Procedure

The prisms were tested in the 400-kip universal testing machine at the Structural Laboratory of Carleton University with two sets of LVDTs mounted on frames. These metal frames were custom-made by a local tool and die company to ensure precise alignment of anchoring points and therefore accurate LVDT recording. Fig. 41 shows the schematic arrangement of prisms and the location of LVDTs for the recording of the stress-strain relationship of the prism, and Photo 16 shows the mounting frame with the LVDT already in place. The mounting frames were spaced consistently at 200 mm by inserting a set of fixed length timber spacers between frames for each prism as shown in Photo 17. Each prism was then tested with 12 mm fibreboard between end platens of the testing machine as shown in Photo 18. Each prism was subjected to the same loading rate of 0.05 kN per second to ensure a consistent loading rate for each prism. The loading and LVDT displacements were recorded at every second interval during the testing period of each prism, with the computer program for LVDT recordings pre-programmed to have 200 step increments per inch accuracy.
Photograph showing the mortar mix was dry-mixed thoroughly before water was added.
Fig. 40  Typical prism arrangement in Phase II test program.
Photo 13  Typical construction progress of prism with flagstone.
Photo 14  Typical construction progress of prism with restoration bricks.
Photo 15 Close-up view of mortar joints showing the moisture absorption process.
Fig. 41  Schematic arrangement of LVDTs on the test prism for strain measurement.
Photo 16 Typical arrangement of LVDTs mounted on the special frames.
Fixed length timber spacers are used to ensure constant LVDT spacing.
Photo 18 Prism arrangement in loading machine.
10 Phase II Test Results

10.1 Properties of Masonry Units

The primary properties of the masonry units used in the final testing program are presented in the next sections.

10.1.1 Restoration Brick

The compression tests were carried out with the Tinius-Olsen 400-kip universal testing machine in the Structural Laboratory of Carleton University. The compressive strength of the restoration brick was 33.0 MPa obtained by averaging the compressive test results of three 50mm cube specimens. The modulus of elasticity of the brick was 14.0 GPa calculated from the stress-strain data recorded with the strain gauges mounted on the brick specimens during compressive strength testing. Fig. 42 shows a typical plot of the stress-strain relationship of the brick. The tensile strength of the brick was 4.0 MPa calculated from split-tensile tests on two 57 mm x 90 mm x 90 mm specimens. This represents 12% of the compressive strength of the brick.
Fig. 42  Stress-strain relationship of restoration brick.
10.1.2 Flagstone

The ultimate compressive strength of this particular flagstone was averaged at 103.0 MPa from compressive testing of three 50 mm cube specimens. The modulus of elasticity was interpreted as 36.3 GPa obtained from strain gauge data on one specimen only. Fig. 43 shows a typical stress-strain relationship of flagstone used in the final testing program. The modulus of rupture, on the other hand, was calculated from split tensile testing of three other 50 mm cube specimens, and the average result was 14.7 MPa, representing 14.3% of the compressive strength.

![Stress-strain relationship of flagstone unit.](image)

Fig. 43  Stress-strain relationship of flagstone unit.
10.2 Properties of Strong Mortar

The 28-day compression test result of the strong mortar (1/0.5/4) revealed that the average compressive strength of six 50 mm cube specimens was 14.4 Mpa; this result was confirmed by compressive testing of 75 mm diameter by 150 mm high cylinders. The stress-strain relationship of the cylinder testing is shown in Fig. 44 and the ultimate strength was 13.7 MPa. The modulus of elasticity of the mortar was interpreted using this stress-strain relationship recorded with the LVDT mounted on the cylinder specimen during the compression test. Photo 19 shows the testing set up of the LVDT on a cylinder specimen; the average modulus of elasticity was determined as 10.8 GPa.

10.3 Properties of Weak Mortar

Fig. 45 shows the stress-strain relationship of the weak mortar; based on 75 mm diameter by 150 mm high cylinders, the compressive strength of this mortar was 1.9 MPa while its Young's modulus was interpreted as 8.0 GPa. The average compressive strength of six 50 mm cube specimens was, however, averaged as 2.1 MPa.
Fig. 44  Stress-strain relationship of strong (1/0.5/4) mortar.

Fig. 45  Stress-strain relationship of weak (1/3/10) mortar.
Photo 19 LVDT mounted on 75 mm diameter x 150 mm mortar specimen.
10.4 Compressive Test Results of Prisms

The final test results of the prisms are plotted in Figs. 46 to 49. The computer program used in the collection of data had fixed increment steps for the LVDT displacements. The accuracy of the LVDT recording by the computer was limited to 200 divisions per inch, i.e. a continuous interpretation of the displacement of the LVDT was not possible. This resulted in data points which were truncated down to the lower increment or rounded up to the next higher increment, and caused slight zigzagging of data points along the curves.

Fig. 46 shows the plots of the stress-strain relationships of the prisms for the flagstone with strong mortar. The average compressive strength of the prisms was 41.0 MPa and the average modulus of elasticity was 11.3 GPa. All five stress-strain curves were extremely similar and so were their ultimate strengths, indicating very low variability among these prisms.

Fig. 47 shows the stress-strain relationships of the prisms for the flagstone and weak mortar. The compressive strength of the prisms was averaged as 34.0 MPa, and the associated initial moduli of elasticity were interpreted as between 4.8 GPa and 10.2 GPa with a mean value of about 7.4 GPa. Although all five stress-strain curves were also very similar with very low variability among these prisms, the interpretation of the modulus of elasticity could be very subjective depending on the individual engineer. The stress-strain curves of these prisms did not exhibit a typical
ascending and slow descending before failure; they followed a fairly consistent smaller modulus of elasticity once the stresses passed beyond 6.0 MPa. Finally, sudden rupture took place at fairly similar ultimate failure stresses.

The average ultimate strength of the brick prisms with strong mortar was 13.8 MPa, and the moduli of elasticity could be interpreted between 4.3 and 7.1 GPa with an average of 5.7 GPa as shown in Fig. 48. Prism 1 was discarded in the averaging due to a pre-cracked condition in one of the bed joints. The bottom sections of the stress-strain curves were discarded in the determination of moduli.

The average compressive strength of the brick prisms with weak mortar was 11.0 MPa, and the moduli of elasticity were between 2.8 GPa and 5.7 GPa depending on the interpretation of the initial tangent of the curves as shown in Fig. 49. This set of data was the least consistent in terms of their ultimate strengths and their moduli of elasticity, although the average was interpreted at 4.3 GPa.
Table 1 summarizes the average compressive strengths of prisms, Table 2 summarizes the average moduli of elasticity of prisms, and Table 3 presents the range of moduli of elasticity of prisms interpreted.

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<th>Table 2 Average moduli of elasticity of prisms</th>
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Fig. 46 Stress-strain curves for flagsstone prisms with strong mortar.

Fig. 47 Stress-strain curves for flagstone prisms with weak mortar.
Fig. 48  Stress-strain curves of brick prisms with strong mortar.

Fig. 49  Stress-strain curves of brick prisms with weak mortar.
Photos 20 and 21 show two typical failed brick prisms, and Photos 22 and 23 show some typical failed stone prisms. In all cases, the splitting of the masonry units is very clear, agreeing with the reported failure mechanism of masonry prisms. Photo 23 further captured the instant of the failure of a prism. The sudden release of energy caused the near explosion of the stone units.
Photo 20  Typical brick prism failure mode.
Photo 21  Typical brick prism exploding at failure.
Photo 22 Typical flagstone prism failure mode.
Photo 23 Typical stone prism explosion at failure.
11 Discussion of Results

11.1 Phase I Testing

Since the phase I test program was only designed to help arrive at the parameters for the phase II test program, there were an insufficient number of prisms built to have any statistical reliability. However, the knowledge gained was still valuable in designing the phase II test program.

First of all, the modulus of elasticity of the masonry units was consistent between different test methods. Therefore, it was not necessary to use strain gauges on the masonry units on the prisms in phase II testing to obtain the modulus of elasticity of the units. The modulus of elasticity of masonry units could be determined from simple compressive testing of units, with strain gauges mounted on the test specimens.

Secondly, the determination of the modulus of elasticity of mortars proved to be much more complicated than this testing program could measure. Therefore, the attempt to measure the modulus of elasticity of the mortar in the prisms was omitted due to lack of suitable instrumentation and equipment available at the time of testing. This phenomenon was later confirmed by the German researcher Stöckl\(^\text{4,5}\) after the testing program for the thesis was completed. Stöckl et al. concluded that there appear to be three different zones across the vertical plane of the mortar joint in a
prism. The top zone, the middle zone and the bottom zone each had its own distinct properties in terms of their compressive strengths and moduli of elasticity. They compared the results by testing the bed joint mortar taken from the actual prism and loaded with steel brush platen to eliminate the frictional effect of the platens and effects from non-absorptive specimens.

It was due to unique "clips" utilised in their testing program they were able to obtain the modulus of elasticity of the mortar across the bed joints in the prisms. They concluded that the modulus of elasticity of the sample taken out from bed joint mortar was 5 to 15 times higher than the results obtained from specimens prepared using regular non-absorptive moulds for the same mortar mix. This explains the unexpected result of the phase I test prism with high strength mortar which actually had a higher modulus of elasticity in the mortar joint than that of the overall prism. The test prism with low strength mortar was so low in compressive strength, the modulus of elasticity of the mortar in the bed joint actually exceeded the overall value of the prism.

Thirdly, the use of 200 mm DEMEC gauges for the measurement of the modulus of elasticity of prisms was confirmed by the weighted average of the 50 mm DEMEC gauges. The results obtained using these two different DEMEC gauges were exactly the same; i.e., the 200 mm DEMEC gauges proved to be just as effective as the weighted average of 50 mm DEMEC gauge measurements in measuring the overall modulus of elasticity of prisms. Therefore, LVDT’s with proper mounting
frames with anchoring points spaced at 200 mm could be used to obtain the modulus of elasticity of the prisms.

Fourthly, the modulus of rupture value of the masonry units was essential for the computer simulation, but neither the compressive strength nor the modulus of rupture of the mortar played a role. The modulus of rupture of masonry units was a piece of important data needed in the computer simulation to determine the failure load of the prisms, but the ultimate compressive strength of mortar derived from the non-absorptive moulds was not needed.

11.2 Computer Simulation Results

The computer simulation provided a valuable tool in understanding the behaviour of the masonry prisms, especially when different parameters could be entered to simulate different situations. Unfortunately, however, there are just not enough research data available to fully utilize the finite element analysis program to understand the behaviour of the mortar and unit interface in the prisms. In their separate respective tests, Beckhaus and Fritsche measured the modulus of elasticity with the same mortar type in the bed joints of prisms and found startling results when compared with the current German code. Beckhaus found the modulus was averaging 4500 MPa while Fritsche found the modulus was between 700 to 1200 MPa depending on whether the voids in the bricks were taken into account or not. Compared to the estimated value of 11 000 MPa that the German code allowed for that combination of brick and mortar, Beckhaus and Fritsche’s results were extremely
low. They representing only approximately 35% to 10% of the code provision, thus making the computer simulation in this thesis very difficult as there are simply too many unknown variables. Until a better understanding of the properties of mortar in the actual bed joint is achieved in terms of its strength and modulus, simulating the true behaviour of prisms in computer programs is virtually impossible.

The ANSYS finite element analysis program has the capability of simulating crushing failure of the elements, and the program checks the stress of every element to find if any reaches the crushing stress that was input. When the element reaches the crushing stress, the program would interpret that element has cracked. Since the computer program does not have the capability to account for the effective lateral compressive pressure provided by the masonry units, the increase in crushing strength of the mortar is not being accounted for in the simulation. Thus when the elements in the mortar reach the specified crushing stress, the program would interpret that the mortar has failed, thus prematurely halting the simulation process. The crushing strength of the mortar was eventually deleted altogether from the input data in order to achieve more realistic prism strengths.

The moduli of elasticity for masonry units and mortars were needed to simulate the deformation of the model. The modulus of rupture of the masonry unit was needed to simulate the splitting (cracking) failure of the model. Since the elements did not have any reinforcing steel, any “cracking” of an element would cause non-convergence failure of the program. Therefore, as far as the simulation
using finite element analysis program is concerned, the modulus of rupture of the unit is the most important piece of data required.

However, it was clear from the simulation results that it was the combination of the properties of masonry unit and mortar that determines the strength and the modulus of elasticity of the prism. Isolating the properties of individual elements in the prism simply did not yield meaningful results. This emphasizes the need to devise testing programs to give proper input data for the computer simulation process to work properly in the future.

11.3 Phase II Testing

The compressive strength of the prisms in the phase II test program depended mainly on the tensile strength of the masonry unit, but not so much on the strength of the mortar. The prism always failed by tensile splitting of the masonry units in laboratory testing; however, the tensile strength of the masonry unit can be correlated with the compressive strength of the units. The flagstone prisms had average compressive strengths between 34.0 to 41.0 MPa with the associated low and high mortar strengths of 2.1 MPa and 14.4 MPa. The ratio of the high strength to low strength mortar is 685%, but the increase of the strength of the prisms is only 20.6%. The low strength restoration brick prisms exhibited a similar relationship. The average compressive strengths of the brick prisms were averaging 11.5 MPa with the low strength mortar and 13.5 MPa with the high strength mortar. An increase of only
14.7% in prism compressive strength was obtained while the mortar strength increased 685% as noted before.

The strengths of the prisms represent 33.0% of the flagstone unit strength when low strength mortar was used and 39.8% when high strength mortar was used; similarly, the strengths of the prisms represent 35.5% of the strength of the restoration bricks with the low strength mortar and 40.9% with the high strength mortar.

This relative minor influence of mortar strength on prism strength is in agreement with the statement made by S. J. Lawrence, and G. T. Cao\(^6\) that \textit{"there is a general misconception among practitioners that increasing the quantity of cement will result in higher strength."} There is the same general misconception that a stronger mortar will result in a stronger structure. In order to increase the strength of the mortar, more cement is usually added to the mortar. This will not only reduce the workability of the mortar, it will also increase the shrinkage of the mortar thus increasing the risk of separation cracking between the mortar and the units at both bed joints and head joints. This separation cracking, if developed, would be harmful to the structure as water may penetrate into the joints. Freeze-thaw problems and other water related problems could occur. Therefore, the compatibility between mortar and masonry unit is much more important than mortar compressive strength alone. Compatibility such as water absorption rate of masonry units and workability of mortar to ensure good bonding is much more important than the increase of the strength of the mortar.
Furthermore, the current Canadian Standard$^{17}$ for unit strengths of over 90 MPa and Type S mortar, provides for a strength of 25 MPa as specified in Table 3 of CSA S304.1-M94, the prisms built with flagstone and the strong mortar achieved 41 MPa which is well in excess of the CSA standard's 25 MPa requirement. Even with the low strength mortar, the stone prism strength was averaging at 34 MPa. Note however, that CSA requirement is intended for solid bricks. The Standard recognizes that stone units may not behave exactly the same as solid bricks and requires the designer to carry out prism testing to find the appropriate compressive strength. The same holds true for the prisms built with restoration brick; the interpolation of Table 3 from CSA S304.1-M94 leads to a specified compressive strength of prisms of 11.5 MPa for 33 MPa bricks and Type S mortar. The test results of the brick prisms made with low strength mortar, which had a compressive strength of less than 2.0 MPa, already achieved this strength. This shows the current Canadian Standard is conservative in terms of assessing the compressive strength of the prisms. This conservative assessment of the ultimate strength of the structure is necessary for the safety of the structure when no prism testing program is conducted for a particular project. Table 4 summarizes the measured results in this thesis and the CSA provisions.
Table 4  Compression strength comparison of CSA estimations and Phase II test results

<table>
<thead>
<tr>
<th>Prism Combination</th>
<th>CSA estimates (MPa)</th>
<th>Phase II test results (MPa)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flagstone</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strong mortar</td>
<td>25.0</td>
<td>41.0</td>
<td>-39.0</td>
</tr>
<tr>
<td>Weak mortar</td>
<td>N/A</td>
<td>34.0</td>
<td>N/A</td>
</tr>
<tr>
<td>Restoration Brick</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strong mortar</td>
<td>11.3</td>
<td>13.5</td>
<td>-16.3</td>
</tr>
<tr>
<td>Weak mortar</td>
<td>N/A</td>
<td>11.5</td>
<td>N/A</td>
</tr>
</tbody>
</table>

By testing prisms with either DEMEC gauges or LVDTs, the overall deformational behaviour of prisms can be studied, but this information still does not yield a proper understanding of the behaviour of the mortar at the interface. In the phase I testing, the modulus of elasticity recorded by the strain gauges mounted in the mortar joints of the 1/0.25/3 mortar actually showed a higher modulus than the Ohio stone, and therefore, the prisms. This phenomenon can only be understood with the aid of Stöckl's recently published paper. Further detailed studies of the bed joint mortar must be developed in order to gain a better understanding of the mortar joint behaviour.

Unlike the strength of the prisms, which were between 30-40% of the strength of the masonry unit, the interpretation of the moduli of elasticity of the prisms was much more subjective. In many cases, the interpretation could be made within a wide range and a single value of modulus for that particular combination of masonry unit and mortar could not be assigned without knowledge of the variability. The variability increased as the compressive strengths of the individual components
decreased. The least variable set of data for the modulus was found in the high strength unit/strong mortar combination, and the greatest variable set was found in the low strength unit/weak mortar combination. However, the average initial modulus of elasticity appears to be the in the same range for a given masonry unit regardless of the mortar strengths.

The wide range of modulus of elasticity of prisms clearly indicates the difficulties researchers face in their attempt to determine the prism modulus accurately. As discussed earlier, the strengths of mortars increased quite significantly in the prisms compared with the traditional cube strengths from non-absorptive specimens. An increased strength of mortars would inevitably increase the modulus of elasticity of the mortar.

Stöckl et. al. concluded that different specimen types or testing methods would produce different moduli of elasticity for the same mortar. The measured values of the modulus of elasticity of mortar made with non-absorptive moulds were very much different from the results obtained from testing the actual mortar joints. The difference was attributed to the moisture absorption process of the mortar in the joints of the prism, and it was this difference which caused the basic properties of the mortar obtained from the non-absorptive moulds to be virtually useless.

Although water absorption appears to be a simple process, it depends on many different factors. Firstly, it depends on the water content of the mortar. Obviously if the mortar is too dry, there is an insufficient amount of water that can be absorbed by
the units. On the other hand, if the mortar is too wet, the mortar will be too soft even after some of the water is absorbed by the units. Water absorption also depends on the rate of absorption of the masonry units, the environmental conditions such as temperature and relative humidity at the job site, and moisture lost by evaporation. C. Beall explains these factors in detail in her book "Masonry Design and Detailing".

The extra strength gained by adding too much Portland cement to the mortar may even have an adverse affect on the overall performance of the prism. An excessive amount of cement usually reduces the workability of the mortar and thereby increases the chance of poor workmanship in both head and bed joints. Furthermore, even if the joints were formed properly, the shrinkage due to an excessive cement content might cause separation cracking in the joints. All these would contribute to a worse overall performance of the structure than with relative low strength mortar but good bonding between the mortar and the masonry units.

The modulus of mortar itself was extremely difficult to measure accurately. L. Binda et al. also stated that the measurement of the mortar modulus was difficult using conventional testing methods and hence developed a totally different method to measure the modulus of elasticity of masonry prisms. Their use of electronic speckle pattern interferometry (ESPI) offered good agreement with the strain gauge measurement on mortar specimens, therefore this technique may become a useful tool in measuring the modulus of elasticity of mortar in view of the fact that direct measurements were so difficult.
J.J. Brooks and B.H. Abu Baker proposed a simple estimating formula for the modulus of elasticity for clay brick prisms. Using their formula, the estimated modulus of elasticity for the brick/strong mortar prisms would have been 11.9 GPa assuming 10% water absorption by the bricks. The measured range for this combination in this thesis was between 4.0 to 6.0 GPa; this represents close to a 295% and 198% over-estimation of the actual measured values. When compared with the brick/weak mortar prisms, the estimating formula would have given the modulus of the prisms as 5.2 GPa, which lies within the measured range of between 3.8 GPa to 6.4 GPa. Appendix B shows the calculation of the moduli of elasticity of different brick/mortar combinations. Stone prisms were not compared as the proposed formula was developed for brick prisms. However, the large error percentage for the brick/strong mortar combination further confirms the need for more systematic research on the modulus of elasticity of masonry.

The current Canadian Standard allows the moduli of elasticity be estimated by multiplying the compressive strength of prisms by 850 for clay bricks. Using this estimating method, the modulus of elasticity of the prisms with the restoration brick/strong mortar combination would be 11.5 GPa (850 \times 13.5\text{MPa}) compared to the test result of 5.7 GPa, and 9.8 GPa for the brick/weak mortar combination with the measured result of 4.3 GPa. The CSA does require prism testing be done for estimating the modulus of elasticity for stone masonry structures. Table 5 shows the comparison of the average moduli of elasticity of prisms from the phase II test results and the CSA estimations. The Canadian Standard's estimating method is clearly not
very valid as it does not take into account the modulus of elasticity of either the mortar or the masonry unit.

Maurenbrecher and Trischuk\textsuperscript{12} conducted many tests on repointing mortar with Ohio sandstone, and in their findings the average strength of the prisms was 26.5 MPa, approximately 40\% of the Ohio’s average compressive strength of 60.0 MPa. The averaged modulus of elasticity of the prisms was 5.5 GPa from over 15 different batches of mortar. If the Canadian Standard’s estimation were to be used, the modulus of elasticity would have been 22.5 GPa (850 x 26.5 MPa), almost four times higher than the actual measured results.

Thus, more innovative testing methods for the measurement of the modulus of elasticity of mortars and prisms are needed to improve the estimation accuracy of the modulus of elasticity.

The effect of the joint thickness has not been studied in this thesis as it has been documented by A. J. Francis, C. B. Horman and L. E. Jerrems\textsuperscript{19} that the thicker
the joints, the lower the strength of the prism; the current Canadian Standard to some extent recognizes this effect as the standard joint thickness is specified as 10 mm.
12 Conclusions

With the relatively recent development of high strength concrete, it is tempting for designers to "jump" to the conclusion that a higher mortar compressive strength will automatically yield a higher overall strength of an element in masonry structures. With the results of this research, however, it is clear that this may not necessarily be true. Based on the results of this research, the following conclusions can be drawn:

1. The compressive strength of prisms is influenced very little by the strength of the mortar. When the compressive strength of the mortar was increased 685%, the increase of prism strengths were only 20.7% and 14.7% for stone prisms and brick prisms respectively. Therefore, the structural engineer should specify the proper mortar for good compatibility with the masonry units, good workability, suitable water retention, and suitable air content, without unnecessarily over-designing the strength of the mortar.

2. The influence of the mortar strength on the modulus of elasticity of prisms is again very small. The brick prisms had virtually the same initial tangent modulus of elasticity for both high and low strength mortars. Furthermore, the initial tangent modulus of elasticity for the stone prisms were also very similar regardless of mortar strengths, although modulus for the prisms with low strength mortar started to fall off quite rapidly at a relatively low stress level. This thesis
shows that the modulus of elasticity of prisms is mainly controlled by the properties of the masonry units.

3. The interpretation of the moduli of elasticity for the prisms is very subjective. Until there is a reliable method of measuring the properties of the mortar in the prism, the interpretation will always be subjected to a high degree of variability even when all the individual components remain unchanged.

4. The process of water absorption during laying and curing reduced the water/binder ratio of the mortar and significantly changed the properties of the hardened mortar in the prisms. The relatively minor increase in compressive strength and the similarity of the moduli of elasticity of the prisms suggests that the properties of the hardened mortars may be very similar despite their difference in properties when tested using non-absorptive moulds. The true properties of the hardened mortars remain very difficult to determine.

5. The computer simulation confirmed the flow of forces and stresses within the prism and the cracking mechanism of the prism in agreement with laboratory observations. The computer simulation confirmed that the modulus of rupture of the masonry unit is the most important piece of information required in initiating the failure of prisms.

6. The computer simulations confirmed that the critical properties of the mortar are Poisson’s ratio and modulus of elasticity, and not the compressive strength of the
mortar. As observed from laboratory testing, the failure mechanism of the prisms is always initiated by the splitting of the masonry units. This observation led to the omission of the mortar's compressive strength in order to simulate the proper failure mechanism. Therefore, the compressive strength is irrelevant as far as the computer simulation is concerned, at least until the effective confining pressure of the mortar can be quantified.

7. Other important factors for the computer simulation are the relative differences of the Poisson's ratio between masonry units and mortar, and the relative difference of the moduli of elasticity between masonry units and mortar. The larger the difference of their relative Poisson's ratios, the lower the compressive strength of the prism. The same holds true for modulus of elasticity, i.e. the larger the difference, the lower the strength of the prism.

8. The lack of measuring techniques of the mortar properties in the bed joints makes it difficult to assign proper values for a computer simulation. Any attempt to assign a simple value to the mortar would be wrong. Although the mortar is understood to be experiencing uniaxial compressive stresses during a simple compressive loading of a prism, the effective confining pressure of the mortar cannot be assigned as there is no quantifiable value available at the present time.

9. Finally, the current CSA Standard, similar to other countries' standards and the Australian Standard reported by Page and Brooks⁶, underestimates the strength of the prism assemblage with certain masonry unit/mortar strength combinations to
ensure safe design of structural elements, yet generally overestimates the modulus of elasticity of the prisms. While the CSA Standard restricts the use of the simple estimating formula for the modulus of elasticity of masonry to clay bricks and concrete unit, the Standard’s formula does not take into account either the modulus of elasticity of the unit or the mortar. The current work has shown that a better understanding of the mortar joint properties is needed to accurately estimate the modulus of elasticity of prisms.

Future research is recommended in the following areas:

1. Since the properties of mortar in the prisms are changed significantly by the moisture absorption process, there is a need for better testing procedures to measure the properties of mortar properly. This includes the measurement of the compressive strength as well as the modulus of elasticity of the hardened mortar.

2. Although it is generally accepted that the mortar in the prism assemblage is subjected to tri-axial compressive pressures when the prism is loaded uni-axially, there appears to be little if any research underway to quantify the equivalent lateral confining pressure. This information, if available, will definitely enhance the understanding of the prism behaviour by either computer simulation or laboratory testing.

3. Poisson’s ratio of both the masonry units and mortars are also important to determine the ultimate strength and modulus of elasticity of prisms in the
computer simulation. Therefore future research should include the study of the variability of Poisson’s ratio especially within a hardened mortar joint close to and away from the unit interface.

4. The recording of strain using LVDT’s was carried out with an accuracy of 200 divisions; this resulted in all the stress-strain curves not being smooth but rather “toothed”. The computer program used for data recording should be refined to achieve better accuracy of the strain data.
Appendix A

Materials used in final test program - Phase I

Masonry Unit:

Ohio stone

Size: 100mm x 100mm x 80 mm

Supplier: Plouffe Park, Public Works Canada Materials Depot

Cement:

Manufacturer: Federal Cement Ltd.

Type: Type 10

Colour: White

Lime:

Manufacturer: Bondcrete

Type: Type SA (Type S with Air entrainment)

Sand:

Manufacturer: George Drummond Ltd.

Type: Aggregate sand for construction

<table>
<thead>
<tr>
<th>Table A.1 Mortar mix proportions for Phase I test program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortar Type</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>Strong mortar</td>
</tr>
<tr>
<td>Weak mortar</td>
</tr>
</tbody>
</table>
Materials used in final test program - Phase II

Masonry Unit:

**Flagstone sandstone**

Size: 90mm x 90mm x 30 mm

Supplier: Merkley Supply Ltd.

**Restoration brick**

Size: 90mm x 90 mm x 27 mm

Supplier: Merkley Supply Ltd.

Cement:

Manufacturer: Federal Cement Ltd.

Type: Type 10

Colour: White

Lime:

Manufacturer: Bondcrete

Type: Type SA (Type S with Air entrainment)

Sand:

Manufacturer: George Drummond Ltd.

Type: Aggregate sand for construction

<table>
<thead>
<tr>
<th>Table A2 Mortar mix proportions for Phase II test program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortar Type</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>Strong mortar</td>
</tr>
<tr>
<td>Weak mortar</td>
</tr>
</tbody>
</table>
Appendix B

Proposed Estimating Formula

J.J. Brooks and B.H. Abu Baker proposed an estimating formula for the modulus of elasticity in their paper published in November 1998. Their formula is as follows:

\[
\frac{1}{E_m} = \frac{2.15}{f_{br}} + \frac{0.175}{\gamma_{wa}}
\]

Where:

- \( E_m \) = Modulus of elasticity of prism.
- \( f_{br} \) = Compressive strength of brick.
- \( \gamma_{wa} \) = Ratio of moduli of elasticity between brick and mortar.
- \( f_m \) = Compressive strength of mortar.

and

\[
\gamma_{wa} = \frac{(1 - 0.016 \ W_a) / (1 - 0.029 \ W_a)}{\text{for masonry cured continuously under polyethylene.}}
\]

Where:

- \( W_a \) = Water absorption percentage (%)
Comparison of moduli of elasticity between Brook's proposed estimation formula and Phase II final test results

The following table summarizes the measured test results of the brick prisms and the estimated value using the proposed formula. The stone prisms were not compared as the water absorption percentage was too low when compared with the brick, and was considered outside the proposed formula's proposed parameters.

<table>
<thead>
<tr>
<th>Prism combination</th>
<th>Brook's estimated moduli (GPa)</th>
<th>Phase II measured moduli range (GPa)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick/Strong Mortar</td>
<td>11.9</td>
<td>4.3 - 7.1</td>
<td>277 - 168</td>
</tr>
<tr>
<td>Brick/Weak Mortar</td>
<td>5.2</td>
<td>2.8 - 5.7</td>
<td>Within range</td>
</tr>
</tbody>
</table>
References


