

THE GROWTH AND YIELD OF TEAK
(*Tectona grandis* Linn F.)
PLANTATIONS IN NORTHERN GHANA

by

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ABSTRACT

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Teak (*Tectona grandis* Linn F.) is a popular exotic species in Ghana, widely grown in industrial plantations and small scale community woodlots. In spite of its importance, limited information exists on the growth and yield of this species. Presented here are the results of a preliminary study aimed at assessing the growth and yield potential and developing provisional yield models for the management of teak in Northern Ghana. Data were collected from 100 temporary sample plots from plantations in this region, ranging in ages from 3 to 40 years. Local, standard and stand volume equations and tables were constructed from the data. Additive above ground biomass and site index equations, and provisional empirical yield models were also developed and presented. Site index curves were used to classify teak plantations in the region into site classes I, II and III, in order of decreasing productivity. The assessment of growth and yield revealed the potential for growing teak to acceptable timber size on good sites. Yield functions, indicate that teak can be grown on biologically optimum rotations of 31, 38 and 48 years on site classes I, II and III respectively. The diameter distribution was modelled by the three-parameter Weibull function, using the maximum likelihood and the percentile parameter estimators. The diameter distribution showed positive skewness indicating there are more trees in smaller diameter classes. Initial planting spacing of 2 by 2 m could be reduced to accommodate initial mortality and to achieve optimum stocking levels in order to improve form and timber quality.

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DEDICATION

To

My wife **Ruth Nunifu** and our two kids; **Yoobat** and **Suguruman Nunifu**,
for their love, patience and the hard times they had to go through without me.

THE GROWTH AND YIELD OF TEAK (*Tectona grandis* Linn F.) PLANTATIONS IN NORTHERN GHANA.

1.0 INTRODUCTION

Teak (*Tectona grandis* Linn F. *Verbenaceae*) is one of the most important plantation species both in the high forest and the savannah zones of Ghana. The species was introduced into Ghana between 1900 and 1910 (FAO and UNEP 1981). Teak has since acclimatized well and has been widely grown in both industrial plantations and small community woodlots.

Large scale plantations of teak in Ghana started in the late 1960s, under a plantation programme that was initiated with the help of the Food and Agriculture Organization (FAO) of the United Nations (Prah 1994). These plantations, estimated to cover about 45,000 ha (Drechsel and Zech 1994) were to supplement the supply of wood products from the indigenous natural forests.

Teak, a high quality deciduous timber species, native to Peninsular India, Burma and Indonesia, has gained importance in Ghana in recent times as a source of electric transmission poles for the rural electrification project. A further increase in teak plantations occurred following the establishment of a 5-year rural afforestation programme in 1989 under the Ghana Forestry Department, which

saw an increase in the extent of existing as well as the establishment of new plantations in Northern Ghana. Apart from electric and telephone transmission poles, the tree is also valued by small scale farmers and local communities as poles for construction, fencing, rafters, fuel wood, stakes and wind breaks. It has also become an important source of income for small scale farmers who plant the species on their farms.

There is a considerable potential for growing teak to timber size on good soils in Northern Ghana (FAO and UNEP 1981) and the economic benefits are undisputed. However, local knowledge on the growth and yield characteristics of the species which will help in realising this potential and assist in making important management decisions is still lacking.

This study was therefore designed as a preliminary investigation, aimed at assessing growth and yield, developing provisional growth and yield models and tables for management, and to serve as a basis for future studies into the growth and yield of teak in Northern Ghana. The specific objectives are:

- 1) to assess the growth and yield of teak,
- 2) to develop volume and biomass tables for teak,
- 3) to develop provisional yield functions and tables for teak in plantations in Northern Ghana.

2.0 LITERATURE REVIEW

2.1 CRITICAL SILVICS OF TEAK

2.1.1 General Description and Natural Distribution of teak

Teak, also known commercially as teek (Spanish) belongs to the family *Verbenaceae*. It grows naturally in Southern Asia, from the Indian Subcontinent, through Burma and Thailand to Laos, approximately 9° and 25°N latitude and 73° to 103°E longitude (Troup 1921). As an exotic species, teak grows in several parts of the world. According to Hedegart (1976), the wide distribution of teak attests to the fact that, teak can survive and grow in a wide range of climatic and edaphic conditions. It is generally drought and heat resistant.

Teak vary in size according to locality and conditions of growth. On favourable sites, it may reach a height of about 40 to 45 m, with a clear bole of up to 25 or 27 m, and a diameter of between 1.8 and 2.4 m (Farmer 1972). According to Kadambi (1972), records from Thailand reported a teak tree, claimed to be the worlds largest tree (1965), with approximately 22 feet (6.6 m) diameter at breast height (dbh) and 151 feet (45 m) total height. In drier regions, trees are generally small. The boles are generally straight, cylindrical and clear when young, but tend to be fluted and buttressed at the base when mature. They tend to fork when grown in

isolation, but are generally shade intolerant.

2.1.2 Site and Soil Requirement

Teak grows on a variety of geological formations and soils (Kadambi 1972, Seth and Yadav 1959), but the quality of growth depends on the depth, structure, porosity, drainage and moisture holding capacity of the soil (Kadambi 1972). Teak grows best on deep, well drained and fertile soils with a neutral or acid pH (Kadambi 1972, Watterson 1971), generally on elevations between 200 and 700 m, but exceptionally on elevations up to 1300 m above sea level (Troup 1921).

Warm tropical, moderately moist climate is best for teak growth. Optimum annual rainfall for teak is 1200 to 1600 mm, but it endures rainfall as low as 500 mm and as high as 5000 mm (FAO 1983, Hedegart 1976, Kadambi 1972, Troup 1921).

2.1.3 Establishment and Early Growth

Plantation grown teak is established using stump plants rather than direct sowing of teak seeds which does not always give satisfactory results (Borota 1991). Depending on desired product (fuelwood, poles, lumber or a mixture of products) and the site quality, the initial planting spacing generally range from 1.8 by 1.8 m

to about 3 by 3 m (Kadambi 1972). When planted in taungya¹, spacing could be as wide as 4.5 m between rows. Generally, on good soils, wider spacing is used. This results in better diameter and height growth, and also reduces nursery, planting and early thinning costs (Kadambi 1972). On sloping terrain, wider spacings have been suggested to encourage ground cover and to avoid erosion (Weaver 1993).

Teak is generally shade intolerant but needs training for improved form. Closer than the normal planting spacing is sometimes adapted to ensure quick canopy closure, thereby achieving training and reducing weeding cost (Adegbeihn 1982, Kadambi 1972). This practice necessitates early thinning.

The time of the first thinning is largely determined by site quality. Lowe (1976) noted that although thinning may be delayed for 10 to 15 years after planting without unduly affecting the growth potential of the final crop, very heavy thinning becomes necessary if the growth of the final tree crop is to be maintained at satisfactory levels.

¹ The practice where by farmers grow food crops with trees on the same piece of land to help raise the tree crop with the agreement that, food crop component be removed when the tree crop gets established.

2.1.4 Growth and Yield

Teak is generally fast growing when young, but its overall growth rates on rotation basis is not outstanding (FAO 1956). It is considered moderate to fast growing (Briscoe and Ybarra-Conorodo 1971). A study of the standing biomass of teak in India, showed height growth to be most rapid between 10 and 50 years after which it declined (Weaver 1993).

The rotation of teak in India is a function of forest type and management systems (Ghosh and Singh 1981). Plantation crops have rotations between 50 and 80 years, whereas in areas where teak occurs in mixed stands, rotation is about 70 to 80 years. Coppice systems or coppice with standards have rotations of between 40 and 60 years (Weaver 1993).

FAO (1985) quotes the peak ages for the mean annual volume increment at 50 and 75 years respectively, for site classes I and II in Kerala, India, based on stemwood volume. In Indian yield tables for teak (Laurie and Ram 1940), the maximum total volume growth occur at ages between 5 and 15 years depending on site class. Similar estimates in Trinidad (Miller 1969) are between 7 and 12 years. At Mtibwa, Tanzania, Malende and Temu (1990) estimated the peak ages of mean and current annual increments for teak to be at 42 and 55 years respectively.

At base age 20, the site index for teak was estimated by Malende and Temu (1990) to be between 16 and 25 m. In Miller (1969), the estimate is between 15 and 23 m. Akindele's (1991) estimate for Northwestern Nigeria was between 10 and 29 m. At the same base age, figures from Laurie and Ram (1940) ranged from 28 m for site class I to 12 m for site class V. Similar results have been reported by Keogh (1982), Friday (1987), and Drechsel and Zeck (1994). In Ghana, a similar study for teak in the high forest zone reported indices ranging from 17 m to 26 m (Anonymous 1992).

Logu *et al.* (1988) estimated the above ground biomass production for teak to be between 2.1 and 273 t/ha for ages ranging from 5 to 97 years respectively. The mean annual biomass increments was estimated to peak at between 10 and 40 years depending on site conditions.

2.2. SAMPLING FOR GROWTH AND YIELD

2.2.1 Permanent and Semi-permanent Sample Plots

Permanent sample plots (PSPs) are considered the most reliable sources of data for estimating and modelling growth and yield (Alder and Synott 1992). Apart from individual tree increments, PSPs provide information on recruitments and mortality. These estimates may not be necessary for monitoring well managed plantations,

but are essential components of growth in mixed natural forests (Alder 1980, Alder and Synott 1992)

PSPs are classified into experimental and passive monitoring plots (Vanclay *et al.* 1995, Alder and Synott 1992). Passive monitoring plots by definition are constrained to existing conditions whereas experimental plots are established to explore novel situations, particularly extreme treatments (Alder and Synott 1992) such as varying intensities of thinning.

The process of obtaining data from PSPs to cover the entire rotation of a stand takes a long time to complete and the stand may get destroyed by fire, disease or other catastrophic agencies (Chapman and Meyer 1949). Besides, it has been argued that, the more times a PSP is measured, the less information it provides as compared with the previous measurement, unless it is growing into an age-site-stand density stratum that has not been well sampled (Alder 1980). In this case, sampling is more efficient if plots are replaced after a few re-measurements. This is particularly true for plantations or even-aged forests (Alder 1980). Semi-permanent plots offer the best alternative in this regard.

Semi-permanent plots are located in stands of different ages, covering the full range of site condition, and remeasured for only a few times at suitable intervals. By the overlapping of the ages chosen, the trend of development is established

(Chapman and Meyer 1949). This method is particularly suitable for plantations or even aged natural forests where records of planting or logging dates are available.

The general disadvantage of PSPs is the high cost of establishment and maintenance. Plot size and sampling intensity is therefore, often low (Shiver and Borders 1996, Sheil 1995). There is also the tendency of treating PSPs differently when they are clearly marked for the purpose of re-locating them for measurements. This brings into question, their representativeness of the population.

2.2.2 Temporary Sample Plots

Temporary sample plots (TSPs) are primarily used for estimating relationships that are not time dependent (Alder 1980). They are used in static inventories to estimate the amount of growing stock in relation to the land area. However, growth can be estimated from TSPs by stem analysis if annual growth rings are present.

Based on the principle of comparison of plots of different ages, TSPs can be used to construct yield models (Chapman and Meyer 1949). Many plots of different ages, covering different site conditions are measured and the averages for stands of the same sites but different ages are combined into a curve, assumed to show the trend of growth (Chapman and Meyer 1949). This way, TSPs are useful

alternatives to PSPs when there is an urgent need. However, many plots, covering the range of site conditions are needed to accurately determine the growth trend.

In recurrent inventory, growth is estimated from TSPs by the simple difference between estimates of a stand or tree attribute on two successive occasions. The standard error of this estimate is high since the estimates on the two occasions are independent (Shiver and Borders 1996, Philip 1994, Schreuder *et al.* 1993, Murchison 1989, Loetsch *et al.* 1973). TSPs however have the advantage of less cost and hence permits higher sampling intensity which can result in accurate estimates.

2.2.3 Sampling with Partial Replacement

The development of this method of sampling in forestry goes back to Bickford (1956) and particularly to Ware and Cunia (1962), who provided a unifying theory for this method and compared it to different growth estimators (Shiver and Borders 1996). The basic aim of the theory was to provide estimators for current stand volume and growth with improved precision.

In sampling with partial replacement (SPR) only a portion of the plots or units are retained for re-measurements on the subsequent occasions. These are called the matched plots and could be permanent or semi-permanent plots. In addition,

temporary (or unmatched) plots are established and are not re-measured. The improvement in precision came first from a direct increase in sample size and second from exploiting the correlation between the matched PSP and the unmatched TSP estimates on both occasions.

The matched plots makes it possible to accurately estimate growth, mortality and recruitments. With many more temporary plots, the estimate of the current growing stock can be accurately determined. Moreover, the improved estimates of current growing stock makes growth estimates even more precise (Shiver and Borders 1996). The problem with this inventory design is the choice of optimum combination of matched and unmatched plots. A combination that minimizes cost and standard error is often the ideal.

2.3 TREE VOLUME AND YIELD ESTIMATION

Several methods have been developed to estimate stand volume and yield, each varying in degree of sophistication and precision depending on the complexity of the system dealt with. For the purpose of this study, stand volume estimation by the mean tree method and volume tables will be discussed in some detail.

2.3.1 The Mean Tree Method of Stand Volume Estimation

The underlying theory of this method is that, the volume obtained by careful measurement of the tree of mean volume can be multiplied by the number of trees in the stand or plot to obtain the estimate of the stand or plot volume (Spurr 1952). The most common approach is to obtain the average volume of sub-sample trees in each plot as the plot mean tree volume. From this and the number of trees, the volume of each plot is calculated and hence the volume per hectare. This approach is in fact, two-stage sampling with the sub-sample trees constituting the second stage sample.

The common problem with this method is the sub-sample size, which is usually small, especially when sub-sample trees are to be felled for detailed measurements. According to Philip (1994), a minimum sub-sample size of about 20 trees per plot is normally needed to provide a precise estimate of the volume of the mean tree. Philip (1994) suggested the pooling together of the sub-sample trees of all plots to get a pooled tree of mean volume. He however warned that a serious bias could result if different plots provide different numbers of trees in the sub-sample and contain different sizes of trees.

Another approach is based on the assumption that, the tree of mean basal area is also the tree of mean volume (Spurr 1952, Crow 1971). Although fairly good

results have been obtained by this method, especially when the tree of mean basal area is also the tree of mean height, the fallacy of the basic assumption has long been recognized (Spurr 1952). The mean tree in this case is a tree with diameter as close as possible, to the quadratic mean diameter of a sample of trees from the target stand. This tree is isolated and its volume carefully determined. The ratio of the volume to basal area of the mean tree can be multiplied by the total basal area of the plot to obtain plot volume estimate (Schreuder *et al.* 1993).

2.3.2. Stand Volume Estimation using Volume Tables

Since it is not possible to measure individual tree volume directly in the field, it must be estimated by the use of auxiliary variables such as diameter and height (Murchison 1984). The use of volume equations and tables which relate these variables to tree volume offers speed and convenience in estimating stand volume. There is no doubt therefore that, the use of volume tables is the most common approach to estimating yield.

Volume tables may be constructed on the basis of single tree or stand volume. Single tree volume tables predict volume per tree and stand volume tables predict volume per unit area (usually per hectare) (Philip 1994). The single tree volume tables can be distinguished into local (single entry), standard (double entry) and form class (multiple entry) volume tables (Husch *et al.* 1982). Local volume tables

give tree volume in terms of diameter at breast height (dbh) only. The term local is used because, such tables may generally be restricted to the local area for which the height - dbh relationship that is hidden in the table is relevant (Husch *et al.* 1982). Avery and Burkhart (1994) however noted that, the terms "local" and "standard" as used to describe the single entry and the double entry volume equations do not suggest the former is inferior to the latter.

Standard volume tables give the volume of the tree in terms of dbh and merchantable or total height. These are normally prepared for single species, or a group of species and specific localities (Husch *et al.* 1982). The third type, the form class volume tables give volume in terms of dbh, merchantable or total height, and some measure of form such as Girard form class or absolute form quotient (Spurr 1952, Husch *et al.* 1982, Avery and Burkhart 1994).

Single tree volume tables are generally prepared by three methods; the graphical, alignment chart and the regression methods. The graphical method is the oldest and requires less mathematical techniques (Spurr 1952). It is however unsatisfactory as it is open to subjectivity and the error in estimated volume cannot be measured (Philip 1994, Spurr 1952).

The alignment chart method is another old technique of volume table construction. It was first introduced by Bruce and Reineke (1931) to correct for curvilinearity in

multiple regression equations (Spurr 1952). It produces satisfactory results, though there are several disadvantages associated with it (Spurr 1952). A common disadvantage is, prepared base charts are needed which are not always available. Moreover, the charts cannot be read very accurately and are subject to error because of dimensional changes in the paper (Spurr 1952).

The graphical and the alignment chart methods have been generally discarded in favour of mathematical functions and models (Husch *et al.* 1982). These methods consist of measuring the volume of selected trees in a representative sample, establishing a relationship between the measurements taken on the tree and volume (usually by regression analysis), choosing the best model and verifying the accuracy of the tables constructed (Philip 1994).

In selecting trees for the construction of volume tables, there is the need to clearly define the population. This could be by species, geographic location or age. Some form of stratification becomes necessary if variation in tree size and growth conditions is high (Demaerschalk and Kozak 1974, Marshall and Demaerschalk 1986). In plantations, age is a useful basis for stratification (Philip 1994).

The choice of appropriate model is based on adequacy of fit as dictated by least squares regression assumptions; normality of regression residuals, uniformity of variance across all predictor variables, and the independence of the predictor

variables and regression residuals.

These assumptions are hardly met in practice and often, some form of transformation is necessary. Commonly, the logarithmic transformation is used, but is shown to have some bias in prediction. Details of this bias and its correction as proposed by Baskerville (1972), are presented in section 2.4.2. The most common problem in volume table construction has been heteroscedasticity of residuals. This is because, larger tree volumes tend to deviate more from the regression line than smaller ones. Cunia (1964) proposed the use of weighted least squares to correct for heteroscedasticity in volume table construction.

Once two or more models demonstrate adequacy of fit in terms of these assumptions, a number of methods exist for evaluating goodness of fit. The common ones are; the coefficient of determination (R^2), standard error of the mean, Furnival index (Furnival 1961) and the mean square difference between predicted and observed volumes (Schlaegel 1981). The Furnival index is calculated as:

$$FI = \frac{SE}{GM_Y} \quad [1]$$

Where FI is the Furnival index, SE is the standard error of the fitted regression, and GM_Y is the geometric mean of the dependent variable. The best model is the

one with high coefficient of determination (R^2), small standard error of the mean in the measured units, and small Furnival index (Furnival 1961).

Stand volume tables are based on stand variables such as basal area, top height, mean height and mean dbh. The most common stand volume tables are based on the regression of volume per hectare on stand basal area per hectare and some measure of height representative of the crop; often the dominant or top height is used (Philip 1994).

The measure of volume per hectare may be obtained by measuring a representative sample from the stand or by measuring the volumes of small plots directly or indirectly by the use of individual tree volume tables. According to Philip (1994), the error of prediction in the latter case must be derived from the sum of error from three sources; residual variance in the single volume table, residual variance in the stand volume table and the variance in the sampling units themselves. In the former case, only the last two sources of variance are included in the error. The criteria for judging adequacy of fit is similar to those shown for single tree volume tables.

2.4 FOREST BIOMASS AND YIELD ESTIMATION

The conventional measure of yield in forestry has been related to volume. This is because of the use of tree stem for wood products such as lumber, plywood, poles, pilings, pulp and paper (Aldred and Alemdag 1988), the value of which are closely related to volume. Consequently, mensuration has been primarily directed towards developing techniques for expressing forest growth and productivity in terms of merchantable log volume (Young 1971).

In contrast however, in many established community forests in developing countries, all the forest components are used - branches, foliage and stems (Applegate *et al.* 1988). In such situations, biomass estimates are the most suitable for quantifying products. By definition, biomass is the amount of living organic matter accumulation on a unit area at a specified point in time (Newbould 1967). This is normally expressed in terms of fresh or oven dry weights on per unit area bases. The usual measure of biomass in forestry has been the above ground tree components, which are easily accessible. However, total tree biomass is defined to include the under ground components (roots).

There are two common techniques for estimating biomass in forestry; the mean tree method and regression analysis. If available, specific gravity can be used to convert volume tables into biomass tables.

2.4.1 The Mean Tree Method of Biomass Estimation.

The basic principle is similar to the mean tree method of stand volume estimation; a tree of mean biomass is isolated and its biomass carefully measured. The stand biomass is then obtained by multiplying this estimate by the number of trees in the stand. This is accomplished by obtaining estimates of stand attributes that approximate those of the tree of mean biomass. Crow (1971) used different measures of stand characteristics to determine the tree of mean biomass, but each was shown to have some amount of bias. Some of these are, the tree of mean total height, tree of mean total height and diameter, tree of mean diameter, tree of mean basal area and tree of mean bole volume.

The difficulty of getting measures that closely approximate those of the tree of mean biomass is the major disadvantage of this method. This results from high variation in tree size, especially in natural stands. Baskerville (1965) recommended the use of a stand table approach in which estimates are based on the weight of a mean tree within each diameter class multiplied by the frequency within the class.

2.4.2 Biomass Equations and Tables.

Perhaps the most widely used and convenient method of quantifying forest biomass is by the use of equations or the tables constructed from them. According to Applegate *et al.* (1988), this is so because of the simplicity in determining estimates and the ease with which results can be applied. This method relates easily measured variables such as the diameter and height to the component biomass of the forest fractions (Baskerville 1972, Madgwick and Satoo 1975).

The principle upon which biomass equations are obtained may be simple; 1) fell representative sample trees and take sub-samples for oven dry weight determination, 2) extrapolate from the sub-samples to the whole component and sum up the various components to obtain the total tree biomass, and 3) develop a predictive mathematical model relating the easily measured variables to the component biomass.

The problems however, are in: (i) selecting representative sample trees and parts, (ii) developing an unbiased predictive model, and (iii) ensuring additivity of the parts to equal the whole tree biomass (Philip 1994).

Aldred and Alemdag (1988) noted that, selecting samples for biomass tables must follow statistically defensible sampling rules to ensure that the population of

interest is properly represented. Simple random sampling of trees or clusters of trees in a highly varied population may not achieve the desired representation, though this may be quite satisfactory for even-aged pure stands. In general, the need for some form of stratification has been recommended for uneven-aged mixed stands (Cunia 1979a).

Sub-sampling of the component fresh biomass is a necessity when trees are large, in which case weighing all tree components become impossible. Reliable sub-sampling methods have been proposed such as, randomised branch sampling (Jessen 1955, Valentine and Hilton 1977) and importance sampling (Rubstein 1981). Valentine *et al.* (1984) presented a combination of these two sampling methods for estimating above ground biomass, woody volume and mineral contents and discusses the theory and principles. The method is shown to be efficient, provides unbiased estimates and avoids the time consuming labourious task of weighing the whole tree.

Most authors have found that total biomass may be predicted satisfactorily from diameter at breast height (dbh) (eg. Cunia and Briggs 1984). These equations commonly take the form of a quadratic in dbh or the allometric form of it. A simple logarithmic transformation such as;

$$\ln(w) = a + b \ln(\text{size}). \quad [2]$$

where size is the dbh or basal area or the combination of dbh and height is also in common use. The problem with the quadratic form has been heteroscedasticity of residuals. It is easily corrected for by weighted least squares (Cunia 1964).

The logarithmic transformation as above (equation 2) has been noted by Meyer (1938) to yield biased estimates, a point emphasized by Satchell *et al.* (1971), Baskerville (1972) and Beauchamp and Olson (1973). The argument in support of this fact has been that, if the residuals of the logarithmic transformed variable are normally distributed, the residuals of the untransformed variable are skewed. Therefore, failure to account for the skewness when transforming the variable into the measured units yields the median rather than the mean estimate (Baskerville 1972, Brownlee 1967, Furnival 1961, Finney 1941). The result of this bias is a systematic underestimation of the dependent variable. Baskerville (1972) proposed a correction for the skewness by the addition of one-half the residual mean square to the estimated logarithmic mean before transformation as;

$$\hat{Y} = e^{(\hat{U} + \sigma^2/2)} \quad [3]$$

$$\sigma_A^2 = e^{(2\hat{U} + 2\sigma^2/2)} - e^{(2\hat{U} + \sigma^2/2)} \quad [4]$$

Where, \hat{Y} = estimated mean in measured units,
 σ_A^2 = estimated variance in measured units,

$\hat{\mu}$ = estimated mean in logarithmic units,

σ^2 = estimated variance in logarithmic units.

Beauchamp and Olson (1973) extended this work and noted that, unless the variance is small, the correction above will still result in a biased estimate.

It is generally desirable for the tree component biomass (e.g, leaves, branches, stem and roots), predicted by their individual equations, to add up to the same value predicted by the whole tree biomass equation. This is referred to as the additivity property (Aldred and Alemdag 1988). This requirement is met only if individual component coefficients add up to the corresponding whole tree coefficients. The additivity property requires that; the same independent variables be used in each equation, transformed variables be linear, and the set of equations be fitted from the same data. The additivity property is generally defeated by nonlinear transformation such as using logarithms (Cunia and Briggs 1984, Aldred and Alemdag 1988).

The problem of forcing additivity was considered by Kozak (1970), and was extended by Chiyenda and Kozak (1984) to a point that excluded the requirement that same independent variables be used in all components and total biomass regressions. Cunia (1979b) and later, Jacobs and Cunia (1980), proposed three methods or procedures for ensuring additivity of biomass regressions, or those of biomass tables generated by them. One method, referred to as method one,

requires the calculation of the regression function for each component separately with the regression function of the total defined and calculated as the sum of the component regressions. This method is simple and convenient to use and achieves the desired additivity. The disadvantage is, no statement of reliability can be made about the prediction using such a model.

The second method, designated as method two, ensures additivity by using the same independent variables in the least squares linear regression of the biomass of each component and that of the total. The same sets of weights must be used if required (Cunia and Briggs 1984). The third approach uses linear regression functions with dummy variables. A dummy variable is defined for each component biomass such that, $u_i = 1$ for component i or total, and $u_i = 0$ otherwise; where u_i is the dummy variable for component i . The dummy variables are used in combination with the independent variables to generate new variables, $x_{ij} = x_j$ for component i or total, and $x_{ij} = 0$ otherwise. The independent variables are then combined to estimate the general equation;

$$\hat{y} = \sum_{i=1}^s \sum_{j=1}^m B_{ij} x_{ij} \quad [5]$$

Where, y is the component biomass, B_{ij} is the regression coefficient of the new predictive variable x_{ij} derived from the product of the i th dummy variable with the j th independent variable.

This method has the advantage of ensuring additivity and providing estimates for the standard error. The disadvantage is the tedious work required, especially when dealing with a large sample. The general equation can be estimated by ordinary weighted least squares (OWLS) or the generalized least squares (GLS)(Cunia and Briggs 1985). Reed and Green (1985) presents an extension of this method to cover nonlinear models.

2.5 YIELD MODELS AND TABLES

Yield tables present the anticipated yields from an even-aged stand at various ages, and is one of the oldest approaches to yield estimation. Modern yield tables often include not only yield, but also, stand height, diameter, number of stems, stand basal area, and current and mean annual increments (Vanclay 1994).

Yield tables are commonly classified into normal, empirical and variable-density (Avery and Burkhart 1994). Normal yield tables are supposed to be based on "normal" or optimal stocking; hence, stand density is not considered. Empirical yield tables are supposed to be based on average or actual rather than normal stocking. Like normal yield tables, empirical yield tables are limited in use to the average stocking condition upon which they are based. Variable density yield tables include some measure of stand density.

Yield models are mathematical functions relating yield to stand age, site index, and some measure of stand density. The basic form of a yield model has been that proposed by Schumacher (1939). In its simplest form, the model relates yield in terms of volume per hectare to stand age and site index. This model has proved to be useful, reliable and widely used for many pure even-aged stands (Vanclay 1994).

Mackinney and Chaiken (1939) built upon this equation by including a measure of stand density as an independent variable to develop what is known to be the first variable density yield model (Clutter *et al.* 1983). Clutter (1963) adapted the general form of this equation, given as;

$$\ln V = \beta_0 + \beta_1 A^{-1} + \beta_2 I + \beta_3 S \quad [6]$$

where V is the stand volume per hectare, A is the stand age, S is the site index, and I is some measure of stand density (usually the logarithm of stand basal area), to develop a compatible growth and yield model. This model ensures the compatibility of estimates of yield from tables on one hand, with figures derived from successive summation of growth estimates on the other hand, based on the same data (Vanclay 1994).

Also of common use in growth and yield modelling is the Chapman-Richards

growth model (Richards 1959, Chapman 1961). The model, supposedly derived from basic biological considerations has proven to be very flexible in application (Clutter *et al.* 1983). The basic form of the Chapman-Richards growth model is;

$$\frac{dY}{dt} = \alpha Y^{\beta} - \gamma Y \quad [7]$$

(Clutter *et al.* 1983)

where α , β , and γ are constants such that, $\alpha > 0$, $0 < \beta < 1$, and $\gamma > 0$. Integrating this equation gives the yield model. Vanclay (1994) noted some doubts about the supposed biological basis of the model, but indicated it had other merits.

Some effort has been made at modelling growth and yield using systems of simultaneous equations (e.g, Furnival and Wilson 1971, Amaites *et al.* 1984, Borders and Bailey 1986, Borders 1989). For this approach, individual components of growth are identified and expressed collectively as a system of equations to predict stand growth and yield (Vanclay 1994). This system of equations are then estimated by indirect, two-stage or three-stage least squares, or seemingly unrelated regression techniques (Johnston 1984).

Many growth and yield models provide rather limited information about the forest stand, but effective management and planning also require information on size

and the species contributing to the stand volume (Vanclay 1994). This problem is solved by the use of diameter-distribution-based yield models.

Many probability density functions can be and are used to describe stand diameter distributions; the Gram-Charlier (Meyer 1930), beta distribution (Prodan 1953, Clutter and Bennett 1965), Weibull distribution (Bailey and Dell 1973), gamma distribution (Nelson 1964), Johnson's S_B distribution (Hafley and Schreuder 1977), Lognormal distribution (Bliss and Reinker 1964). For the purpose of this study, the Weibull distribution will be discussed.

The Weibull distribution was developed by Weibull (1951) to model the probability of material failure. Bailey and Dell (1973) are credited as the first to introduce the Weibull distribution into forestry to model diameter distributions. The three-parameter Weibull distribution function is defined by the probability density function (pdf);

$$f(x) = \frac{c}{a} \left(\frac{x-a}{b} \right)^{c-1} \text{EXP} \left[- \left(\frac{x-a}{b} \right)^c \right] \quad [8]$$

Where, x is a specified diameter and a , b and c are constants such that, $x \geq 0$, $a > 0$, $b > 0$, and $c > 0$. The parameter a , commonly termed the location parameter, identifies the lower bound of the diameter distribution. For fixed values of b and c , changes in parameter a simply shifts the entire distribution along the x -axis.

Parameter b is the scale parameter, and the point $x=a+b$ corresponds approximately to the 63rd percentile of the distribution (Shifley and Lentz 1985). Parameter c indicates the shape of the Weibull distribution. When $c \leq 1$, the distribution has a reverse J-shape, for $c > 1$, the distribution is mound-shape, approximating normal distribution for $c = 3.6$. When c is between 1 and 3.6, the Weibull distribution is positively skewed, and negatively skewed for $c > 3.6$ (Bailey and Dell 1973). The cumulative distribution function derived as an integral of the pdf (appendix XIV) is given by;

$$F(x) = 1 - \text{EXP}\left[-\left(\frac{x-a}{b}\right)^c\right] \quad [9]$$

where, a , b and c are parameters as defined before, $F(x)$ is the relative frequency of a diameter class between a and x . The general expression of the above equation is given by;

$$P = \text{EXP}\left[-\left(\frac{L-a}{b}\right)^c\right] - \text{EXP}\left[-\left(\frac{U-a}{b}\right)^c\right] \quad [10]$$

where P is the probability that a diameter is found between two limits L (lower limit) and U (upper limit). For any diameter class, L and U are the lower and upper class boundaries respectively. Therefore, by multiplying P by N , the number of trees in the stand, the frequency of each diameter class is obtained.

There are three common methods of estimating the parameters of the Weibull distribution; the maximum likelihood method (Cohen 1965, Bailey 1974, Schreuder *et al.* 1978, Gove and Fairweather 1989), the percentile method (Zankis 1979, Clutter *et al.* 1983) and the method of moments (Shifley and Lentz 1985). The maximum likelihood estimation is the most efficient. The moment estimators offer speed and ease in exchange for some loss in precision. The percentile estimators are also easy to obtain and are even more accurate than the maximum likelihood estimators when the shape parameter c , is less than or close to 2 (Zankis 1979).

2.6 SITE INDEX AND SITE QUALITY EVALUATION

For meaningful growth and yield forecasting, effective evaluation of the site productivity is required. A lot of effort has been made in this regard to the development of techniques for quantifying site quality. Clutter *et al.* (1983) classified these methods into direct and indirect.

Direct methods make use of historical yield records, stand volume and height data, which are often not available for most species. The indirect methods make use of overstorey interspecies relationships, lesser vegetation characteristics, and topographic, climatic and edaphic factors. The direct methods most invariably, provide better evaluation than the indirect methods (Clutter *et al.* 1983).

Site index is the oldest indirect and the most widely used concept for evaluating site productivity (Husch *et al.* 1982). Site index is conveniently defined as the total height of specified trees in a stand at an arbitrary base age (Powers 1973). Of common usage is the top or dominant height, defined as the average height of the 100 fattest or tallest trees per hectare. Site index curves or equations relate dominant or top height to age. Tree height growth is used because, theoretically, it is sensitive to site quality differences, little affected by varying density levels and tree composition, relatively stable under varying thinning intensities, and strongly correlated with volume (Avery and Burkhart 1994).

Data for the development of site index equations commonly come from three sources; temporary plots (TSPs), PSPs or stem analysis. TSPs provide the most inexpensive and the quickest source of data, but are based on the assumption that, full range of site indices are well represented in all age classes (Alder 1980, Clutter *et al.* 1983, Avery and Burkhart 1994). This is hardly met in practice. PSPs and stem analysis offer the most reliable data for site indices, but are relatively slow and expensive in providing data.

Site index curves may be constructed by graphical methods or by regression analysis. Statistically, there are three broad approaches; the guided or proportional curve, the difference equation and the parameter prediction methods (Clutter *et al.* 1983). The most frequently used equation forms are Schumacher's (1939) and

Chapman-Richards' (Richard 1959, Chapman 1961).

The critical silvics of teak, as well as different approaches to estimating and modelling yield have been reviewed. The best option for this study, given the time and resource constraints, was to make use of temporary plot data from plantations of different ages since no PSP data base exists for these stands. However, the disadvantages of this approach are generally recognized and acknowledged.

3.0 MATERIALS AND METHODS

3.1 THE STUDY AREA

The study was carried out in the Northern Region of Ghana and centered on Tamale. Sample plantations were selected from four forest districts of the region between May and July, 1996. These are; the Tamale, Yendi, Savelugu and Damongo forest districts (see map in figure 1). These districts are all located in the Guinea Savannah vegetation zone of the country (see figure 2).

3.1.1 The Natural Vegetation Zones of Ghana

Ghana is a tropical country with about 238,549 km² land area. The country lies between latitudes 4°45" and 11° 11" north and longitude 1° 14" and 3° 07" west. Ghana is divided into six vegetation zones as shown in figure 2.

The rainforest and the semi-deciduous forest zones are broadly classified as the high forest zone. This zone occupies the southwestern third of the country and covers an area of about 81,342 km². The remaining 157,198 km² which constitutes two-thirds of the country is mainly the savannahs. These are classified as the southern and the northern savannahs, based on the location.

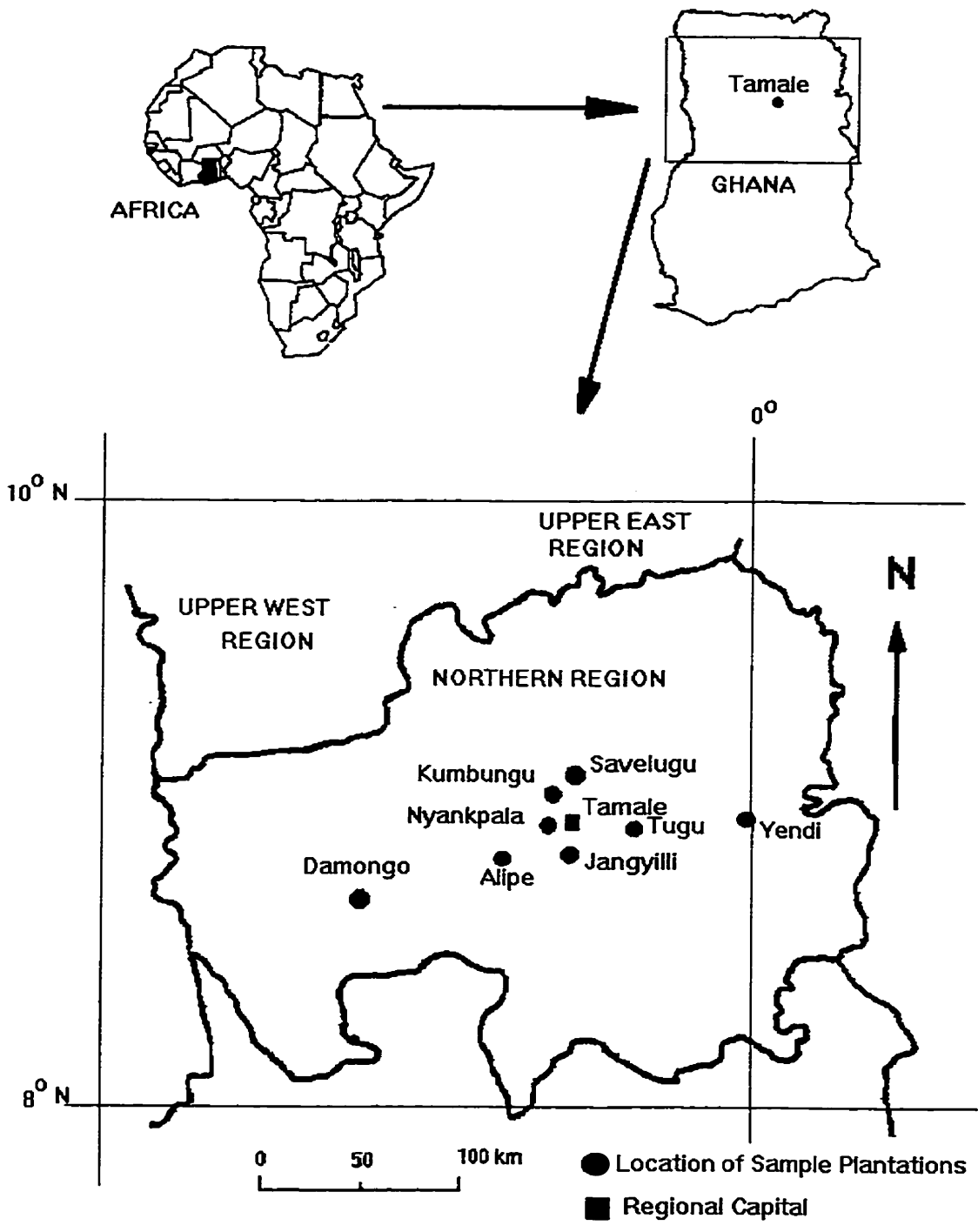


Figure 2. The study area.

The southern savannah type consists of the coastal scrub and grasslands whereas the northern type is made up of the Sudan savannah and the Guinea savannah. The Guinea savannah is by far, the largest vegetation zone of the country (Lawson, 1968).

3.1.2 The Guinea Savannah Vegetation zone

The Guinea savannah is characterised by two distinct seasons of approximately equal length; the wet (rainy) and the dry seasons. The dry season is characterised by the harmattan winds, generally called the north-east trade winds. This is a very dry airmass, the inception of which marks the beginning of the dry season. Characteristic of the wet season is the south Atlantic airmass, referred to as the south-west monsoons, which are moisture laden and are known to bring about rains. The mean annual rainfall is between 960 mm and 1200 mm, and falls between March and October, with the peak in July and August. The mean annual temperature which is 28.3°C does not vary significantly during the seasons.

The characteristic vegetation of the Guinea savannah is made up of short deciduous, widely spaced and heavily branched fire resistant trees. They seldom form a closed canopy and overtop an abundance of ground flora of grasses and shrubs of varying height (Taylor, 1952). The characteristic species are, *Butyrospermum paradoxum* and *Parkia clappertoniana*, found mostly on farmland.

Other common species are, *Daniellia oliveri*, *Burkia africana*, *Terminalia spp.*

The underlying geology of the zone is varied. The common types however, are the voltaian sandstone, shales and granites (Boateng, 1966). These geological formations give rise to two broad groups of soils; the Savannah Ochrosols and the Groundwater Laterites. The Savannah Ochrosols are found on the voltaian sandstones (Boateng, 1966). These consist of well drained porous loams. These soils are among the best in the zone in spite of their deficiency in nutrients such as phosphorous and nitrogen.

The Groundwater Laterites are the most extensive soils and are found on the voltaian shales and granites. These are underlain by iron pans or mottled clay layers, so rich in iron that it hardens to form an iron pan on exposure (Boateng, 1966). Their drainage is very poor and they tend to get waterlogged in the rainy season and become extremely dry in the dry season. They constitute the poorest type of soils in the zone.

3.2 DATA COLLECTION

The list of all teak plantations in the region was obtained from the Regional forestry offices and stratified into one year age classes. Plantations were sampled from these groups with an effort to equal allocation of three sample plantations to each

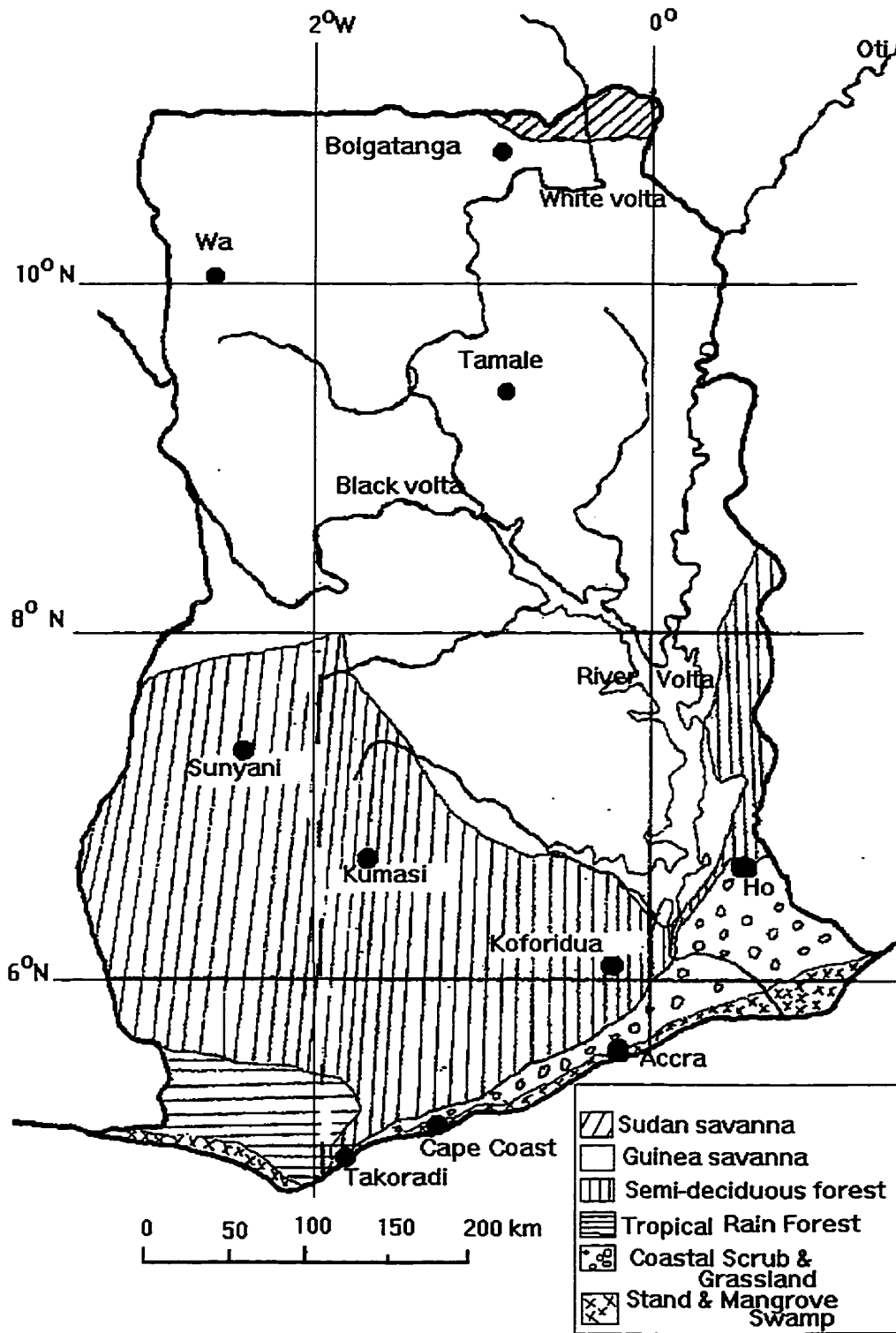


Figure 2. Natural vegetation zones of Ghana
 (Source: Traced from the Atlas for Ghana p12)

age group, except for those ages in which the number of plantations was less than three. For each age group, an effort was also made to cover the full range of site conditions (from the poorest to the best). A pre-sample inspection was done to assess the conditions of each plantation considered for sampling. Plantations that were found to be badly understocked due to mortality or harvesting were discarded and replaced.

In all 25 plantations were sampled, ranging in age from 3 to 40 years, with a total of 100 temporary sample plots. For each sample plantation, the following operations and measurements were carried out;

a) Four circular plots each of radius 7 m (approximately, 0.015 ha) were selected at random. Circular plots were used to avoid directional bias.

b) For each plot, all teak trees enclosed were measured for diameter at breast height (dbh) (in centimeters), total height (in meters) and numbered to facilitate relocation. For trees forking below breast height, the diameter of each leader was measured separately and their quadratic mean calculated; the height of the tallest leader was recorded to correspond with the quadratic mean diameter.

c) With the help of a random number generator, three (3) trees were selected at random from each plot as sub-sample trees and felled for detailed measurements.

The small sub-sample size was a compromise between data adequacy and destruction of tree value.

d) For each sub-sample tree;

i) Detailed measurements were taken for diameter at the base or stump level (D1), breast height (dbh), at half the height above breast height (D3), at the top 1 cm (D4) and the total height of the main stem.

ii) All leaves and branches were separated from the bole and weighed and sub-samples taken for oven dry weight determination;

iii) The main stem was weighed and sub-samples taken for oven dry weight determination. Samples were taken at strategic positions to minimize error due to the variation in moisture content along the stem;

iv) The sub-samples of parts taken in ii) and iii) above were clearly labelled by tree number, plot, and plantation location, and dried in an oven at a temperature of about 70°C to constant weight.

For trees that were too large to be weighed directly in the field, cross-sectional discs were taken from the base and the top sections of the bole and each

weighed. The volume measurements of each disc were taken and by simple proportions, the fresh weight of the whole bole was determined. The equipment used in weighing fresh biomass was a load cell suspended from a tree with a motorcycle battery as the source of power. Small samples (sub-samples) were weighed using an electronic scale.

Growth and yield survey data for teak plantations in the high forest zone of Ghana were obtained from the Ghana Forestry Department Planning Branch, Kumasi, Ghana for comparison. A summary of part of this data is presented in appendix IX. The ages of these plantations ranged from 13 to 26 years and were all from the Offinso Forest District of the Ashanti Region.

3.3 DATA ANALYSIS

3.3.1 Volume Estimations

The volume of each sub-sample tree was computed using Smalian's formula. The stand volume was estimated by three different methods and compared:

a] As two-stage simple random sampling, the estimated total volume per hectare and the corresponding variance was calculated from (Cochran 1977) as;

$$Y = \frac{N}{n} \sum_{i=1}^n \frac{M_i}{m_i} \sum_{j=1}^{m_i} V_{ij} \quad N = \frac{1}{a} \quad [11]$$

$$\text{var}(Y) = \frac{N^2}{n} \left(\frac{N-n}{N} \right) S_1^2 + \frac{N}{n} \sum_{i=1}^n M_i^2 \left(\frac{M_i - m_i}{M_i} \right) \frac{S_2^2}{m_i} \quad [12]$$

Where Y is the total volume per hectare, n the number of sample plots per plantation, a is the individual plot area in hectares, M_i the number of trees per sample plot, m_i the number of sub-sample trees per plot, V_{ij} the volume of the j th tree in the i th plot, S_1^2 and S_2^2 are the variance for the first and second stage simple random samples respectively.

b] As two-stage sampling with the second stage sample with probability proportional to basal area, the mean volume per plot based on the sample plots was obtained from (Murchison 1984) as:

$$Y_{Pi} = \frac{1}{n} \left(\sum_{i=1}^n \left(\sum_{j=1}^{M_i} BA_{ij} \frac{1}{m_i} \sum_{j=1}^{m_i} \frac{V_{ij}}{BA_{ij}} \right) \right) \quad [13]$$

Stand volume estimate per hectare as;

$$Y = \frac{N}{n} \left(\sum_{i=1}^n \left(\sum_{j=1}^{M_i} BA_{ij} \frac{1}{m_i} \sum_{j=1}^{m_i} \frac{V_{ij}}{BA_{ij}} \right) \right) \quad [14]$$

where, V_{ij} , M_i , m_i , Y , N and n are as defined before and BA_{ij} is the basal area of the j th tree from the i th sample plot and Y_{pi} is the total volume of the i th plot. The variance was calculated using the formula from Murchison (1984, page 63), given as;

$$\text{Var}(Y) = \frac{N(N-n)}{n} S_1^2 + \frac{N}{n} \sum_{i=1}^{i=n} \left(\frac{(M_i - m_i)}{M_i m_i} \right) S_2^2 \quad [15]$$

Where S_1^2 and S_2^2 are the first and second stage sample variances, given as;

$$S_1^2 = \frac{1}{n-1} \sum_{i=1}^{i=n} \left(\sum_{j=1}^{j=M_i} BA_{ij} \frac{1}{m_i} \sum_{j=1}^{j=m_i} \frac{V_{ij}}{BA_{ij}} - \frac{1}{n} \sum_{i=1}^{i=n} \left(\sum_{j=1}^{j=M_i} BA_{ij} \frac{1}{m_i} \sum_{j=1}^{j=m_i} \frac{V_{ij}}{BA_{ij}} \right) \right)^2 \quad [16]$$

$$S_2^2 = \frac{1}{(m_i - 1)} \left(\sum_{j=1}^{j=M_i} BA_{ij} \right) \sum_{j=1}^{j=m_i} \left(\frac{V_{ij}}{BA_{ij}} - \frac{1}{m_i} \sum_{j=1}^{j=m_i} \frac{V_{ij}}{BA_{ij}} \right)^2 \quad [17]$$

It should be noted that, S_2^2 is a measure of the variation between individual sub-sample tree estimates of plot totals; not the variation between individual sub-sample tree volumes (Murchison 1984).

c] A standard volume equation was constructed using the individual tree dbh, height and volume measurements. The fifteen most commonly used equations presented in Unnikrihnan and Singh (1984) were each tested. The model that

produced the best fit for the data was the weighted version of the equation [18] below, with $(D^2H)^{-1}$ as the weight.

$$V = \beta_0 + \beta_1 D + \beta_2 D^2 + \beta_3 D^2 H \quad [18]$$

Where D is the diameter at breast height (dbh), H is the total tree height, V is tree volume and $\beta_0, \beta_1, \beta_2, \beta_3$, are regression coefficients.

Equation [18] above was estimated by ordinary weighted least squares. The use of $(D^2H)^{-1}$ as weights was investigated using SPSS². The volume equation was then used to estimate plot volumes using the diameter and height measurements in each plot. This was extrapolated for stand volumes per hectare.

A single entry volume equation and table for the full range of data collected was considered. Test for coincidence showed differences in trends for different sites. The data set was split into site classes, using the site index curves presented in appendix XII. The data set for each site class was then fitted with a single entry volume equation of the form;

$$V = \alpha + \beta_1 D + \beta_2 D^2 \quad [19]$$

² Weight estimation in SPSS for Windows 3.1 release 6.1.

Heteroscedasticity was corrected for each equation by weighting. The appropriate weighting variable was determined as a function of dbh, D using SPSS. The variable D^{-6} produced well behaved residuals and was thus used as weights for each site class.

3.3.2 Biomass Computations

The oven dry weight of each component of the sub-sample trees was determined by simple proportions from the weights of the oven dry samples. These were summed up to give the above ground biomass of each tree. Based on the sample tree dry weights, biomass equations were developed for the construction of biomass tables. To ensure additivity, the dummy variable method (Jacobs and Cunia 1980, Cunia and Briggs 1984) was used to estimate the biomass equations by weighed least squares (WLS). Dummy variables U_i were defined such that; $U_i = 1$ for component i and total tree biomass, and $U_i = 0$ otherwise. The components of biomass were leaves, branches and stem. For instance, if the leafy component biomass is considered, its dummy variable took the value 1 for leaves and total biomass, and 0 otherwise.

Prior to estimating the general equation, the equation of each component was estimated to determine the regression standard errors for each component. The standard error for the individual components were used in combination with the

transformation vector $(D)^{-5}$ as weights.

The independent variables that were statistically significant for at least one biomass component were dbh and the square of dbh. The dummy variables were used in combination with these independent variables to generate new variables. By the backward elimination stepwise model selection criterion, the insignificant independent variables were eliminated. The final general equation [20] was estimated by WLS.

$$\hat{y} = \sum_{i=1}^{i=3} \sum_{j=1}^{j=3} \beta_{ij} X_{ij} \quad [20]$$

Where, X_{ij} = the independent variable generated by the combination of the i th dummy variable with the j th independent variable,

β_{ij} = the regression coefficient of the X_{ij} independent variable,

y = the dependent variable; the biomass value of the i th component.

From equation [20], the individual component equations were determined by selecting the appropriate values for the dummy variables, and used to construct the respective tables. Equation [20] became the total tree biomass equation if all the dummy variables took the value of 1. The additive biomass equations were used to estimate the stand biomass per hectare.

3.3.3 Yield Models

Plots were classified using the proportional curves method described by Alder (1980). A single equation was fitted to the plot level top height³ - age data, using the logarithmic transformation of Schumacher's (1939) equation:

$$T_0 = T_{\max} \exp(\beta A^k) \quad [21]$$

Where T_0 is the mean top height, T_{\max} is the maximum height the species could reach on the site, A is the age of the stand, β is regression coefficient and k is a constant.

By nonlinear regression, the value of k was determined iteratively for the value that minimized the sum of squared errors. The value of k was found to be $\frac{1}{2}$. This was used to transform the age variable and by ordinary least squares, the equation;

$$\ln T_0 = \ln T_{\max} + \beta A^{-\frac{1}{2}} \quad [22]$$

was estimated. Site classes were determined by allowing the regression constant (which in this case is $\ln T_{\max}$) to vary to produce curves with the same gradient and

³ The average height of the largest 100 trees per hectare.

different intercepts (anamorphic curves). This method has been recommended for use when only temporary sample plot data such as in this study, are available (Alder 1980).

Based on the site index curves estimated above, plantations were sorted according to site classes, and fitted with a general yield equation of the form;

$$\ln Q = \beta_0 + \beta_1 A^k + \beta_2 S + \beta_3 I \quad [23]$$

Where Q is some measure of yield (mean dbh, mean Height, stand volume per hectare, basal area or biomass per hectare), A is plot age, S is the site index, I is some measure of stand density and β_0 , β_1 , β_2 , β_3 , and k are constants.

The value of k equal to $\frac{1}{2}$ was used. Basal area was used as a measure for stand density for the volume yield equation, but was found to be statistically insignificant and was dropped from the model. Thus equation [23] was reduced to equation [24] for each site class.

$$\ln Q = \beta_0 + \beta_1 A^{-1/2} \quad [24]$$

This was estimated as the yield equation for each measure of yield for each site

class. The volume yield model was divided by age to obtain the model for mean annual volume increment (MAI) (equation [25]). The current annual increment (CAI) model was obtained by taking the derivative of the volume yield model with respect to age (equation [26]). The ages of maximum MAI and CAI respectively were determined by taking the derivatives of each of equations [25] and [26], setting them to zero and solving for A.

$$\text{MAI} = Q/A = (A^{-1}) \text{EXP} (\alpha + \beta A^{-k}) \quad [25]$$

$$\text{CAI} = -k\beta A^{-k-1} \text{EXP} (\alpha + \beta A^{-k}) \quad [26]$$

Asymptotic height - dbh relationship was estimated for the different site classes, using the Chapman-Richards function (Richards 1959). The asymptotic model was considered because, by its mathematical form, it offers flexibility for extrapolations beyond the empirical data set (Garman *et al.* 1995). Estimation of the equation parameters was done by nonlinear regression using SPSS. The equation as presented by Garman *et al.* (1995) is given as:

$$H = 1.37 + \left[\beta_0 (1 - \text{EXP}(\beta_1 D))^{\beta_2} \right] \quad [27]$$

where, H is the total height, β_0 , β_1 , β_2 are regression constants, β_0 is the asymptotic height. A regression equation was generated for each site class.

3.3.4 Diameter Distribution Models

Tree diameters within each age class were grouped into one centimetre diameter classes and fitted with the three-parameter Weibull distribution function (Weibull, 1951). One centimetre classes were considered because of the relatively small tree sizes in the smaller age groups. Parameter estimation was performed using, the maximum likelihood and the percentile methods. The maximum likelihood estimates (MLE) of the location parameter a , was obtained by the formula given by Zankis (1979), given as:

$$\hat{a} = \frac{(x_1 x_n - x_2^2)}{(x_1 + x_n - 2x_2)}, \quad \text{if: } x_2 - x_1 < x_n - x_2; \text{ else: } \hat{a} = x_1 \quad [28]$$

where x_1 = smallest diameter in the sample,
 x_2 = the second smallest diameter,
 x_n = the largest diameter in the sample.

The scale and the shape parameters were estimated from equations given by Gove and Fairweather (1989). The shape parameter c , was estimated from the nonlinear equation;

$$\left[\frac{\sum_{i=1}^n f_i x_i^c \ln x_i}{\sum_{i=1}^n f_i x_i^c} - \frac{1}{\hat{c}} \right] - \frac{1}{n} \sum_{i=1}^n f_i \ln x_i = 0 \quad [29]$$

iteratively, and substituted into the equation;

$$\hat{b} = \left[\frac{1}{n} \sum_{i=1}^n f_i x_i^{\hat{c}} \right]^{\frac{1}{\hat{c}}} \quad [30]$$

for the estimate of b;

Where, f_i is the i th diameter class frequency and x_i is the corresponding class midpoint. The parameter estimates obtained by this method were cross checked by estimating the three parameter Weibull function iteratively by nonlinear regression (SPSS).

The percentile estimates (PE) of the parameters were obtained using the equations proposed by Zankis (1979). The parameter a was estimated by equation [28] above. The parameters c and b were estimated from:

$$\hat{c} = \frac{\ln \left[\frac{\ln(1-p_k)}{\ln(1-p_i)} \right]}{\ln \left[\frac{x_{[np_k]}^{-\hat{a}}}{x_{[np_i]}^{-\hat{a}}} \right]} \quad [31]$$

and

$$\hat{b} = -\hat{a} + x_{[0.63]} \quad [32]$$

respectively.

Where, $p_i = 0.16731$, $p_k = 0.97366$ and $0.63n = 63\text{rd percentile in the sample}$ and n is sample size.

Goodness of fit was tested by using the Kolmogorov-Smirnov (KS) criterion (Daniel 1978). This criterion utilizes the maximum absolute differences between the cumulative observed and predicted diameter probabilities to determine goodness of fit. These differences are compared with statistics (KS statistics) given in tables at various probability levels. A hypothesis is rejected if the maximum absolute difference exceeds the tabulated KS-statistic at the chosen probability level and sample size.

4.0 RESULTS

4.1 DIAMETER AND HEIGHT GROWTH

Table 1 presents the summary statistics for diameter and height estimates of sample plantations by age classes. The summaries by individual plantations are presented in appendix I. The mean annual diameter and height growth ranged between 0.49 and 1.10 cm, and 0.38 to 1.09 m respectively, per year.

Table 1. Summaries of diameter and height measurements.

AGE	DIAMETER (cm)			HEIGHT (m)		
	MEAN	SE	RANGE	MEAN	SE	RANGE
3	2.76	0.051	1.43-6.05	2.88	0.051	1.36-5.55
4	3.25	0.068	1.47-8.59	2.84	0.051	1.46-5.62
6	5.95	0.093	2.23-11.62	5.51	0.081	2.90-9.40
7	7.67	0.078	3.66-11.78	7.61	0.066	4.00-9.70
8	7.69	0.073	5.09-11.46	7.62	0.068	4.25-10.50
9	7.82	0.105	4.14-13.69	6.78	0.078	4.00-10.60
17	10.78	0.144	5.14-18.06	8.07	0.076	5.00-11.95
26	18.21	0.337	12.41-24.82	13.86	0.165	10.80-17.30
31	21.06	0.401	14.64-28.80	14.76	0.132	12.80-18.90
38	23.58	0.726	15.60-33.50	19.85	0.409	14.30-26.40
40	19.53	0.461	12.41-26.50	15.13	0.203	13.10-20.20

Based on individual plantations, the ranges were 0.49 to 1.26 cm and 0.38 to 1.19

m respectively. The mean annual diameter growth recorded for teak plantations in the high forest zone of Ghana ranged from 1.1 to about 2.0 cm per year (appendix IX).

Table 2. The estimates of the Weibull parameters by the Maximum likelihood (MLE) and the Percentiles (PE) methods

AGE (YRS)	MAXIMUM LIKELIHOOD ESTIMATES			PERCENTILE ESTIMATE		
	A	B	C	A	B	C
3	1.438	1.556	1.653	1.438	1.582	1.569
4	1.255	1.804	1.883	1.255	2.085	1.917
6	2.325	4.381	2.078	2.325	4.195	2.012
7	3.600	4.757	3.343	3.600	4.360	3.549
8	5.086	4.609	3.531	5.086	3.034	2.169
9	4.129	4.441	2.634	4.129	4.141	2.232
17	5.374	8.237	3.177	5.374	6.086	2.696
26	12.401	8.889	3.506	12.401	6.739	2.245
31	14.638	9.089	2.719	14.638	7.642	1.475
38	14.161	10.286	2.775	14.161	9.999	2.298
40	12.155	8.556	2.631	12.155	8.535	2.389

The parameter estimates of the Weibull distribution for teak in Northern Ghana and the coefficients of the asymptotic height-dbh function are presented in tables 2 and 3 respectively. The summaries of observed and predicted diameter frequencies by the two parameter estimation methods are presented in appendix XI. Ten out

of the eleven hypotheses tested were acceptable for the maximum likelihood estimates and eight out of the 11 for the percentile estimates at 0.05 probability level. This is generally an indication of good fit. As indicated by the estimates of the parameter c , the general shape of the diameter distribution curve is mound-shaped, and since none of the estimates exceeds 3.6, the distributions are positively skewed.

In Table 3, the regression constants β_0 , represent the maximum total heights on the sites, β_1 is a measure of steepness of the curve and β_2 is curvature parameter. As shown in Table 3, if teak is allowed to grow for a long period of time, the estimated maximum (asymptotic) height is 32.84 m on site class I, 22.50 m on site class II and 15.91 on site class III.

Table 3. Coefficients and standard errors of height - dbh equation.

SITE CLASS	ESTIMATES OF COEFFICIENTS		
	β_0	β_1	β_2
I	32.84	-0.038	1.265
II	22.50	-0.091	1.653
III	15.91	-0.070	1.354

4.2 VOLUME ESTIMATION

The double entry single tree and stand volume equations are given by equations 33 and 34 respectively. The double entry volume table is given in appendix V.

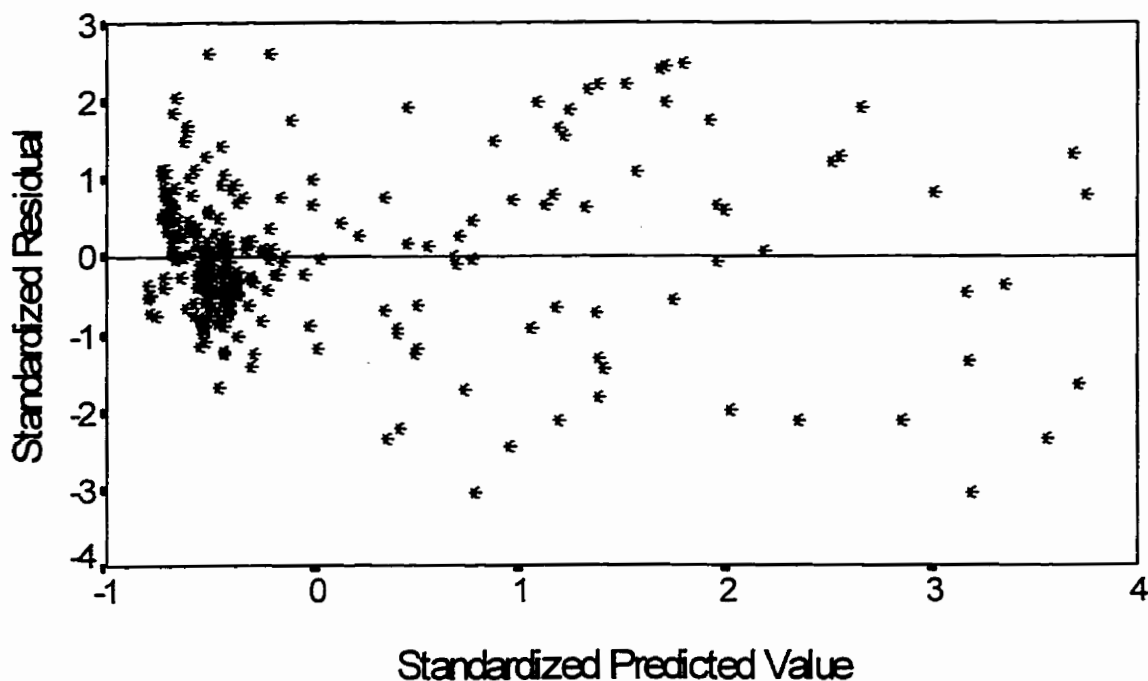


Figure 3. The scatterplot of the standardized residuals from the double entry volume equation against predicted values.

Figure 3 presents the scatterplot of the residuals of the double entry volume equation. The plot shows that the residual variance is fairly uniform across the range of predicted values, with no sign of serious bias. The coefficients of the single entry volume equations for the different site classes are given in Table 4.

Equation [35] represents a general single entry volume equation for all site classes. Though test of coincidence showed that this equation generally gives a poor fit, it is useful for quick volume estimation with some loss in precision. The table is presented in appendix VI.

$$V = -0.36 + 0.96D - 0.13D^2 + 0.05D^2H \quad [33]$$

$$SE = 0.11 \quad R^2 = 0.99 \quad wt = (D^2H)^{-1}$$

$$Y = -0.52 + 4.22G - 0.19G^2 + 0.02G^2T \quad [34]$$

$$SE = 1.55 \quad R^2 = 0.99 \quad wt = (BA)^{-1}$$

$$V = 0.98 - 0.67D + 0.43D^2 \quad [35]$$

$$SE = 0.01 \quad R^2 = 0.95 \quad wt = D^{-6}$$

where, V = tree volume, D = tree diameter at breast height, H = total tree height, G = stand basal area, T = stand top height, SE = regression standard error, wt = weight, R^2 = coefficient of determination, and Y = stand volume in cubic metres per hectare.

Table 4. Coefficients of local volume equations by site classes.

SITE CLASS	COEFFICIENTS				
	CONSTANT	D	D ²	SE	R ²
I	0.75	-0.52	0.40	0.01	0.95
II	4.42	-2.63	0.65	0.03	0.93
III	0.65	-0.50	0.39	0.05	0.97

The summaries of the artificial and absolute cylindrical form factors, and the absolute form quotients are presented in Table 5. The absolute form factors are based on the diameter at the stump level whereas the artificial form factors are based on diameters at breast height. The absolute form factors indicates that, tree volumes tend to be higher than, but closer to those of cones, than they are to cylinders, of the same basal areas. The artificial form factors range from 0.62 to 1.59.

The absolute form quotients are fairly stable and vary from 0.57 to 0.77. This is the ratio of the diameter at half the height above breast height to the dbh. The values show that, there is in general, a 23 to 43% decrease in diameter from breast height to the point half the height above breast height.

Table 5. Absolute⁴ and artificial⁵ form factors of teak in plantations in Northern Ghana.

Age	ABSOLUTE FORM FACTOR		ARTIFICIAL FORM FACTOR		ABSOLUTE FORM QUOTIENT	
	MEAN	SE	MEAN	SE	MEAN	SE
3	0.41	0.012	1.59	0.088	0.74	0.017
4	0.38	0.009	1.58	0.114	0.69	0.016
6	0.35	0.008	0.80	0.029	0.61	0.016
7	0.32	0.006	0.63	0.009	0.57	0.009
8	0.35	0.008	0.66	0.010	0.61	0.008
9	0.34	0.007	0.68	0.012	0.57	0.016
17	0.33	0.006	0.64	0.015	0.58	0.014
26	0.39	0.018	0.63	0.007	0.73	0.018
31	0.42	0.014	0.64	0.016	0.77	0.021
38	0.43	0.023	0.62	0.011	0.77	0.022
40	0.42	0.008	0.62	0.004	0.75	0.008

The summary of the stand volume estimates is presented in appendix II. The results indicate a generally comparable precision for the two-stage simple random (2SRS) and the sample with probability proportional to basal area (PPG), for the young plantations. With increasing age, the PPG estimates appear more precise. The estimates by the use of volume equation may be regarded as the closest approximation to the true population values since the equation is based on the same population.

⁴The ratio of the volume of a tree to that of a cylinder of the same height and diameter equal to the stump level diameter of the tree (Philip 1994).

⁵The ratio of the volume of a tree to that of a cylinder of the same height and diameter equal to dbh of the tree(Philip 1994).

4.3 BIOMASS EQUATIONS AND TABLES

Table 6 presents the coefficient of the additive biomass equations for the various components. The tables are presented in appendix VII. All biomass estimates are measured in terms of oven dry weights in kilograms.

Table 6. The coefficients of additive biomass models.

COMPONENT BIOMASS	REGRESSION COEFFICIENTS			SE	R ²
	CONST.	D	D ²		
STEM	1.113	-0.985	0.248	0.015	0.94
BRANCHES	0.171	-0.186	0.050	0.006	0.86
LEAVES	-	0.112	0.016	0.008	0.89
TOTAL	1.284	-0.969	0.314	0.012	0.96

The means of oven dry weights expressed as proportions of fresh weights are presented by components and age classes in appendix VIII. The trend shows a general increase in percentage oven dry weight as the trees age, with a general fall in standard error. Thus, variability of these estimates reduces with age.

4.4 YIELD MODELS AND TABLES.

Table 7. Regression coefficients for component yield models for plantation teak in northern Ghana from equation [25].

YIELD VARIABLE	SITE CLASS I			SITE CLASS II			SITE CLASS III		
	α	β	SE	α	β	SE	α	β	SE
MEAN dbh	4.20	-4.88	0.07	3.94	-5.50	0.09	3.58	-6.20	0.06
MEAN HEIGHT	3.67	-4.31	0.08	3.60	-4.99	0.08	3.10	-5.23	0.09
BASAL AREA	6.10	-9.20	0.17	5.73	-10.73	0.19	4.95	-11.75	0.18
VOLUME	8.10	-11.13	0.11	7.87	-12.33	0.12	7.20	-14.12	0.13
BIOMASS	7.69	-10.68	0.20	7.19	-12.23	0.19	6.62	-12.85	0.19

The regression coefficients of the yield models are given in table 7. The yield table is presented in appendix X. The mean and current annual increment are estimated to peak at ages; 14 and 31, 17 and 38, and 21 and 48 respectively for Site classes I, II and III (see appendix X).

The site index curves are presented in appendix XII. The site index equation is;

$$\ln S = \ln T + 3.69 \left(A^{-\frac{1}{2}} - 0.05^{-\frac{1}{2}} \right) \quad [36]$$

where T is the top height of the plantation, S is the site index of the plantation at the base age 20 years and A is the stand age.

A plot of the average top heights predicted for site class I in the study area with figures from the Indian yield tables for site class III (Laurie and Ram 1940) and Trinidad site class II (Miller 1969) are given in Figure 4. The three curves generally show a similar trend.

5.0 DISCUSSION

5.1 ASSESSMENT OF GROWTH AND YIELD OF TEAK IN NORTHERN GHANA

The major limitation of this study was time and budget constraints which resulted in a small data size. This is a general disadvantage of the use of temporary sample plots in yield modelling. Besides, there was poor representation of the older age classes in the sample. This was a result of the general lack of plantations within these age classes. Thus, it is possible that, the average growth trend might not have been adequately defined.

Also of notable limitation is the sub-sample size, which was small. This was due to concerns about the value of teak and the amount of destruction that may be associated with large sub-samples. The bias that might result from the small sub-sample size may be minimized by the relatively uniform growth conditions reflected in the small standard errors observed in appendix I. Also, sub-samples were selected at random, which should result in unbiased data.

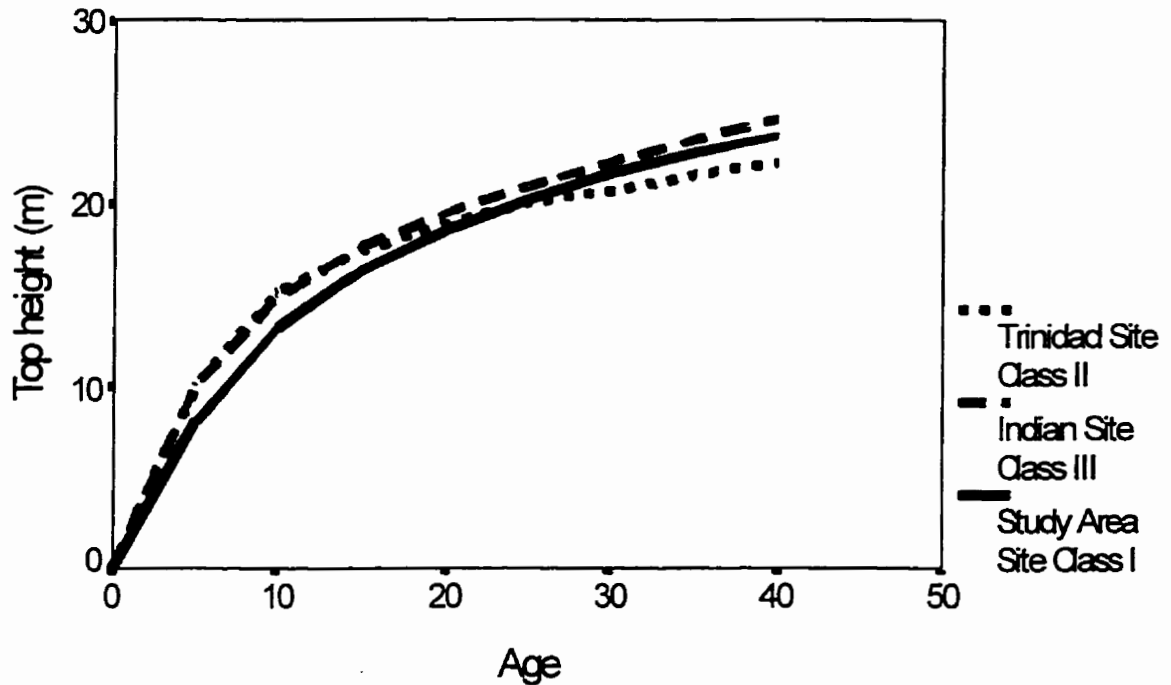


Figure 4. The graph of top height against age for site class I in the study area, Indian yield tables site class III, and site class II for Trinidad.

The assessment of growth and yield was based on the assumption that, the various age classes were well represented and that, the yield estimates were unbiased. Though this cannot be guaranteed, the growth trend observed in this study appears consistent with studies conducted elsewhere in the tropics (see Figure 4).

Based on the site indices calculated from equation [36], the most productive site class in the study area (site indices of 16 and above at base age 20) was comparable in productivity with sites classified as V (site index 16 to 20 m) in the

high forest zone of Ghana (appendix IX). This is not surprising because soils and climatic conditions in this zone compare much more favourably with average requirements for the growth of teak than the conditions in the study area. This site class was also comparable in productivity to site class III in the Indian yield tables (Laurie and Ram 1940) and to site class II for Trinidad (Miller 1969) (see figure 4). Thus, if the site indices truly reflect the productive potential for teak in Northern Ghana, then the potential for teak in the region is considerable.

An examination of the stocking levels indicates that, plantations in the high forest zone rarely exceeded 400 stems per hectare. The spacing factors, calculated by expressing average plant spacing as a percentage of average top height, were generally between 30 and 40%, as compared to 15 and 20 observed in the study area for plantations with ages above 17 years. Though the former may represent slight understocking, it is outside the scope of this study to comment. The projected mean diameters from the yield table in appendix X at age 40, based on these growth conditions are, 30.83, 21.55 and 13.46 for site classes I, II, and III respectively. The corresponding yields on per hectare basis are, 566.91, 372.58 and 143.66 m³ respectively. Though these yields look encouraging, the generally small mean diameters indicate that, most of the yield will be trees of smaller sizes. This is supported by the positive skewness in diameter distribution. This indicates the lack of and the need for thinning in such plantations.

The spacing factors of the very young plantations were very high (above 50%). This is partly attributed to the time it takes for the species to fully “capture” a site, due to the relatively low growth rates, and partly due to initial mortality. Mortality is mainly caused by unfavourable climatic conditions such as lack of rains during the time of planting. Initial mortality creates gaps, which tend to defeat the “training” of the species in the early stage of the growth, resulting in poor forms. This is a probable explanation for the generally low absolute form factors observed, with trees boles tending to be more conical than cylindrical.

It is clear from this assessment that, teak has a potential on good sites in the study area. Considerable improvements in the quality and volume of yield can be achieved by adopting appropriate management practices such as thinning, which is not done currently.

5.2 STAND VOLUME ESTIMATION

The double entry volume equation presented in this study has been shown to be unbiased. The sample size was also large enough (about 289 trees) to adequately define the volume equations. There is the need for testing of this equation with an independent set of data to establish the presence of and the nature of any prediction bias for appropriate correction. It is generally dangerous to extrapolate for volume measurements outside the range of data used in constructing the

volume tables and this is not recommended.

The major limitation of the stand volume equation is that, if plantation management regimes differ from what is observed in this study, estimates may not be accurate. Besides, estimates of the input variables are seldom available and will have to be derived or based on expected optimums. The error in yield estimates will thus come from two sources; sampling and measurement errors in estimating input variables and error in estimating volume using the equation. This error could be quite large, affecting the accuracy and reliability of the yield estimates.

The general single entry equation (equation 33) can be used if only rough estimates are desired, with some loss in accuracy. A casual examination of the coefficients of these equations (Table 4) shows that, the coefficients for site classes I and III are close to those of the general model, with those of site class II appearing much different. The general local volume equation may produce good results for site classes I and III, though no effort was made to investigate this.

When estimates of higher precision are desired, it is recommended that, the double entry volume equation be used. Diameter and height data are fed into the model to generate the individual tree volumes. These are then summed to produce an estimate for the stand.

Using two-stage sampling with probability proportional to basal area at the second stage (PPG) has been shown to result in improved precision over the mean tree method (two-stage simple random sampling). The sub-sample size of three appeared not to be adequate for the mean tree method especially for the higher age classes. Apparently, these findings are consistent with those of Murchison (1990), who found out that, a sub-sample size of 3 to 6 trees per plot was adequate for the PPG but not for the mean tree method. If absolutely necessary, the PPG method should be given preference over the mean tree method. The use of the volume equations is recommended if a compromise is sought between data adequacy and tree value, limiting the sub-sample size to figures less than or equal to three.

5.3 BIOMASS EQUATIONS AND TABLES.

The biomass models were developed to ensure additivity. This may be useful for future studies and research which may require this property. However, for quantifying products for valuation purposes, this property may not be important.

The method chosen to ensure additivity provides estimates for the standard error for the total above ground biomass model. This allows for the statement of reliability to be made. One disadvantage with this method is that, the component biomass models may not be individually, the best models in terms of meeting the

regression assumptions. Presumably, by using the individual component standard errors as weights the component additive models should approach their individually best models.

5.4 YIELD MODELS AND TABLES.

The yield models and tables presented in this study are empirical as they represent average conditions of stocking only. No consideration is given to stand density. Plantations included in this study have a history of 2 by 2 m planting spacing. No thinning has been done in these plantations and missing trees are assumed to be the result of natural mortality. Though some random tree cuttings were observed in some plantations, basal area as a measure of stand density was statistically insignificant in estimating the volume yield model. This may be an indication that these random cuttings did not significantly influence yield estimates.

The mean annual volume increments are estimated to peak at the ages; 31, 38 and 48 respectively for site classes I, II and III. These may represent the biologically optimum rotation ages for teak on the various site classes in Northern Ghana. At these ages, it may be possible to obtain some trees of timber size and for electric transmission poles on site classes I and II. The estimated mean annual increments at these ages are; 14.40, 9.32 and 4.05 m³ per hectare per year.

Yield can be estimated using either the tables or the equations presented in table 7. To estimate yield, there is the need to know the plantation age. With the site class determined using the site index curves presented in appendix XII, the appropriate yield table is chosen. The volume yield estimate is obtained by taking the product of the stocking factor and the volume yield presented in the tables.

Care must be taken when using the tables, not to extrapolate beyond the range of the data used in the study as this may result in serious bias. The nature of this bias is not known at present. Bias may be due to under representation of plantations in the older age classes. The danger is that, if the older plantations included in the study were generally located on good sites, the tables will tend to over-estimate yields in those age classes. The reverse is true if they were found on poor sites. Though ocular observation of the plantation during sampling did not reveal any of the above, this danger cannot be ruled out.

There is the need for permanent or semi-permanent plots to validate and to determine the nature of the bias that may be associated with the use of these tables. Until this is done, these tables remain provisional.

The site index curves were also based on the assumption that, all site conditions have equal likelihood of being represented in each age class. Though a conscious effort was made to include all sites in each age class, some age classes were

lacking. This assumption cannot therefore be guaranteed. The curves in Figure 4 however show consistency of results of this study with those of other studies. Thus, the site index curves presented here can be considered useful until curves based on larger data sets are available. The functions presented here need validation with broader data sets. The curves will be useful for classifying sites in future studies, especially when permanent plots are to be established to represent all site classes.

5.5 DIAMETER DISTRIBUTION MODELS

The results of the study show that, the three-parameter Weibull distribution model is adequate for fitting diameter distribution of teak in Northern Ghana, under the current management regimes. The maximum likelihood method was superior to the percentile method in estimating the parameters.

For the younger plantations, the percentile estimates were equally as good in fit as the maximum likelihood estimates. Zankis (1979) has indicated the percentile estimators are simple and more accurate than the maximum likelihood estimators when c , the shape parameter is less than 2 and the sample size is small. Zarnoch and Dell (1985) have also shown that the percentile estimators were comparable to or even better than the maximum likelihood estimators when c is close to or less than 2; but inferior for most forestry applications where c is generally greater than

2. For positively skewed distributions as observed for this study, the predictive ability of the percentile estimators will generally be good.

Zarnoch and Dell (1985) have also demonstrated the insensitivity of the Weibull function to variation in parameter estimates. Their results showed that though the parameter estimates may be considerably inaccurate, the resulting percentile estimates are generally accurate. Therefore, though the percentile estimates may not be precise, their frequency prediction can be expected to be good.

6.0 CONCLUSIONS AND RECOMMENDATIONS

Teak plantations in the study area are generally less productive than plantations in the high forest zone of Ghana. The most productive sites in the study area are comparable only to sites classified as class V in the high forest zone in terms of productivity. The potential of teak on such sites is considerable.

Teak can be grown on an estimated biologically optimum rotations of 31, 38 and 48 years on site classes I, II and III respectively. However, if timber quality is desired, this period may have to be extended. Maximum current annual increments are estimated to occur at 14, 17 and 21 years respectively.

Site indices of 19, 14 and 10, at the base age of 20 years, are estimated for teak in Northern Ghana, to correspond with site classes I, II, and III respectively.

The three parameter Weibull probability density function is adequate for describing diameter distributions of teak plantations in Northern Ghana. The maximum likelihood and the percentile methods can be used for parameter estimation.

The site index curves and yield models are provisional and must be regarded as such. There is the need for validating these in further studies, with data from

permanent or semi-permanent growth plots.

It is recommended that the current planting spacing of 2 by 2 m be reduced to about 1.8 by 1.8 m to accommodate initial mortality. Otherwise, "beating up" should be encouraged to maintain optimum stocking levels. Thinning and other management practices should be considered seriously especially for plantations on good soils.

The sub-sample size of three trees per plot was admittedly, too small for estimating stand volume by the two-stage simple random sampling method. It is recommended that, in future studies, the sample size be increased if possible; otherwise the use of the volume tables developed in this study be considered.

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APPENDICES

APPENDIX I

SUMMARIES OF STAND CHARACTERISTICS BY PLANTATIONS

Plantation No.	Age	DBH (cm)		Height (m)	
		Mean	Standard Error	Mean	Standard Error
1	3	2.08	0.05	1.93	0.04
2	3	2.99	0.08	3.12	0.06
3	3	3.02	0.08	3.30	0.08
4	4	3.41	0.14	3.06	0.10
5	4	3.01	0.08	2.67	0.08
6	4	3.35	0.14	2.82	0.09
7	6	4.33	0.10	3.77	0.07
8	6	6.04	0.10	5.64	0.08
9	6	7.54	0.17	7.16	0.10
10	7	7.98	0.13	8.11	0.09
11	7	7.41	0.14	7.12	0.12
12	7	7.53	0.13	7.47	0.12
13	8	7.10	0.12	6.89	0.10
14	8	8.15	0.11	8.24	0.10
15	8	7.71	0.12	7.59	0.11
16	9	7.91	0.19	7.15	0.12
17	9	7.78	0.18	6.30	0.13
18	9	7.76	0.17	6.77	0.14
19	17	10.73	0.27	8.28	0.13
20	17	10.76	0.20	8.31	0.12
21	17	10.86	0.29	7.59	0.13
22	26	18.21	0.34	13.86	0.16
23	31	21.06	0.40	14.76	0.13
24	38	23.58	0.73	19.85	0.41
25	40	19.53	0.46	15.13	0.20

APPENDIX II

COMPARASON OF STAND VOLUME ESTIMATES [m³/ha] BY;
TWO-STAGE SIMPLE RANDOM SAMPLING (2SRS), PROBABILITY
PROPORTIONAL TO BASAL AREA (PPG) AND VOLUME TABLES (VT).

PLANTATION NUMBER	AGE	2SRS		PPG		VT	
		VOL	SE	VOL	SE	VOL.	SE
1	3	1.89	0.30	1.79	0.14	1.83	0.00
2	3	6.57	2.03	4.83	0.32	5.20	0.01
3	3	5.04	0.85	4.74	0.60	5.36	0.02
4	4	6.48	3.38	6.32	1.61	5.91	0.04
5	4	8.42	0.80	5.32	0.65	5.00	0.01
6	4	7.53	3.03	6.57	1.53	5.56	0.03
7	6	11.51	2.00	12.88	1.36	11.48	0.04
8	6	31.56	6.13	26.93	4.78	30.51	0.11
9	6	39.06	9.03	38.22	3.08	46.47	0.08
10	7	55.22	2.80	48.03	2.76	58.06	0.06
11	7	31.76	2.29	30.43	3.37	34.86	0.12
12	7	40.42	3.59	33.82	4.92	40.06	0.15
13	8	24.42	4.44	26.37	2.31	29.88	0.07
14	8	46.11	7.23	47.17	4.85	55.31	0.12
15	8	36.14	8.08	37.76	6.09	44.17	0.17
16	9	41.81	14.82	37.83	7.24	44.92	0.34
17	9	25.89	5.91	26.46	3.15	28.55	0.12
18	9	31.43	6.27	31.79	4.59	40.81	0.33
19	17	53.53	12.84	59.81	5.28	74.57	0.44
20	17	66.47	18.06	73.94	9.48	81.51	0.41
21	17	51.18	12.71	59.20	7.08	65.15	0.32
22	26	303.56	87.17	293.75	21.71	339.42	1.05
23	31	428.39	101.42	437.74	27.73	472.28	1.52
24	38	650.31	155.70	675.60	21.12	776.72	0.18
25	40	160.93	26.05	185.49	19.65	220.33	2.01

APPENDIX III

ABOVEGROUND STAND BIOMASS ESTIMATES [t/ha]

PLANTATION NUMBER	AGE	COMPONENT BIOMASS (t /ha)			
		STEM	BRANCH	LEAVE S	TOTAL
1	3	0.27	0.01	0.44	0.72
2	3	0.95	0.16	0.83	1.94
3	3	0.92	0.16	0.81	1.90
4	4	1.46	0.28	0.88	2.62
5	4	1.00	0.18	0.90	2.08
6	4	1.45	0.27	0.87	2.60
7	6	3.41	0.69	1.58	5.68
8	6	12.55	2.59	3.52	18.65
9	6	16.63	3.43	3.47	23.54
10	7	19.16	3.96	3.91	27.03
11	7	12.17	2.51	2.69	17.37
12	7	14.65	3.03	3.21	20.88
13	8	7.44	1.54	1.77	10.74
14	8	18.27	3.77	3.69	25.73
15	8	14.88	3.07	3.18	21.13
16	9	16.90	3.49	3.36	23.75
17	9	12.04	2.49	2.50	17.02
18	9	15.57	3.21	3.21	22.00
19	17	30.40	6.25	4.60	41.25
20	17	32.37	6.66	5.00	44.03
21	17	29.77	6.12	4.48	40.36
22	26	95.90	19.58	10.60	126.09
23	31	131.28	26.76	13.54	171.58
24	38	156.86	31.94	15.40	204.20
25	40	113.32	23.12	12.13	148.57

APPENDIX IV

THE LIST OF STANDARD MODELS FOR VOLUME TABLE CONSTRUCTION

NUMBER	EQUATION
1	$V = a + b_1D$
2	$V = a + b_1D + b_2D^2$
3	$V = a + b_1D^2$
4	$V = a + b_1D^2H$
5	$V = a + b_1D^2 + b_2H + b_3D^2H$
6	$V = a + b_1D^2 + b_2DH + b_3D^2H$
7	$\ln V = a + b_1 \ln D$
8	$\ln V = a + b_1 \ln D + b_2 \ln H$
9	$V/D^2 = a + b_1(1/D) + b_2(1/D^2)$
10	$V/D^2 = a + b_1(1/D^2)$
11	$V/D^2H = a + b_1(1/D^2H)$
12	$V/D^2 = a + b_1(1/D^2) + b_2(H/D^2) + b_3H$
13	$V/D^2H = a + b_1(1/H) + b_2(1/D^2) + b_3(1/D^2H)$
14	$V/D^2 = a + b_1(1/D^2) + b_2(H/D) + b_3H$
15	$V/D^2H = a + b_1(1/H) + b_2(1/D) + b_3(1/D^2H)$

APPENDIX V

STANDARD VOLUME TABLE FOR PLANTATION TEAK IN NORTHERN
GHANA [VOLUME IN CUBIC DECIMETERS]

DIAMETER (cm)	HEIGHT (m)					
	2	4	6	8	10	12
2	1.45	1.88	2.32			
4	3.06	4.79	6.52	8.25		
6		8.36	12.25	16.14		
8		12.59	19.51	26.42	33.33	
10		17.49	28.29	39.09	49.89	
12			38.61	54.16	69.71	85.26
14			50.45	71.62	92.78	113.95
16			63.82	91.47	119.11	146.76
18			78.72	113.71	148.70	183.69
20			95.14	138.34	181.54	224.74
22			113.10	165.37	217.64	269.91
24				194.79	256.99	319.20
26				226.60	299.60	372.61
28				260.80	345.47	430.14
30				297.39	394.59	491.79
32				336.38	446.97	557.56
34				377.76	502.60	627.45
36				421.53	561.49	701.46

	HEIGHT (m)					
	14	16	18	20	22	24
14	135.12					
16	174.41	202.06				
18	218.68	253.68	288.67			
20	267.94	311.14	354.34	397.54		
22	322.18	374.46	426.73	479.00	531.27	583.54
24	381.41	443.62	505.83	568.03	630.24	692.45
26	445.62	518.63	591.64	664.64	737.65	810.66
28	514.81	599.49	684.16	768.83	853.50	938.17
30	588.99	686.19	783.39	880.59	977.79	1075.00
32	668.15	778.75	889.34	999.93	1110.50	1221.10
34	752.30	877.15	1002.00	1126.80	1251.70	1376.50
36	841.43	981.40	1121.40	1261.30	1401.30	1541.30

APPENDIX VI

LOCAL VOLUME TABLE FOR PLANTATION TEAK IN NORTHERN GHANA

Diameter class (cm)	VOLUME (dm ³)
2	1.36
4	5.15
6	12.37
8	23.02
10	37.08
12	54.58
14	75.49
16	99.83
18	127.60
20	158.78
22	193.40
24	231.43
26	272.89
28	317.78
30	366.08
32	417.82
34	472.97
36	562.13

APPENDIX VII

ABOVEGROUND BIOMASS TABLES FOR TEAK IN PLANTATIONS
IN NORTHERN GHANA [OVEN DRY WEIGHTS IN KILOGRAMS]

DIAMETER (cm).	COMPONENT BIOMASS (Kg)			
	LEAVES	BRANCHES	BOLE	TOTAL
2	0.29	0.00	0.32	0.60
4	0.70	0.23	1.50	2.42
6	1.23	0.85	4.67	6.75
8	1.89	1.88	9.83	13.59
10	2.68	3.30	16.96	22.94
12	3.59	5.12	26.09	34.80
14	4.62	7.35	37.19	49.16
16	5.78	9.97	50.28	66.03
18	7.06	12.99	65.36	85.41
20	8.47	16.41	82.42	107.30
22	10.01	20.23	101.46	131.69
24	11.66	24.45	122.49	158.60
26	13.45	29.07	145.50	188.01
28	15.36	34.08	170.49	219.93
30	17.39	39.50	197.47	254.35
32	19.55	45.31	226.43	291.29
34	21.83	51.53	257.38	330.73
36	24.24	58.14	290.31	372.69

APPENDIX VIII

MEAN OVEN DRY BIOMASS AS PROPORTIONS OF FRESH BIOMASS OF THE ABOVEGROUND TREE COMPONENTS.

AGE	STEM		BRANCH		LEAVES	
	MEAN	St. Dev	MEAN	St. Dev	MEAN	St. Dev
3	0.400	0.036	0.341	0.080	0.380	0.060
4	0.411	0.044	0.342	0.071	0.360	0.034
6	0.452	0.043	0.418	0.063	0.360	0.073
7	0.491	0.032	0.475	0.041	0.420	0.032
8	0.502	0.035	0.484	0.035	0.350	0.054
9	0.513	0.045	0.485	0.034	0.400	0.052
17	0.542	0.087	0.512	0.050	0.401	0.091
26	0.561	0.005	0.532	0.013	0.411	0.023
31	0.562	0.005	0.532	0.012	0.412	0.024
38	0.563	0.005	0.533	0.012	0.412	0.025
40	0.568	0.005	0.535	0.010	0.413	0.025

APPENDIX IX

SUMMARY OF GROWTH AND YIELD DATA FOR SOME TEAK PLANTATIONS
IN THE OFFINSO DISTRICT IN THE HIGH FOREST ZONE OF GHANA

Pl'n no.	Age	Number of Stems /ha	Mean dbh (cm)	Basal Area/ha (m ²)	Top height (m)	Volume m ³ /ha	Site Index (m)
1	13	291	21.00	10.38	16.80	84.00	20.00
2	14	265	27.00	13.03	19.90	114.00	23.00
3	15	223	26.00	11.70	18.00	95.00	20.00
4	15	399	19.00	11.72	16.90	88.00	19.00
5	16	259	29.00	16.12	22.00	152.00	24.00
6	16	248	28.00	13.67	19.90	115.00	22.00
7	16	245	28.00	14.63	23.70	146.00	26.00
8	16	188	28.00	11.34	18.70	92.00	20.00
9	16	285	29.00	18.26	22.20	175.00	24.00
10	16	432	22.00	16.39	19.70	140.00	21.00
11	17	200	34.00	17.83	23.10	172.00	24.00
12	17	320	27.00	16.34	20.30	139.00	21.00
13	17	200	32.00	15.90	24.60	162.00	26.00
14	17	198	26.00	11.20	16.60	84.00	18.00
15	17	229	29.00	13.86	21.20	127.00	22.00
16	17	308	28.00	18.30	21.50	169.00	23.00
17	18	298	27.00	15.25	22.10	146.00	23.00
18	18	294	29.00	18.18	21.20	164.00	22.00
19	18	163	29.00	10.89	19.10	90.00	20.00
20	18	229	27.00	12.00	20.20	107.00	21.00
21	19	177	30.00	11.91	19.40	100.00	20.00
22	20	326	25.00	15.28	18.60	124.00	19.00
23	20	270	30.00	17.65	22.30	165.00	22.00
24	20	216	30.00	14.83	21.60	138.00	22.00
25	20	245	32.00	16.74	20.70	153.00	21.00
26	20	262	30.00	16.66	20.90	160.00	21.00
27	21	276	29.00	17.75	22.90	174.00	23.00
28	21	197	33.00	16.36	22.90	158.00	23.00
29	21	203	32.00	15.65	24.00	161.00	24.00
30	21	296	28.00	17.41	22.60	165.00	22.00
31	22	255	32.00	19.05	23.90	192.00	23.00
33	22	287	30.00	20.60	24.90	217.00	24.00

APPENDIX X

PROVISIONAL YIELD TABLES FOR TEAK PLANTATIONS IN
NORTHERN GHANA

SITE CLASS I

AGE	dbh(cm)		HEIGHT(m)		BA m ² /ha	VOLUME (m ³)			BIOMASS (t/ha)		
	MEAN	TOP	MEAN	TOP		GROSS	CAI	MAI	TOTAL	LEAVES	WOODY
2	2.12	3.13	1.86		0.67	1.26	2.48	0.63	0.70	0.37	0.33
4	5.81	6.72	4.55		4.48	12.62	8.78	3.15	5.64	1.56	4.08
6	9.10	9.43	6.75		10.42	35.03	13.26	5.84	14.20	2.70	11.50
8	11.88	11.54	8.55		17.24	64.39	15.84	8.05	24.63	3.58	21.05
10	14.25	13.24	10.04		24.31	97.55	17.17	9.76	35.87	4.23	31.63
12	16.30	14.66	11.31		31.32	132.56	17.75	11.05	47.34	4.72	42.62
14	18.10	15.86	12.40		38.14	168.24	17.87	12.02	58.73	5.07	53.66
16	19.69	16.90	13.36		44.70	203.87	17.73	12.74	69.88	5.33	64.55
18	21.11	17.82	14.21		50.99	239.04	17.42	13.28	80.70	5.52	75.18
20	22.39	18.63	14.97		56.99	273.49	17.02	13.67	91.15	5.65	85.50
22	23.56	19.36	15.66		62.71	307.07	16.56	13.96	101.22	5.74	95.48
24	24.63	20.02	16.28		68.17	339.71	16.08	14.15	110.91	5.80	105.11
26	25.61	20.62	16.85		73.39	371.38	15.59	14.28	120.22	5.84	114.39
28	26.52	21.17	17.38		78.36	402.07	15.10	14.36	129.18	5.85	123.32
30	27.36	21.68	17.87		83.12	431.79	14.62	14.39	137.79	5.86	131.93
32	28.14	22.15	18.32		87.68	460.57	14.16	14.39	146.07	5.85	140.23
34	28.88	22.58	18.74		92.04	488.44	13.71	14.37	154.05	5.83	148.22
36	29.57	22.99	19.13		96.22	515.43	13.28	14.32	161.73	5.80	155.93
38	30.22	23.37	19.50		100.2	541.57	12.87	14.25	169.13	5.77	163.36
40	30.83	23.73	19.85		104.1	566.91	12.47	14.17	176.27	5.73	170.54

SITE CLASS II

AGE	dbh(cm)		HEIGHT (m)		BA m ² /ha	VOLUME (m ³)			BIOMASS (t/ha)		
	MEAN	TOP	MEAN	TOP		GROSS	CAI	MAI	TOTAL	LEAVE S	WOOD
2	1.05	2.32	1.08		0.16	0.43	0.93	0.21	0.40	0.21	0.19
4	3.29	4.98	3.03		1.44	5.50	4.24	1.38	3.46	0.96	2.50
6	5.44	6.98	4.78		3.86	17.05	7.15	2.84	8.97	1.70	7.27
8	7.36	8.55	6.28		6.93	33.47	9.12	4.18	15.82	2.30	13.52
10	9.03	9.81	7.56		10.35	53.03	10.34	5.30	23.31	2.75	20.56
12	10.51	10.86	8.68		13.91	74.49	11.05	6.21	31.02	3.09	27.93
14	11.82	11.75	9.65		17.50	97.00	11.42	6.93	38.74	3.34	35.40

...more

16	13.00	12.52	10.52	21.06	120.00	11.56	7.50	46.34	3.53	42.81
18	14.06	13.20	11.30	24.55	143.14	11.56	7.95	53.75	3.67	50.08
20	15.03	13.80	12.00	27.96	166.16	11.45	8.31	60.94	3.78	57.16
22	15.92	14.34	12.64	31.26	188.90	11.29	8.59	67.89	3.85	64.04
24	16.73	14.83	13.23	34.46	211.27	11.08	8.80	74.59	3.90	70.69
26	17.49	15.28	13.77	37.55	233.20	10.84	8.97	81.06	3.94	77.12
28	18.19	15.68	14.26	40.54	254.64	10.60	9.09	87.28	3.96	83.33
30	18.84	16.06	14.73	43.42	275.58	10.34	9.19	93.29	3.96	89.32
32	19.45	16.41	15.16	46.21	296.00	10.08	9.25	99.07	3.97	95.11
34	20.02	16.73	15.56	48.90	315.90	9.82	9.29	104.65	3.96	100.69
36	20.56	17.03	15.94	51.50	335.29	9.57	9.31	110.03	3.95	106.08
38	21.07	17.31	16.30	54.02	354.18	9.32	9.32	115.22	3.93	111.29
40	21.55	17.58	16.64	56.45	372.58	9.08	9.31	120.24	3.91	116.33

SITE CLASS III

AGE	dbh(cm)	HEIGHT (m)		BA m ² /ha	VOLUME (m ³)			BIOMASS (t/ha)		
	MEAN	TOP	MEAN		GROSS	CAI	MAI	TOTAL	LEAVE S	WOOD
2	0.45	1.72	0.58	0.03	0.06	0.15	0.03	0.24	0.13	0.11
4	1.62	3.69	1.72	0.40	1.15	1.02	0.29	2.16	0.60	1.56
6	2.85	5.17	2.79	1.17	4.20	2.02	0.70	5.75	1.09	4.65
8	4.01	6.33	3.71	2.22	9.10	2.84	1.14	10.29	1.49	8.79
10	5.05	7.27	4.51	3.44	15.41	3.44	1.54	15.31	1.81	13.50
12	5.99	8.04	5.21	4.75	22.74	3.86	1.89	20.53	2.05	18.48
14	6.84	8.70	5.83	6.11	30.76	4.15	2.20	25.79	2.23	23.56
16	7.61	9.28	6.38	7.48	39.25	4.33	2.45	30.99	2.36	28.63
18	8.32	9.78	6.87	8.85	48.03	4.44	2.67	36.09	2.47	33.62
20	8.97	10.23	7.32	10.20	56.98	4.50	2.85	41.05	2.54	38.51
22	9.57	10.63	7.73	11.53	66.00	4.52	3.00	45.86	2.60	43.26
24	10.12	10.99	8.10	12.83	75.02	4.50	3.13	50.52	2.64	47.88
26	10.63	11.32	8.45	14.09	84.00	4.47	3.23	55.02	2.67	52.34
28	11.12	11.62	8.77	15.32	92.90	4.43	3.32	59.36	2.69	56.67
30	11.57	11.90	9.07	16.52	101.70	4.37	3.39	63.55	2.70	60.85
32	11.99	12.15	9.35	17.69	110.38	4.30	3.45	67.60	2.71	64.89
34	12.39	12.39	9.61	18.82	118.92	4.23	3.50	71.51	2.70	68.80
36	12.76	12.62	9.86	19.92	127.32	4.16	3.54	75.28	2.70	72.58
38	13.12	12.83	10.09	20.99	135.56	4.09	3.57	78.93	2.69	76.24
40	13.46	13.02	10.31	22.02	143.66	4.01	3.59	82.46	2.68	79.78

APPENDIX XI

OBSERVED AND PREDICTED DIAMETER CLASS FREQUENCIES BY AGE CLASSES

AGE CLASS	DIAMETER CLASS (cm)	DIAMETER FREQUENCIES (PER HACTARE)		
		OBSERVED	PRED (PT)	PRED. (ML)
3 YEAR	1	27	28	21
	2	650	682	649
	3	567	558	553
	4	260	234	265
	5	63	67	79
	6	16	14	16
	TOTAL	1583	1583	1583
4 YEAR	1	22	18	16
	2	314	501	310
	3	643	617	717
	4	449	349	424
	5	173	117	130
	6	22	25	25
	7	5	3	6
	9	2	0	2
	TOTAL	1630	1630	1630
6 YEAR	2	22	18	23
	3	135	165	136
	4	449	363	429
	5	406	455	455
	6	384	437	407
	7	384	346	320
	8	271	233	227
	9	89	135	136
	10	65	68	70
	11	49	30	45
	12	5	12	13
	TOTAL	2260	2260	2260

7 YEAR	4	32	3	17
	5	65	77	105
	6	238	276	280
	7	520	499	437
	8	401	518	454
	9	276	290	297
	10	184	76	122
	11	32	8	35
	TOTAL	1748	1748	1748

8 YEAR	5	65	19	68
	6	309	277	237
	7	352	489	441
	8	541	462	492
	9	330	287	322
	10	65	124	119
	11	33	38	17
	TOTAL	1695	1695	1695

9 YEAR	4	16	30	16
	5	114	122	127
	6	281	272	270
	7	368	351	350
	8	302	334	334
	9	261	249	255
	10	139	149	143
	11	97	72	80
	13	10	9	14
	14	2	2	2
	TOTAL	1591	1591	1591

17 YEAR	5	5	19	15
	6	32	34	35
	7	65	68	71
	8	108	142	148
	9	265	213	197

... More

10	238	256	237
11	222	256	237
12	152	216	208
13	168	153	163
14	141	92	104
15	76	46	59
17	6	6	7
18	5	2	2
TOTAL	1484	1484	1484

26 YEAR

12	19	16	42
13	41	23	71
14	72	75	85
15	89	128	113
16	210	169	141
17	227	191	170
18	162	193	170
19	152	177	170
20	130	148	155
21	117	113	113
22	81	80	85
23	61	52	57
24	32	31	28
25	19	17	14
TOTAL	1413	1413	1413

31 YEAR

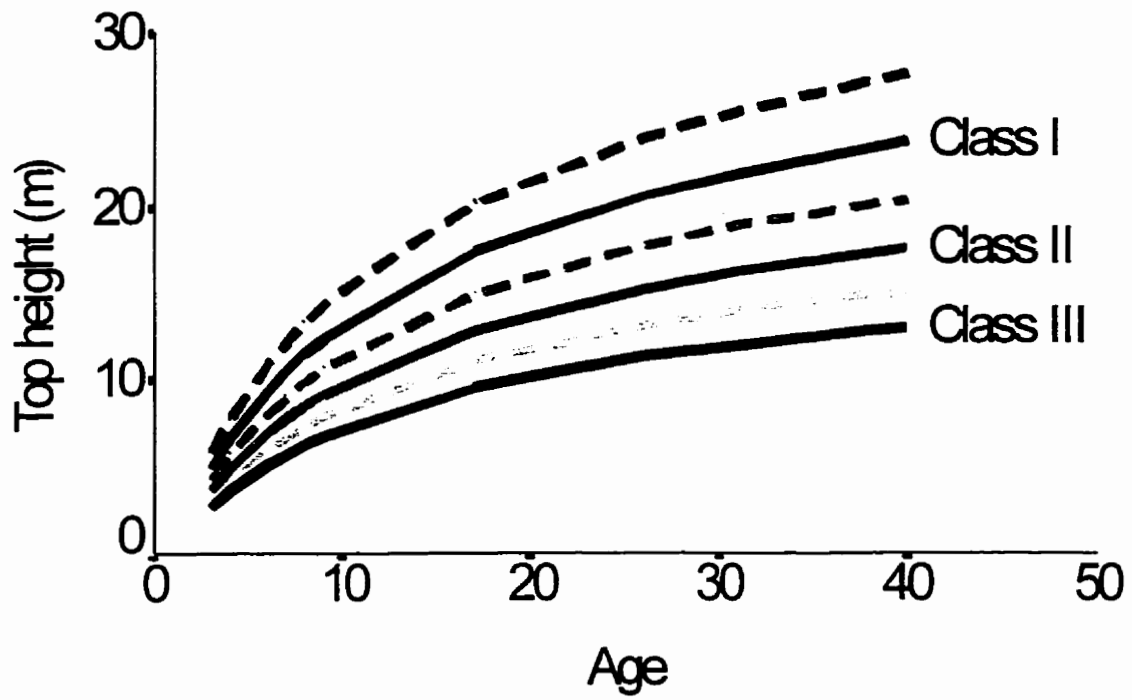
15	32	63	59
16	95	110	92
17	111	129	105
18	131	135	118
19	141	133	131
20	114	126	131
21	149	115	131
22	122	102	118
23	106	90	114
24	65	77	92

... more

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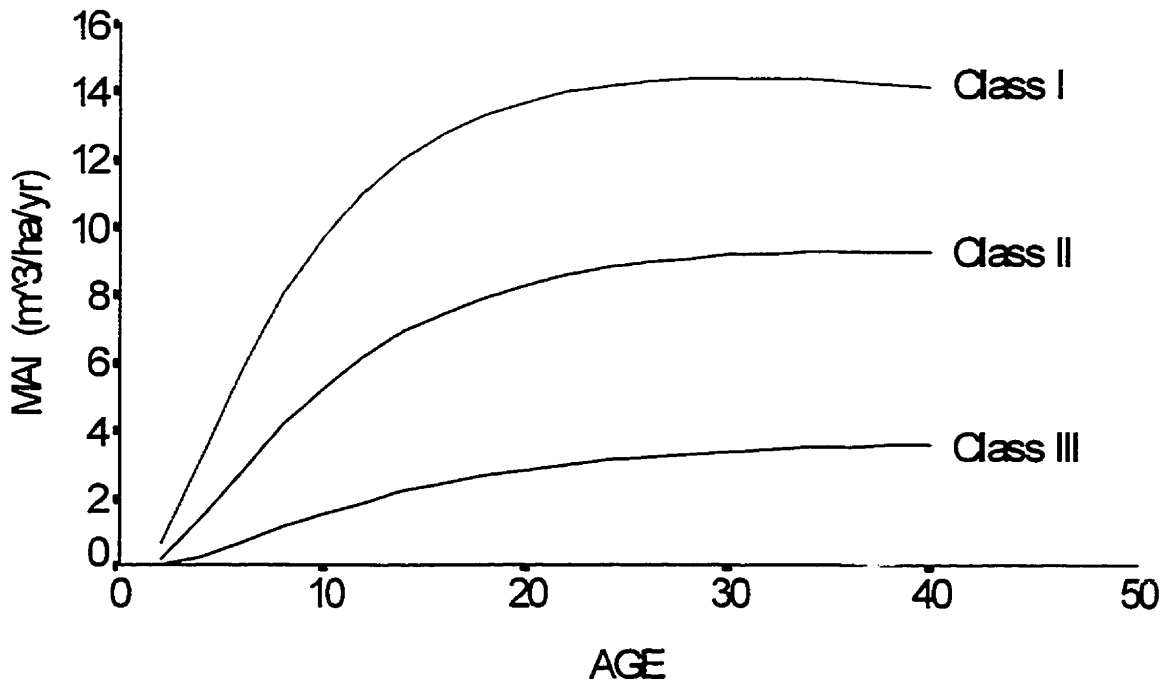
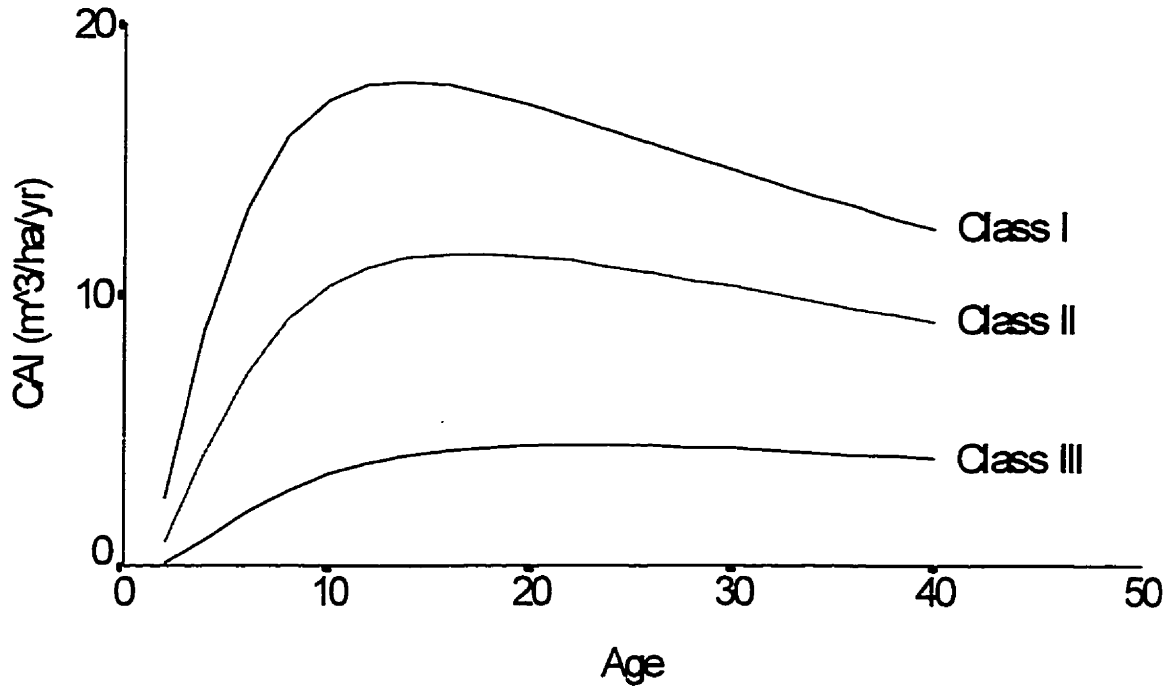
	25	77	65	79
	26	20	54	55
	27	32	44	42
	28	97	36	26
	29	16	29	18
	TOTAL	1309	1309	1309
38 YEAR	16	32	32	35
	17	34	55	61
	18	96	77	78
	19	95	96	85
	20	97	111	99
	21	132	120	104
	22	124	123	106
	23	142	120	126
	24	101	112	110
	25	97	99	100
	26	66	85	90
	27	65	70	81
	28	65	55	71
	32	27	14	25
	33	7	9	9
	TOTAL	1180	1180	1180
40 YEAR	12	22	19	41
	14	55	46	54
	15	85	81	81
	16	107	114	113
	17	137	141	137
	18	162	158	152
	19	157	163	166
	20	166	157	148
	21	145	141	135
	22	115	119	111
	23	96	95	99
	24	65	71	66
	25	42	50	51
	TOTAL	1354	1354	1354

APPENDIX XII

SITE INDEX CURVES OF TEAK PLANTATIONS IN
NORTHERN GHANA

APPENDIX XIII

VOLUME GROWTH CURVES FOR TEAK PLANTATIONS
IN NORTHERN GHANA



APPENDIX XIV

DERIVATION OF THE CUMULATIVE WEIBULL FUNCTION
FROM THE WEIBULL PROBABILITY DENSITY FUNCTION

Let $F(x)$ = Cumulative Weibull function,
 $f(x)$ = Weibull probability density function.

The probability of a diameter falling between a and x is given by

$$F(x) = \int_0^{x-a} \frac{c}{b} \left(\frac{t}{b} \right)^{c-1} \text{EXP} \left[- \left(\frac{t}{b} \right)^c \right] dt; \quad t = x-a \quad \text{A1}$$

$$\text{Let } M = \left(\frac{t}{b} \right)^c, \text{ then } \frac{dM}{dt} = \frac{c}{b} \left(\frac{t}{b} \right)^{c-1} \quad \text{A2}$$

Substituting this into A2 gives;

$$F(x) = \int_0^M \text{EXP}(-M) dM \quad \text{A3}$$

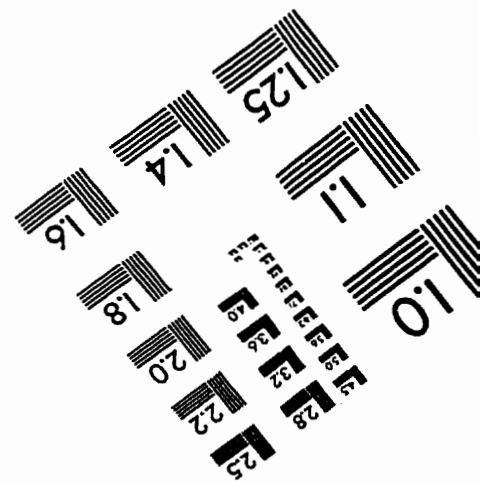
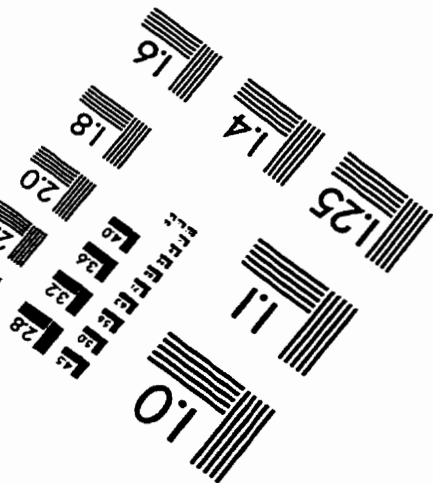
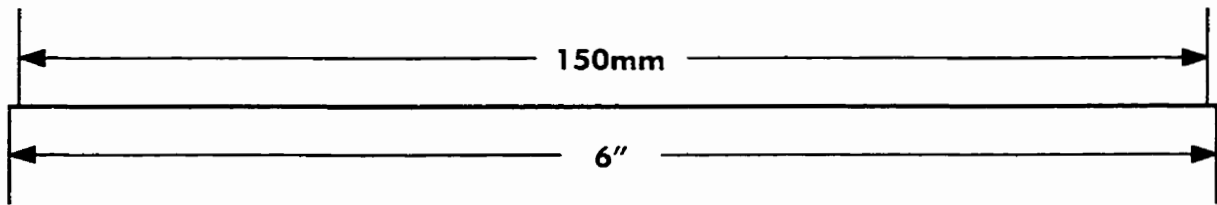
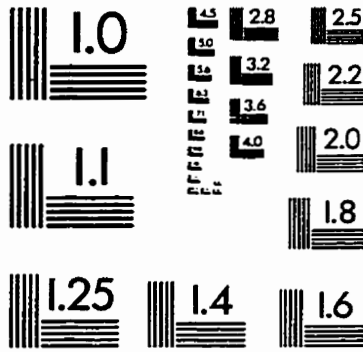
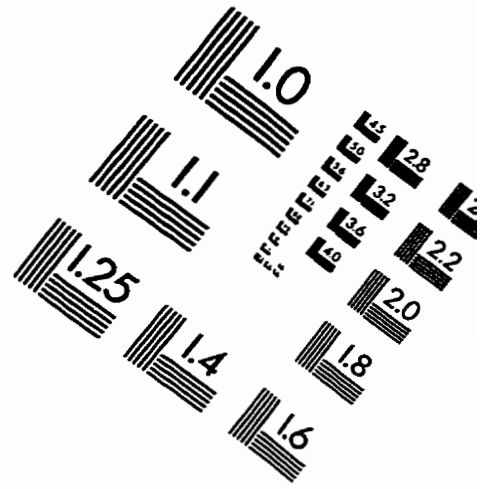
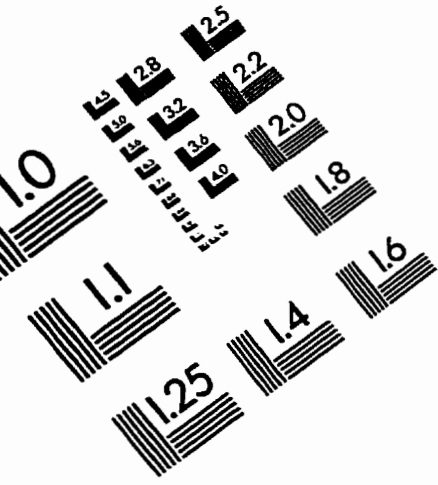
Therefore;

$$F(x) = 1 - [-\text{EXP}(-M)] \quad \text{A4}$$

Hence

$$F(x) = 1 - \text{EXP} \left[\left(\frac{x-a}{b} \right)^c \right] \quad \text{A5}$$

IMAGE EVALUATION TEST TARGET (QA-3)



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