The Flight Dynamics of a Full-Scale Ornithopter

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science Graduate Department of Aerospace Science and Engineering in the University of Toronto

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ABSTRACT

This thesis investigates the non-linear flight dynamics of a full-scale flapping-wing aircraft (ornithopter). Using simplifying assumptions, the equations of motion were developed for a 2-wing-panel and 3-wing-panel model. A computer program was written to examine the longitudinal and lateral stability of the ornithopter. The program was first tested using inputs for a model ornithopter known as "Mr. Bill" and then using the inputs for a full-scale ornithopter. The results indicate that both "Mr. Bill" and the full-scale ornithopter are longitudinally and laterally stable. The accelerations and displacements are much less for the 3-panel model than for the 2-panel case because the 3-panel design serves to balance the time-varying forces seen by the fuselage.
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LIST OF SYMBOLS

Subscripts

b - body

t - tail

f - fin

cp - center panel

w1 - port wing

w2 - starboard wing

Greek Symbols

α - angle of attack

β - sideslip angle

ε - downwash angle

Φ - roll angle

Θ - pitch angle

Ψ - yaw angle

γ - flapping angle

ρ - density of air
English Symbols

AR - aspect ratio
b - wing span
t - mean wing chord
CD - drag coefficient
CD0 - drag coefficient at zero angle of attack
CL - lift coefficient
CLA - lift - curve slope
CM - pitching - moment - curve slope
CY - side - force stability derivative
CN - yawing - moment stability derivative
CL - rolling - moment stability derivative
D - drag force
dxb - x distance from body cg. to center panel cg (3 - panel)
   x distance from body cg. to point on root of wing aligned with wing cg (2 - panel)
dyb - y distance from body cg. to center panel cg (3 - panel)
   y distance from body cg. to point on root of wing aligned with wing cg (2 - panel)
dzb - z distance from body cg. to center panel cg (3 - panel)
   z distance from body cg. to point on root of wing aligned with wing cg (2 - panel)
dx - x component of center panel velocity w.r.t. body (3 - panel)
dy - y component of center panel velocity w.r.t. body (3 - panel)
dz - z component of center panel velocity w.r.t. body (3 - panel)
ax - x component of center panel acceleration w.r.t. body (3 - panel)
ay - y component of center panel acceleration w.r.t. body (3 - panel)
az - z component of center panel acceleration w.r.t. body (3 - panel)
D xp - x distance from wing cg to pivot point
D yp - y distance from wing cg to pivot point
D zp - z distance from wing cg to pivot point
D xw - x distance from root of wing to cg of wing panel
D yw - y distance from root of wing to cg of wing panel
D zw - z distance from root of wing to cg of wing panel
- Oswald efficiency factor
- $F_x$ - $x$ direction reaction force at wing pivot point
- $F_y$ - $y$ direction reaction force at wing pivot point
- $F_z$ - $z$ direction reaction force at wing pivot point
- $H_{xb}$ - $x$ distance from body cg to pivot point on wing
- $H_{yb}$ - $y$ distance from body cg to pivot point on wing
- $H_{zb}$ - $z$ distance from body cg to pivot point on wing
- $I_{xx}, I_{yy}, I_{zz}, I_{xz}, I_{yz}$ - moment of inertias
- $\dot{I}_{xx}, \dot{I}_{yy}, \dot{I}_{zz}, \dot{I}_{xz}, \dot{I}_{yz}$ - derivatives of moment of inertias
- $J_L$ - $x$ direction reaction moment at wing pivot point
- $J_M$ - $y$ direction reaction moment at wing pivot point
- $J_N$ - $z$ direction reaction moment at wing pivot point
- $L$ - lift force
- $L_{roll}$ - rolling moment
- $L_{aero}$ - total aerodynamic moments in the $x$ direction
- $m$ - mass
- $M$ - pitching moment
- $M_{Lx}$ - $x$ direction reaction moment between the outer wing panel and center panel (3-panel)
- $M_{Lx}$ - $x$ direction reaction moment between the wing and body (2-panel)
- $M_{Mx}$ - $y$ direction reaction moment between the outer wing panel and center panel (3-panel)
- $M_{My}$ - $y$ direction reaction moment between the wing and body (2-panel)
- $M_{Nz}$ - $z$ direction reaction moment between the outer wing panel and center panel (3-panel)
- $M_{Nz}$ - $z$ direction reaction moment between the wing and body (2-panel)
- $M_{aero}$ - total aerodynamic moments in the $y$ direction
- $N$ - yawing moment
- $N_{aero}$ - total aerodynamic moments in the $z$ direction
P - roll angular velocity rate
\dot{P} - roll angular acceleration rate
Q - pitch angular velocity rate
\dot{Q} - pitch angular acceleration rate
R - yaw angular velocity rate
\dot{R} - yaw angular acceleration rate
r - distance from wing root to wing segment
Re - Reynolds' number
Rx - x direction reaction force between the outer wing panel and center panel (3-panel)
     x direction reaction force between the wing and body (2-panel)
Ry - y direction reaction force between the outer wing panel and center panel (3-panel)
     y direction reaction force between the wing and body (2-panel)
Rz - z direction reaction force between the outer wing panel and center panel (3-panel)
     z direction reaction force between the wing and body (2-panel)
S - reference area
Vol - volume of body
U - surging velocity
\dot{U} - surging acceleration
V - sideslip velocity
\dot{V} - sideslip acceleration
W - plunging velocity
\dot{W} - plunging acceleration
X_{aero} - total aerodynamic forces in the x direction
Y - sideslip force
Y_{aero} - total aerodynamic forces in the y direction
Z_{aero} - total aerodynamic forces in the z direction
1. INTRODUCTION

The study of nature has often led to surprising breakthroughs in science and technology. One such example is the flapping-wing flight of birds. For centuries, humanity has been fascinated by the flight of birds and has often made numerous attempts at imitating them. One of the first known accounts of human flight comes from Greek mythology in which Daedulus and Icarus escape from their prison on the Mediterranean island of Crete using wings constructed of wax. Unfortunately, Icarus flew too close to the sun which melted his wings and he fell to his death. According to Chinese legend, Shun, the emperor of China around 2000 B.C., was taught to fly using wings. Among the Norsemen, a blacksmith named Illmienen fashioned metal wings to rise from the Earth. It was considered for a long time that the only way humans could fly would be to imitate the birds. However, this notion was shattered in 1799 when Sir George Cayley introduced the fixed-wing aircraft concept. Cayley's idea was the first of its kind which abandoned bird flight and introduced a new concept in which the lifting surfaces, the wings, were separated from the mechanism of thrust. This revolutionary idea eventually lead to the first successful engine-powered flight at Kitty Hawk in 1903 by the Wright brothers. Cayley's principle is still used today in all modern aircraft.
With the success of fixed-wing aircraft, flapping-wing flight was all but abandoned as a viable field of research. While fixed-wing aircraft have dominated the aeronautical research field, it is becoming apparent that a flapping-wing aircraft (ornithopter) holds great promise as an efficient means of generating lift and thrust simultaneously.[Ref. 17] Also, the boundary layer does not develop to its full potential in a non-stationary flow about an oscillating airfoil, which means that separation is delayed and the form drag is reduced. The difficulties of designing of such a flapping-wing device would have to consider the aerodynamic, mechanical, and structural aspects. Many attempts have been made to design ornithopters, the first of which was by Leonardo da Vinci. Leonardo designed ornithopters in which the pilot sat in an inclined position and used pulleys which required both hands and feet to flap the wings. [Ref.19] This concept was more sophisticated than the ones that preceded it because it involved more than strapping wings to arms. No one is certain if Leonardo’s designs were actually constructed. Most ornithopters were small rubber-powered models which derived from the 1874 model of Alphonse Penaud [Ref.5]. More sophisticated rubber-powered ornithopters used to study bird flight were built by von Holst [Ref.5]. Alexander Lippisch designed and built a human-powered ornithopter in 1929 which achieved powered glides from catapult launches [Ref.17]. In 1959, Emil Hartman built a human-powered ornithopter which was capable of extended glides but not sustained flight [Ref.17]. One successful example of a human-powered aircraft which was not
an ornithopter was the flight of the Gossamer Condor in 1977 [Ref.19]. The 70 lb. airplane with a 29 m wingspan had a large propeller in the back and was pedaled by a pilot weighing approximately 140 lbs. Although rubber-powered and human-powered ornithopters have existed, successful examples of motorized flapping-wing aircraft are few with the notable exception of the 18 ft span robot pterosaur built by AeroVironment Inc. of Monrovia California [Ref.5]. However, this model was not able to sustain flight.

The first successful flight of a motorized radio-controlled flapping-wing aircraft, known as “Mr. Bill”, was made on September 4, 1991 by Dr. James DeLaurier and Mr. Jeremy Harris at the University of Toronto. This model aircraft was able to sustain flight for about 3 minutes and was landed successfully. This ornithopter’s wing design is different from that for birds in that it does not have the complex motions such as feather spreading, fore-and-aft swinging, semispan variation, etc. (See Appendix G). The wing consists of 3 panels: 2 outer panels and a center panel (See Figure 1).

Figure 1: 3-Panel model ornithopter, "Mr. Bill"
The motion is such that the center panel moves in a direction which is opposite to the flapping of the outer panels. This three-panel design serves to balance the time-varying lift seen by the fuselage and evens out the power required from the engine during the flapping cycle. With the two-panel design, the power required for the downstroke is greater than for the upstroke. The flapping is a simple harmonic motion driven by a lightweight transmission which reduces the high rotational speed of the engine down to the low flapping frequencies which are required. A linear twist is experienced by the wing and is 90 degrees out of phase with the flapping. No ailerons are present, thus turning is accomplished by yaw/roll coupling produced by the rudder deflection in conjunction with the wing's average dihedral angle. The dihedral angle is accomplished by making the upstroke flapping angle larger than the downstroke angle.

Although the major motivation behind such an undertaking has been an interest in flapping-wing flight, the relative aeroacoustic silence of flapping wings can eventually be used for surveillance applications.

Research is now continuing into constructing a full-scale motorized ornithopter capable of carrying a human being. This enormous task can be divided into several different sub-areas some of which include: wing design, landing and take-off simulations, drive mechanism design, and flight dynamic analysis.
2. PROJECT DEFINITION

The purpose of this study is to investigate the complete non-linear flight dynamics of a full-scale ornithopter during cruising flight. In conventional fixed-wing aircraft, the equations of motion can be linearized and then uncoupled into longitudinal and lateral equations. The longitudinal equations examine forward velocity ($U$), plunging velocity ($W$), pitch ($\Theta$), and pitch rate ($Q$), while the lateral equations look at the roll ($\Phi$), yaw ($\Psi$), and sideslip velocity ($V$) (See Figure 2).

Notation. $L =$ rolling moment; $M =$ pitching moment; $N =$ yawing moment; $P =$ rolling velocity; $Q =$ pitching velocity; $R =$ yawing velocity; ($X$, $Y$, $Z$) = components of resultant aerodynamic force; ($U$, $V$, $W$) = components of velocity of $C$.

Figure 2: Conventional notation
However, for the ornithopter, it is more difficult to linearize the equations of motion and thus the non-linearity was maintained. Also, if the lateral variables ($\Phi, \Psi, V$) are set to zero, the complete set of equations do break down to the longitudinal case. However, the longitudinal variables ($\Theta, U, W, Q$) cannot be set to zero to obtain the lateral case. For the lateral case, the complete set of equations must be used. The equations of motion provide valuable insight into unstable flight regimes.

3. LITERATURE SEARCH

In dealing with the problem of examining the non-linear flight dynamics of the ornithopter, the first step was to become familiar with the field of flight dynamics, along with stability and control. Several excellent books have been written on this subject. The most recognized and widely used is written by Etkin, [Ref.6], a pioneer in this field. This book contains all the basics of flight dynamics including a detailed derivation of the non-linear equations of motion, which are applicable to any aircraft. The equations are linearized and decoupled into longitudinal and lateral equations. The stability derivatives are presented along with an examination of the control aspects of the aircraft. Linearization was not in the scope of this thesis and was only studied for informational purposes. A previous edition by Etkin [Ref.7] was useful, but it dealt with the equations using a wind-axes system. Two other books written by
Ashley [Ref.2] and Perkins and Hage [Ref.16] were excellent complements to Etkin and proved to be valuable reference sources.

Once the topic of flight dynamics was mastered, the next step was to find literature that dealt with the ornithopter. The best resource for this was Dr. James DeLaurier of the University of Toronto and Jeremy Harris of Battelle Memorial Institute. A joint paper by DeLaurier and Harris [Ref.5] proved extremely valuable because it showed the feasibility of motorized flapping-wing flight. The work concentrated on a model ornithopter, named "Mr. Bill", showing various aspects such as wing design, drive mechanism, stability and control, and flight tests. The importance of this work is that it showed the analytical and structural design considerations are applicable in constructing a full-scale ornithopter capable of carrying a human being. Two papers written by DeLaurier [Ref.3&4] showed the complexities involved in the aerodynamics and structural dynamics for an ornithopter.

Other ornithopter designs worth mentioning are by Fowler [Ref.8] and Skarsgard [Ref.17] because they also dealt with the possibility of piloted flight. Fowler dealt with the more traditional approach of a human-powered ornithopter whereas Skarsgard examined a motorized ornithopter. Both show the complexities in the aerodynamic, structural, and mechanical design, looking at such aspects as wing design, powerplant, stability and control, landing gears, and weight considerations.
With a good grasp of flight dynamics and a sound knowledge of ornithopters, the next step was to search for literature which combined the two, i.e. the flight dynamics of an ornithopter. Although, no such article was found, projects written by McGuire [Ref.13] and Grant [Ref.9] were the closest and best resource. Both projects developed the equations of motion for the Canada Goose. Grant used the traditional formulation while McGuire used a multibody approach normally considered for bodies which have parts moving relative to one another. Both projects examined only the longitudinal stability, looking at the perturbed and unperturbed motions. The model consisted of three separate bodies: the fuselage and the two flapping wing panels. (See Figure 3)

![Diagram of a 2-panel model](image.png)

**Figure 3: 2-panel model**
Since the vehicle was symmetric about the $xz$ plane, only half of the goose was considered. Grant used the traditional body-fixed axis system at the body center, and an axis system at the wing which was aligned with the body axis even though the wing was moving relative to the body. In each case linear aerodynamics was assumed, twisting of the wing was considered, and sinusoidal flapping was implemented. This two-panel formulation was an excellent prelude to the three-panel case shown in Figure 4.

![Diagram of 3-panel model](image)

Figure 4: 3-panel model

On a final note, a unique and interesting concept was presented by Pantham [Ref.15]. In his thesis on the dynamics of variable swept-wing aircraft, Pantham develops the non-linear equations of motion by using a body-fixed axis system at an arbitrary point rather than at the body's center of gravity. This
formulation is powerful because it can be used even if the center of gravity changes, which is often the case in most aircraft. However, due to the complexities of this model, it was decided to remain with the conventional center-of-gravity based non-linear dynamic equations of motion. Another point of interest was the fact that this was the only other study, to the author's knowledge, where the equations were developed for an aircraft where the wings moved(sweep) relative to the body. This proved interesting because the center panel and the outer flapping panels of the ornithopter also move relative to the body.

4. DEVELOPMENT OF THE EQUATIONS OF MOTION

After completing the intensive literature search on aircraft dynamics and flapping-wing flight, the complete equations of motion were developed. The equations were first developed for the 2-panel design similar to Grant and then for the 3-panel design. Grant's 2-panel model provided a reference base from which further work could be carried on. Since Grant only developed the longitudinal case, the first step was to rework those equations to include the complete set of equations of motion. As a check, the lateral variables $\Phi, P, V, R$ were set to zero and the resulting equations were exactly the same as Grant's longitudinal case. After the fundamental equations were developed, the longitudinal and lateral aerodynamic terms were dealt with. Next, a computer
program was written to solve the equations and, finally, all the required data for "Mr. Bill" and the full-scale ornithopter (FSO) was input into the program to determine the stability. For a detailed development of the equations of motion, see Appendices B, C, D, E.

4.1 2-Panel Model

The 2-panel design consists of the body, which includes the tail+fin and the two wing panels. The wings are coupled to the body with the reaction forces and moments at the joints (Figure 5).

Figure 5: Schematic 2-panel model with reaction forces and moments
The equations of motion were developed for the body and each of the wing halves separately, and then matched through their kinematics. This resulted in a set of 18 linear equations with 18 unknowns, which were the state variables \((U,V,W,P,Q,R)\) and the reaction forces and moments at the interface of the body and wings. A body-fixed axis system was implemented for the fuselage while an axis system which is continually aligned with the body's axis system was used for the wing (Figure 6). This unique axis system for the wing was chosen because it proves to be convenient in the kinematic analysis.

![Figure 6: Axis system of wing and body](image)

4.1.1 Dynamic Model

Dynamically, the wing and the body were both assumed to be rigid, with the wing chosen to be a thin flat plate. The flapping was considered to be sinusoidal and the effects of the twisting of the wing were ignored in the dynamic analysis (See Appendix F). However, in the aerodynamic analysis, twisting had to be considered and the wing was given a sinusoidally-varying linear twist.
Twisting occurred at the same frequency as the flapping but was 90 degrees out of phase to achieve optimum thrust.

4.1.2 Body Equations Of Motion

The body includes everything but the wing panels. As mentioned before, a body-fixed axis is used at the center of gravity. The distance $D_b$ is the distance from the body center of gravity to the root of the wing. The following are the body equations of motion:

\[
\begin{align*}
X_{\text{seno}} &= m_b g \sin \Theta_b + R_{z\text{b1}} + R_{x\text{b2}} = m_b (\dot{U}_b + Q_b W_b - R_b V_b) \\
Y_{\text{seno}} &= m_b g \cos \Theta_b \sin \Phi_b + R_{y\text{b1}} + R_{y\text{b2}} = m_b (\dot{V}_b + R_b U_b - P_b W_b) \\
Z_{\text{seno}} &= m_b g \cos \Theta_b \cos \Phi_b + R_{z\text{b1}} + R_{z\text{b2}} = m_b (\dot{W}_b + P_b V_b - Q_b U_b) \\
L_{\text{seno}} &= d_{z\text{b1}} R_{y\text{b1}} + d_{y\text{b1}} R_{z\text{b1}} - d_{z\text{b2}} R_{y\text{b2}} + d_{y\text{b2}} R_{z\text{b2}} + M_{L\text{b1}} + M_{L\text{b2}} \\
&= I_{x\text{zb}} \ddot{P}_b - I_{x\text{zb}} \dot{R}_b + (I_{x\text{zsb}} - I_{y\text{zsb}}) Q_b R_b - I_{x\text{zb}} P_b Q_b \\
M_{\text{seno}} &= d_{z\text{b1}} R_{x\text{b1}} - d_{x\text{b1}} R_{z\text{b1}} + d_{z\text{b2}} R_{x\text{b2}} - d_{x\text{b2}} R_{z\text{b2}} + M_{N\text{b1}} + M_{N\text{b2}} \\
&= I_{y\text{yb}} \ddot{Q}_b + (I_{x\text{ybz}} - I_{z\text{ybz}}) P_b R_b + I_{x\text{zb}} (P_b^2 - R_b^2) \\
N_{\text{seno}} &= d_{y\text{b1}} R_{x\text{b1}} + d_{x\text{b1}} R_{y\text{b1}} - d_{y\text{b2}} R_{x\text{b2}} + d_{x\text{b2}} R_{y\text{b2}} + M_{N\text{b1}} + M_{N\text{b2}} \\
&= I_{z\text{zb}} \ddot{R}_b - I_{x\text{zb}} \dot{P}_b + (I_{x\text{ybz}} - I_{z\text{ybz}}) P_b Q_b + I_{x\text{zb}} Q_b R_b
\end{align*}
\]

4.1.3 Wing Equations Of Motion

The axis system which was attached to the wing center of gravity is a pseudo-stability-axis frame. This means that this axis system is aligned with the
body-fixed system of the body. The equations of motion for the port (left) wing are:

\[
\begin{align*}
X_{\text{aero-}w} &= -m_{w_1} g \sin \Theta_{w_1} + R_{w_1} = m_{w_1} (\ddot{U}_{w_1} + Q_{w_1} W_{w_1} - R_{w_1} V_{w_1}) \\
Y_{\text{aero-}w} &= m_{w_1} g \cos \Theta_{w_1} \sin \phi_{w_1} + R_{y_{w_1}} = m_{w_1} (\ddot{V}_{w_1} + R_{w_1} U_{w_1} - P_{\text{wing}_1} W_{w_1}) \\
Z_{\text{aero-}w} &= m_{w_1} g \cos \Theta_{w_1} \cos \phi_{w_1} + R_{z_{w_1}} = m_{w_1} (\ddot{W}_{w_1} + P_{\text{wing}_1} V_{w_1} - Q_{w_1} U_{w_1}) \\
L_{\text{aero-}w} &= -D_{w_1}{R}_{y_{w_1}} + D_{y_{w_1}} R_{z_{w_1}} + M_{L_{w_1}} = I_{x_{w_1}} \dot{P}_{\text{wing}_1} + (I_{z_{w_1}} - I_{y_{w_1}}) Q_{w_1} R_{w_1} \\
&\quad + I_{y_{w_1}} (R_{w_1}^2 - Q_{w_1}^2) \\
M_{\text{aero-}w} &= +D_{w_1} R_{x_{w_1}} - D_{x_{w_1}} R_{w_1} + M_{M_{w_1}} = I_{y_{w_1}} \dot{Q}_{w_1} + \dot{\dot{I}}_{y_{w_1}} Q_{w_1} - \dot{I}_{y_{w_1}} \dot{R}_{w_1} - \dot{I}_{y_{w_1}} R_{w_1} \\
&\quad + (I_{x_{w_1}} - I_{z_{w_1}}) P_{\text{wing}_1} R_{w_1} + I_{y_{w_1}} P_{\text{wing}_1} Q_{w_1} \\
N_{\text{aero-}w} &= -D_{y_{w_1}} R_{x_{w_1}} + D_{x_{w_1}} R_{y_{w_1}} + M_{N_{w_1}} = I_{z_{w_1}} \dot{R}_{w_1} + \dot{\dot{I}}_{z_{w_1}} R_{w_1} - \dot{I}_{y_{w_1}} \dot{Q}_{w_1} - \dot{I}_{y_{w_1}} Q_{w_1} \\
&\quad - I_{y_{w_1}} P_{\text{wing}_1} R_{w_1} + (I_{y_{w_1}} - I_{x_{w_1}}) P_{\text{wing}_1} Q_{w_1}
\end{align*}
\]

The equations for the starboard (right) wing are:

\[
\begin{align*}
X_{\text{aero-}w_2} &= -m_{w_2} g \sin \Theta_{w_2} + R_{w_2} = m_{w_2} (\ddot{U}_{w_2} + Q_{w_2} W_{w_2} - R_{w_2} V_{w_2}) \\
Y_{\text{aero-}w_2} &= m_{w_2} g \cos \Theta_{w_2} \sin \phi_{w_2} + R_{y_{w_2}} = m_{w_2} (\ddot{V}_{w_2} + R_{w_2} U_{w_2} - P_{\text{wing}_2} W_{w_2}) \\
Z_{\text{aero-}w_2} &= m_{w_2} g \cos \Theta_{w_2} \cos \phi_{w_2} + R_{z_{w_2}} = m_{w_2} (\ddot{W}_{w_2} + P_{\text{wing}_2} V_{w_2} - Q_{w_2} U_{w_2}) \\
L_{\text{aero-}w_2} &= -D_{w_2}{R}_{y_{w_2}} + D_{y_{w_2}} R_{z_{w_2}} + M_{L_{w_2}} = I_{x_{w_2}} \dot{P}_{\text{wing}_2} + (I_{z_{w_2}} - I_{y_{w_2}}) Q_{w_2} R_{w_2} \\
&\quad + I_{y_{w_2}} (R_{w_2}^2 - Q_{w_2}^2) \\
M_{\text{aero-}w_2} &= +D_{w_2} R_{x_{w_2}} - D_{x_{w_2}} R_{w_2} + M_{M_{w_2}} = I_{y_{w_2}} \dot{Q}_{w_2} + \dot{\dot{I}}_{y_{w_2}} Q_{w_2} - \dot{I}_{y_{w_2}} \dot{R}_{w_2} - \dot{I}_{y_{w_2}} R_{w_2} \\
&\quad + (I_{x_{w_2}} - I_{z_{w_2}}) P_{\text{wing}_2} R_{w_2} + I_{y_{w_2}} P_{\text{wing}_2} Q_{w_2} \\
N_{\text{aero-}w_2} &= -D_{y_{w_2}} R_{x_{w_2}} + D_{x_{w_2}} R_{y_{w_2}} + M_{N_{w_2}} = I_{z_{w_2}} \dot{R}_{w_2} + \dot{\dot{I}}_{z_{w_2}} R_{w_2} - \dot{I}_{y_{w_2}} \dot{Q}_{w_2} - \dot{I}_{y_{w_2}} Q_{w_2} \\
&\quad - I_{y_{w_2}} P_{\text{wing}_2} R_{w_2} + (I_{y_{w_2}} - I_{x_{w_2}}) P_{\text{wing}_2} Q_{w_2}
\end{align*}
\]
4.1.4 Kinematic Analysis

After obtaining the equations of motion for the body and wing, a relationship was required which would link the two different sets together. This could only be accomplished by relating the kinematics of the body and wing, i.e., the velocities, accelerations, angular velocities, and angular accelerations had to be found (See Appendix C for complete development). If the axes systems shown in Figure 7 are aligned at all times, then it can be shown that for the port wing, the velocities and accelerations are:

![Axes systems](image)

Figure 7: Axes systems
port wing velocities:

\[
\begin{align*}
U_{w1} &= U_b + Q_b(d_{zbl} - D_{zwl}) + R_b(D_{yw1} - d_{ybl}) \\
V_{w1} &= V_b + P_b(D_{zw1} - d_{zbl}) + R_b(d_{zbl} - D_{zw1}) + P_{w1}D_{zw1} \\
W_{w1} &= W_b + P_b(d_{ybl} - D_{yw1}) + Q_b(D_{zw1} - d_{zbl}) - P_{w1}D_{yw1}
\end{align*}
\]

port wing accelerations:

\[
\begin{align*}
\dot{U}_{w1} &= \dot{U}_b + \dot{Q}_b(d_{zbl} - D_{zwl}) + \ddot{R}_b(D_{yw1} - d_{ybl}) + Q_b^2(D_{zw1} - d_{zbl}) \\
&\quad + R_b^2(D_{zw1} - d_{zbl}) + P_b Q_b(d_{ybl} - D_{yw1}) + P_b R_b(d_{zbl} - D_{zw1}) \\
&\quad + Q_b W_b - R_b V_b - 2P_{w1}(Q_b D_{yw1} + R_b D_{zw1}) \\
\dot{V}_{w1} &= \dot{V}_b + \dot{P}_b(D_{zw1} - d_{zbl}) + \ddot{R}_b(d_{zbl} - D_{zw1}) + P_b^2(D_{yw1} - d_{ybl}) \\
&\quad + R_b^2(D_{yw1} - d_{ybl}) + P_b Q_b(d_{ybl} - D_{zw1}) + Q_b R_b(d_{zbl} - D_{zw1}) \\
&\quad - P_b W_b + R_b U_b + 2P_b P_{w1} D_{yw1} + \dot{P}_{w1} D_{zw1} + P_{w1}^2 D_{yw1} \\
\dot{W}_{w1} &= \dot{W}_b + \dot{P}_b(d_{ybl} - D_{yw1}) + \ddot{Q}_b(D_{zw1} - d_{zbl}) + P_b^2(D_{zw1} - d_{zbl}) \\
&\quad + Q_b^2(D_{zw1} - d_{zbl}) + P_b R_b(d_{zbl} - D_{zw1}) + Q_b R_b(d_{ybl} - D_{yw1}) \\
&\quad + P_b V_b - Q_b U_b + 2P_b P_{w1} D_{zw1} - \dot{P}_{w1} D_{yw1} + P_{w1}^2 D_{zw1}
\end{align*}
\]

The starboard wing velocities and accelerations are:

starboard wing velocities:

\[
\begin{align*}
U_{w2} &= U_b + Q_b(d_{zbl2} - D_{zw2}) + R_b(D_{yw2} - d_{ybl2}) \\
V_{w2} &= V_b + P_b(D_{zw2} - d_{zbl2}) + R_b(d_{zbl2} - D_{zw2}) + P_{w2}D_{zw2} \\
W_{w2} &= W_b + P_b(d_{ybl2} - D_{yw2}) + Q_b(D_{zw2} - d_{zbl2}) - P_{w2}D_{yw2}
\end{align*}
\]
starboard wing accelerations:

\[
\begin{align*}
\ddot{U}_{w2} &= \dot{U}_b + \dot{Q}_b (d_{zb2} - D_{zw2}) + \dot{R}_b (D_{yw2} - d_{yb2}) + Q_b^2 (D_{zw2} - d_{zb2}) \\
&\quad + R_b^2 (D_{zw2} - d_{zb2}) + P_b Q_b (d_{yb2} - D_{yw2}) + P_b R_b (d_{zb2} - D_{zw2}) \\
&\quad + Q_b W_b - R_b V_b - 2P_{w2} (Q_b D_{yw2} + R_b D_{zw2}) \\
\dot{V}_{w2} &= \dot{V}_b + \dot{P}_b (D_{zw2} - d_{zb2}) + \dot{R}_b (d_{xb2} - D_{zw2}) + P_b^2 (D_{yw2} - d_{yb2}) \\
&\quad + R_b^2 (D_{yw2} - d_{yb2}) + P_b Q_b (d_{zb2} - D_{zw2}) + Q_b R_b (d_{yb2} - D_{yw2}) \\
&\quad - P_b W_b + R_b U_b + 2P_b P_{w2} D_{yw2} + \dot{P}_{w2} D_{zw2} + P_{w2}^2 D_{yw2} \\
\dot{W}_{w2} &= \dot{W}_b + \dot{P}_b (d_{yb2} - D_{yw2}) + \dot{Q}_b (D_{zw2} - d_{xb2}) + P_b^2 (D_{zw2} - d_{xb2}) \\
&\quad + Q_b^2 (D_{zw2} - d_{xb2}) + P_b R_b (d_{xb2} - D_{zw2}) + Q_b R_b (d_{yb2} - D_{yw2}) \\
&\quad + P_b V_b - Q_b U_b + 2P_b P_{w2} D_{zw2} - \dot{P}_{w1} D_{yw2} + P_{w2}^2 D_{zw2}
\end{align*}
\]

4.1.5 Complete Equations Of Motion (2-Panel)

Substituting the above kinematic relationships and noting that the reaction forces and moments on the wing are equal and opposite to the body, the equations of motion now become:
\[
X_{\text{aero}} = m_b g \sin \Theta - m_b (Q_b W_b - R_b V_b) = m_b \dot{U}_b - R_{xb1} - R_{xb2}
\]
\[
Y_{\text{aero}} + m_b g \cos \Theta \sin \Phi - m_b (R_b U_b - P_b W_b) = m_b \dot{V}_b - R_{ybl} - R_{ybl}
\]
\[
Z_{\text{aero}} + m_b g \cos \Theta \cos \Phi - m_b (P_b V_b - Q_b U_b) = m_b \dot{W}_b - R_{zbl} - R_{zbl}
\]
\[
L_{\text{aero}} = (I_{zzb} - I_{yyb}) Q_b R_b + I_{xzb} P_b Q_b
\]
\[= I_{xzb} \dot{P}_b - I_{zzb} \dot{R}_b + d_{zbl} R_{xb1} + d_{ybl} R_{xb1} + d_{zbl} R_{xyb2} - d_{ybl} R_{xyb2} - M_{Lb1} - M_{Lb2}
\]
\[
M_{\text{aero}} = (I_{xzb} - I_{zzb}) P_b R_b - I_{xzb} (P_b^2 - R_b^2)
\]
\[= I_{ybl} \dot{Q}_b - d_{zbl} R_{zbl} + d_{ybl} R_{zbl} - d_{zbl} R_{zbl} + d_{ybl} R_{zbl} - M_{Mbl} - M_{Mb2}
\]
\[
N_{\text{aero}} = (I_{yyb} - I_{xzb}) P_b Q_b - I_{xzb} Q_b R_b
\]
\[= -I_{xzb} \dot{P}_b + I_{zzb} \dot{R}_b + d_{ybl} R_{xb1} - d_{ybl} R_{xb1} + d_{ybl} R_{zbl} - d_{ybl} R_{zbl} - M_{Nb1} - M_{Nb2}
\]
\[
X_{\text{aero}} = -m_w g \sin \Theta - m_w [2Q_b^2 (D_{xwl} - d_{xbl}) + 2R_b^2 (D_{xwl} - d_{xbl})]
+ 2P_b Q_b (d_{ybl} - D_{ywl}) + 2P_b R_b (d_{zbl} - D_{zwl}) + 2Q_b W_b - 2R_b V_b
- 3P_w (Q_b D_{ywl} + R_b D_{zwl})
\]
\[= m_w \dot{U}_b + m_w (d_{ybl} - D_{xwl}) \dot{Q}_b + m_w (D_{ywl} - d_{ybl}) \dot{R}_b + R_{xbl}
\]
\[
Y_{\text{aero}} = m_w g \cos \Theta \sin \Phi - m_w [2P_b^2 (D_{ywl} - d_{ybl}) + 2R_b^2 (D_{ywl} - d_{ybl})]
+ 2P_b Q_b (d_{xbl} - D_{xwl}) + 2Q_b R_b (d_{xbl} - D_{xwl}) - 2P_b W_b + 2R_b U_b
- P_w (P_b - 4D_{ywl}) + \dot{P}_w D_{zwl} + 2P_w (D_{ywl} - P_w W_b - P_w Q_b (D_{xwl} - d_{xbl}))
\]
\[= m_w \dot{V}_b + m_w (D_{zwl} - d_{zbl}) \dot{P}_b + m_w (d_{ybl} - D_{ywl}) \dot{R}_b + R_{ybl}
\]
\[
Z_{\text{aero}} = m_w g \cos \Theta \cos \Phi - m_w [2P_b^2 (D_{zwl} - d_{zbl}) + 2Q_b^2 (D_{zwl} - d_{zbl})]
+ 2P_b R_b (d_{ybl} - D_{xwl}) + 2Q_b R_b (d_{ybl} - D_{xwl}) + 2P_b V_b - 2Q_b U_b
+ P_w (P_b - 4D_{zwl} - d_{zbl}) + \dot{P}_w D_{ywl} + 2P_w (D_{zwl} + P_w V_b + P_w R_b (d_{ybl} - D_{ywl}))
\]
\[= m_w \dot{W}_b + m_w (d_{ybl} - D_{ywl}) \dot{P}_b + m_w (D_{zwl} - d_{zbl}) \dot{Q}_b + R_{zbl}
\]
\[
L_{\text{aero}} = -I_{xwl} \dot{P}_b - (I_{zzw} - I_{yyw}) Q_b R_b - I_{ywl} (R_b^2 - Q_b^2)
\]
\[= I_{xwl} \dot{P}_b - D_{zwl} R_{yb1} + D_{ywl} R_{zbl} + M_{Lb1}
\]
\[
M_{\text{aero}} = I_{ywl} \dot{Q}_b - I_{ywl} \dot{R}_b + D_{xwl} R_{xb1} - D_{xwl} R_{xb1} + M_{Mb1}
\]
\[
N_{\text{aero}} = I_{zzw} \dot{R}_b + I_{yzw} Q_b + I_{zzw} (P_b + P_w) R_b - (I_{yzw} - I_{zzw}) (P_b + P_w) Q_b
\]
\[= I_{zzw} \dot{R}_b - I_{yzw} \dot{Q}_b - D_{ywl} R_{xbl} + D_{xwl} R_{ybl} + M_{Nbl}
\]
These are a set of 18 simultaneous differential equations with 18 unknowns (U, V, W, P, Q, R, R_s, R_y, R_z, M_L, M_M, M_N, R_s, R_y, R_z, M_L, M_M, M_N). Given the initial conditions, these equations were solved numerically using a Gauss-Jordan elimination routine to obtain the derivatives of
the state variables. These were integrated using a second-order Runge-Kutta routine to obtain the desired variables.

4.2 3-Panel Model

The 3-panel design consists of the body (which includes the tail and fin), the center panel, and the two outer wing panels. The outer wings are coupled to the center panel with the reaction forces and moments at the joints (Figure 8).

![Figure 8: 3-panel model with reaction forces and moments](image)

Note that the x moment ($M_{LH}$) is non-existent at the interface of the outer and center panel because there is a hinge connection, which means that the outer panel is free to rotate about the x-axis. The center panel only plunges up and down while the outer panels are pivoted to operate in a rocking "seesaw" manner. There are reaction forces and moments present at the pivot point as well. However, there no moments in the x and y directions (i.e. no $J_{LB}, J_{MB}$).
The x direction moment, $J_{lb}$, is non-existent because the outer wing panel is free to turn about the $x$-axis and thus no moment is resisting its motion. The same can approximately be said for the $y$ direction moment, $J_{mb}$, because the chordwise location of the pivot point is such that it provides no pitching constraint. This is not completely true because there is a moment, $M_{hw}$, at the hinge connection between the center panel and outer panel, which is resisting the motion. However, an engineering decision was made to ignore $J_{mb}$ to reduce the number of unknowns.

The equations of motion were developed for the body, center panel, and both wing halves separately and then related through their kinematics. This resulted in a set of 24 linear equations with the 24 unknowns being the state variables $(U, V, W, P, Q, R)$ and the reaction forces and moments at the hinge connecting the center panel and outer panels as well as at the pivot point. As was the case with the 2-panel model, a body-fixed axis system was used for the fuselage. The center panel and outer panel both had an axis system which is continually aligned with the body's axis system.
4.2.1 Body Equations Of Motion

The body equations of motion are:

\[
\begin{align*}
X_{\text{aero}} &= -m_b g \sin \Theta_b + F_{xb1} + F_{xb2} = m_b (\ddot{U}_b + Q_b W_b - R_b V_b) \\
Y_{\text{aero}} &= m_b g \cos \Theta_b \sin \Phi_b + F_{yb1} + F_{yb2} = m_b (\dot{V}_b + R_b U_b - P_b W_b) \\
Z_{\text{aero}} &= m_b g \cos \Theta_b \cos \Phi_b + F_{zb1} + F_{zb2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b) \\
L_{\text{aero}} &= -H_{zb1} F_{yb1} + H_{yb1} F_{zb1} - H_{zb2} F_{yb2} + H_{yb2} F_{zb2} \\
&= I_{xb} \ddot{P}_b - I_{xzb} \dot{R}_b + (I_{zxb} - I_{yxb}) Q_b R_b - I_{yxb} P_b Q_b \\
M_{\text{aero}} &= H_{xb1} F_{xb1} + H_{xzb} F_{zxb} + H_{zb2} F_{xb2} - H_{xzb} F_{zxb} \\
&= I_{yyb} \dot{Q}_b + (I_{xzb} - I_{yxb}) P_b R_b + I_{yxb} (P_b^2 - R_b^2) \\
N_{\text{aero}} &= -H_{yb1} F_{yb1} + H_{yxb} F_{yxb} - H_{yxb} F_{yb2} + H_{yb2} F_{yxb2} + J_{nb1} + J_{nb2} \\
&= I_{xzb} \ddot{R}_b - I_{xzb} \dot{P}_b + (I_{yyb} - I_{xzb}) P_b Q_b + I_{yxb} Q_b R_b
\end{align*}
\]

4.2.2 Center Panel Equations Of Motion

![Figure 9: Center panel](image.png)
The center panel equations of motion are:

\[
\begin{align*}
X_{\text{new}} & = -m_{\text{c}} g \sin \Theta_{\text{cp}} + R_{\text{zcp}} + R_{\text{zcp2}} = m_{\text{cp}} (\dot{U}_{\text{cp}} + Q_{\text{cp}} W_{\text{cp}} - R_{\text{cp}} V_{\text{cp}}) \\
Y_{\text{new}} & = m_{\text{c}} g \cos \Theta_{\text{cp}} \sin \Phi_{\text{cp}} + R_{\text{ycp}} + R_{\text{ycp2}} = m_{\text{cp}} (\dot{V}_{\text{cp}} + R_{\text{cp}} U_{\text{cp}} - P_{\text{cp}} W_{\text{cp}}) \\
Z_{\text{new}} & = m_{\text{c}} g \cos \Theta_{\text{cp}} \cos \Phi_{\text{cp}} + R_{\text{zcp}} + R_{\text{zcp2}} = m_{\text{cp}} (\dot{W}_{\text{cp}} + P_{\text{cp}} V_{\text{cp}} - Q_{\text{cp}} U_{\text{cp}}) \\
L_{\text{new}} & = -(b_{\text{cp}}/2) R_{\text{zcp}} + (b_{\text{cp}}/2) R_{\text{zcp2}} = I_{\text{xxcp}} \dot{\Phi}_{\text{cp}} + (I_{\text{zcp}} - I_{\text{yycp}}) Q_{\text{cp}} R_{\text{cp}} \\
M_{\text{new}} & = M_{\text{ncp}} + M_{\text{ncp2}} = I_{\text{yyycp}} \dot{\Theta}_{\text{cp}} + (I_{\text{xxcp}} - I_{\text{zcp}}) P_{\text{cp}} R_{\text{cp}} \\
N_{\text{new}} & = (b_{\text{cp}}/2) R_{\text{zcp}} -(b_{\text{cp}}/2) R_{\text{zcp2}} + M_{\text{ncp}} + M_{\text{ncp2}} = I_{\text{zcp}} \dot{\Phi}_{\text{cp}} + (I_{\text{yycp}} - I_{\text{xxcp}}) P_{\text{cp}} Q_{\text{cp}}
\end{align*}
\]

4.2.3 Port Wing Equations Of Motion

![Diagram of Port Wing](image)

Figure 10: Port wing
4.2.4 Starboard Wing Equations Of Motion

\[X_{\text{aero}_{sl}} - m_w g \sin \Theta_{wl} + R_{xw1} + F_{zw1} = m_w (\dot{U}_{wl} + Q_{wl} W_{wl} - R_{wl} V_{wl})\]

\[Y_{\text{aero}_{sl}} + m_w g \cos \Theta_{wl} \sin \Phi_{wl} + R_{yw1} + F_{yw1} = m_w (\dot{V}_{wl} + R_{wl} U_{wl} - P_{\text{wing}1} W_{wl})\]

\[Z_{\text{aero}_{sl}} + m_w g \cos \Theta_{wl} \cos \Phi_{wl} + R_{zw1} + F_{zw1} = m_w (\dot{W}_{wl} + P_{\text{wing}1} V_{wl} - Q_{wl} U_{wl})\]

\[L_{\text{aero}_{sl}} = D_{xw1} R_{yw1} + D_{yw1} R_{zw1} - D_{xpl} F_{yw1} + D_{ypl} F_{zw1}\]

\[= I_{xw1} \dot{\theta}_{\text{wing}1} + (I_{zzw1} - I_{ywl}) Q_{wl} R_{wl} + I_{xw1} (R_{wl}^2 - Q_{wl}^2)\]

\[M_{\text{aero}_{sl}} = D_{xw1} R_{xw1} - D_{xw1} R_{zw1} + D_{zpl} F_{xw1} - D_{zpl} F_{zw1} + M_{Mwl}\]

\[= I_{yyw1} \dot{\varphi}_{wl} + I_{yyw1} Q_{wl} - I_{ywl} \dot{\varphi}_{wl} - I_{ywl} R_{wl} + (I_{zzw1} - I_{xw1}) P_{\text{wing}1} R_{wl} + I_{ywl} P_{\text{wing}1} Q_{wl}\]

\[N_{\text{aero}_{sl}} = D_{yw1} R_{yw1} + D_{yw1} R_{zw1} - D_{ypl} F_{yw1} + D_{ypl} F_{zw1} + M_{Nwl} + J_{Nwl}\]

\[= I_{zzw1} \dot{\phi}_{wl} + I_{zzw1} R_{wl} - I_{ywl} \dot{Q}_{wl} - I_{ywl} Q_{wl} - I_{ywl} P_{\text{wing}1} R_{wl} + (I_{yyw1} - I_{xw1}) P_{\text{wing}1} Q_{wl}\]

---

**Figure 11:** Starboard wing
4.2.5 Kinematic Analysis

The kinematic relationship between the body and the wing is similar to the 2-panel model. However, an extra relationship is necessary between the body and the center panel.

Using the axis system shown in Figure 12, the following equations can be developed (See Appendix D for a complete development):

\[
X_{nwo} - m_w g \sin \Theta_{w2} + R_{xw2} + F_{xw2} = m_w (\dot{U}_{w1} + \dot{Q}_{w1} W_{w1} - R_{w1} V_{w1})
\]
\[
Y_{nwo} + m_w g \cos \Theta_{w2} \sin \Phi_{w2} + R_{yw2} + F_{yw2} = m_w (\dot{V}_{w2} + R_{w2} U_{w2} - P_{wing2} W_{w2})
\]
\[
Z_{nwo} + m_w g \cos \Theta_{w2} \cos \Phi_{w2} + R_{zw2} + F_{zw2} = m_w (\dot{W}_{w2} + P_{wing2} V_{w2} - Q_{w2} U_{w2})
\]
\[
L_{nwo} - D_{xw2} R_{yw2} + D_{yw2} R_{zw2} - D_{zp2} F_{yw2} + D_{yp2} F_{zw2} = I_{xxw2} \dot{\gamma}_{wing2} + (I_{zzw2} - I_{yyw2}) Q_{w2} R_{w2} + I_{yzw2} (R_{w2}^2 - Q_{w2}^2)
\]
\[
M_{nwo} + D_{zw2} R_{yw2} - D_{xw2} R_{zw2} + D_{zp2} F_{zw2} - D_{zp2} F_{xw2} + M_{Mw2} = I_{yyw2} \dot{Q}_{w2} + I_{yyw2} Q_{w2} - I_{yzw2} \dot{R}_{w2} - I_{yzw2} R_{w2} + (I_{xz2} - I_{zzw2}) P_{wing2} R_{w2} + I_{yzw2} P_{wing2} Q_{w2}
\]
\[
N_{nwo} - D_{yw2} R_{yw2} + D_{xw2} R_{zw2} - D_{yp2} F_{xw2} + D_{yp2} F_{yw2} + M_{Nw2} + J_{Nw2} = I_{zzw2} \dot{R}_{w2} + I_{zzw2} R_{w2} - I_{yzw2} \dot{Q}_{w2} - I_{yzw2} Q_{w2} - I_{yzw2} P_{wing2} R_{w2} + (I_{yyw2} - I_{zzw2}) P_{wing2} Q_{w2}
\]
port wing velocities:

\[
U_{wl} = U_b + Q_b (H_{zbl} - D_{zpl}) + R_b (D_{yp1} - H_{ybl}) \\
V_{wl} = V_b + P_b (D_{zpl} - H_{zbl}) + R_b (H_{zbl} - D_{xpl}) + P_{wl} D_{zpl} \\
W_{wl} = W_b + P_b (H_{ybl} - D_{yp1}) + Q_b (D_{xpl} - H_{zbl}) - P_{wl} D_{yp1}
\]

port wing accelerations:

\[
\dot{U}_{wl} = \dot{U}_b + \dot{Q}_b (H_{zbl} - D_{zpl}) + \dot{R}_b (D_{yp1} - H_{ybl}) + Q_b^2 (D_{zpl} - H_{zbl}) \\
\quad + R_b^2 (D_{xpl} - H_{zbl}) + P_b Q_b (H_{ybl} - D_{yp1}) + P_b R_b (H_{zbl} - D_{zpl}) \\
\quad + Q_b W_b - R_b V_b - 2P_{wl} (Q_b D_{yp1} + R_b D_{zpl}) \\
\dot{V}_{wl} = \dot{V}_b + \dot{P}_b (D_{xpl} - H_{zbl}) + \dot{R}_b (H_{zbl} - D_{xpl}) + P_b^2 (D_{zp1} - H_{ybl}) \\
\quad + R_b^2 (D_{yp1} - H_{ybl}) + P_b Q_b (H_{zbl} - D_{xpl}) + Q_b R_b (H_{zbl} - D_{zpl}) \\
\quad - P_b W_b + R_b U_b + 2P_b P_{wl} D_{xpl} + \dot{P}_{wl} D_{zpl} + P_{wl}^2 D_{yp1} \\
\dot{W}_{wl} = \dot{W}_b + \dot{P}_b (H_{ybl} - D_{yp1}) + \dot{Q}_b (D_{xpl} - H_{xbl}) + P_b^2 (D_{zpl} - H_{zbl}) \\
\quad + Q_b^2 (D_{zpl} - H_{zbl}) + P_b R_b (H_{xbl} - D_{xpl}) + Q_b R_b (H_{ybl} - D_{yp1}) \\
\quad + P_b V_b - Q_b U_b + 2P_b P_{wl} D_{xpl} - \dot{P}_{wl} D_{yp1} + P_{wl}^2 D_{zpl}
\]

starboard wing velocities:

\[
U_{w2} = U_b + Q_b (H_{zbl} - D_{zpl}) + R_b (D_{yp2} - H_{ybl}) \\
V_{w2} = V_b + P_b (D_{xpl} - H_{zbl}) + R_b (H_{zbl} - D_{xpl}) + P_{w2} D_{xpl} \\
W_{w2} = W_b + P_b (H_{ybl} - D_{yp2}) + Q_b (D_{xpl} - H_{zbl}) - P_{w2} D_{yp2}
\]
starboard wing accelerations:

\[
\begin{align*}
\dot{U}_w &= \dot{U}_b + \dot{Q}_b (H_{xb} - D_{xp}) + \dot{R}_b (D_{yp} - H_{yb}) + Q^2_b (D_{xp} - H_{xb}) \\
&+ R^2_b (D_{xp} - H_{xb}) + P_b Q_b (H_{yb} - D_{yp}) + P_b R_b (H_{xb} - D_{xp}) \\
&+ Q_b W_b - R_b V_b - 2P_w (Q_b D_{yp} + R_b D_{xp}) \\
\dot{V}_w &= \dot{V}_b + \dot{P}_b (D_{xp} - H_{xb}) + \dot{R}_b (H_{xb} - D_{xp}) + P^2_b (D_{yp} - H_{yb}) \\
&+ R^2_b (D_{yp} - H_{yb}) + P_b Q_b (H_{xb} - D_{xp}) + Q_b R_b (H_{xb} - D_{xp}) \\
&- P_b W_b + R_b U_b + 2P_b P_w D_{yp} + \dot{P}_w D_{xp} + P^2_w D_{yp} \\
\dot{W}_w &= \dot{W}_b + \dot{P}_b (H_{yb} - D_{yp}) + \dot{Q}_b (D_{xp} - H_{xb}) + P^2_b (D_{xp} - H_{xb}) \\
&+ Q^2_b (D_{xp} - H_{xb}) + P_b R_b (H_{xb} - D_{xp}) + Q_b R_b (H_{yb} - D_{yp}) \\
&+ P_b V_b - Q_b U_b + 2P_b P_w D_{xp} - \dot{P}_w D_{yp} + P^2_w D_{xp}
\end{align*}
\]

Using the axis system shown in Figure 13, the velocities and accelerations of the center panel are:

![Figure 13: Axes for body and center panel](image)

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The x and y components, \( \Delta_{xb}, \Delta_{yb} \), of the distance from the center panel c.g. to the body c.g., along with the derivatives, \( \dot{\Delta}_{xb}, \dot{\Delta}_{yb} \), are equal to zero, but are added to the equations for the sake of completeness.

4.2.6 Complete Equations Of Motion (3-Panel)

Making all the appropriate substitutions yields the complete non-linear equations of motion for a 3-panel ornithopter:
\[ X_{\text{aero}} = -m_b g \sin \Theta - m_b (Q_b W_b - R_b V_b) = m_b \dot{U}_b + F_{xw1} + F_{xw2} \]

\[ Y_{\text{aero}} + m_b g \cos \Theta \sin \Phi - m_b (R_b U_b - P_b W_b) = m_b \dot{V}_b + F_{yw1} + F_{yw2} \]

\[ Z_{\text{aero}} + m_b g \cos \Theta \cos \Phi - m_b (P_b V_b - Q_b U_b) = m_b \dot{W}_b + F_{zwl} + F_{zwb} \]

\[ L_{\text{aero}} - (I_{zbz} - I_{yby}) Q_b R_b + I_{xb} P_b Q_b = I_{xb} \dot{P}_b - I_{xb} \dot{R}_b - H_{zbl} F_{yw1} + H_{ybl} F_{zwb} \]
\[ - H_{zbl} F_{yw2} + H_{ybl} F_{zw2} \]

\[ M_{\text{aero}} - (I_{xzb} - I_{xbz}) P_b R_b - I_{xzb} (P_b^2 - R_b^2) = I_{yby} \dot{Q}_b + H_{zbl} F_{xw1} - H_{xbl} F_{zw1} \]
\[ + H_{zbl} F_{xw2} - H_{xbl} F_{zw2} \]

\[ N_{\text{aero}} - (I_{yby} - I_{xbz}) P_b Q_b - I_{xzb} Q_b R_b = -I_{xbz} \dot{P}_b + I_{xbz} \dot{R}_b - H_{ybl} F_{xw1} + H_{xbl} F_{yw1} \]
\[ - H_{ybl} F_{xw2} + H_{xbl} F_{yw2} + J_{Nw1} + J_{Nw2} \]

\[ X_{\text{aero}} = -m_w g \sin \Theta - m_w [2Q_w^2 (D_{xpl} - H_{zbl}) + 2R_w^2 (D_{xpl} - H_{zbl}) \]
\[ + 2P_w Q_b (H_{ybl} - D_{ypl}) + 2P_w R_b (H_{zbl} - D_{zpl}) + 2Q_b W_b - 2R_b V_b - 3P_w (Q_b D_{ypl} + R_b D_{zpl})] \]
\[ = m_w \dot{U}_b + m_w (H_{zbl} - D_{zpl}) \dot{Q}_b + m_w (D_{xpl} - H_{ybl}) \dot{R}_b - R_{xw1} - F_{xw1} \]

\[ Y_{\text{aero}} + m_w g \cos \Theta \sin \Phi - m_w [2P_w^2 (D_{ypl} - H_{ybl}) + 2R_w^2 (D_{ypl} - H_{ybl}) \]
\[ + 2P_w Q_b (H_{zbl} - D_{zpl}) + 2Q_b R_b (H_{zbl} - D_{zpl}) - 2P_b W_b + 2R_b U_b - P_w P_b (H_{ybl} - 4D_{ypl}) \]
\[ + P_w D_{ypl} + 2P_w^2 D_{ypl} + P_w V_b - P_w Q_b (D_{ypl} - H_{zbl})] \]
\[ = m_w \dot{V}_b + m_w (D_{ypl} - H_{zbl}) \dot{P}_b + m_w (H_{ybl} - D_{ypl}) \dot{R}_b - R_{yw1} - F_{yw1} \]

\[ Z_{\text{aero}} + m_w g \cos \Theta \cos \Phi - m_w [2P_w^2 (D_{zpl} - H_{zbl}) + 2Q_w^2 (D_{zpl} - H_{zbl}) \]
\[ + 2P_w R_b (H_{zbl} - D_{zpl}) + 2Q_w R_b (H_{zbl} - D_{zpl}) + 2P_w V_b - 2Q_w U_b + P_w P_b (4D_{zpl} - H_{zbl}) \]
\[ - \dot{P}_w D_{ypl} + 2P_w^2 D_{zpl} + P_w V_b + P_w R_b (H_{zbl} - D_{zpl})] \]
\[ = m_w \dot{W}_b + m_w (H_{ybl} - D_{ypl}) \dot{P}_b + m_w (D_{zpl} - H_{zbl}) \dot{Q}_b - R_{zwl} - F_{zwl} \]

\[ L_{\text{aero}} = -I_{xwl} \dot{P}_b - (I_{xwl} - I_{ywl}) Q_b R_b - I_{ywl} (R_b^2 - Q_b^2) \]
\[ = I_{xwl} \dot{P}_b + D_{zwl} R_{yw1} - D_{yw1} R_{zwl} + D_{xpl} F_{yw1} - D_{ypl} F_{zwl} \]

\[ M_{\text{aero}} - \dot{I}_{ywl} Q_b + \dot{I}_{yw1} R_b - (I_{xwl} - I_{xwl})(P_b + P_{w1}) R_b - I_{yw1} (P_b + P_{w1}) Q_b \]
\[ = I_{yw1} \dot{Q}_b - I_{xwl} \dot{R}_b - D_{zwl} R_{xw1} + D_{xwl} R_{xw1} - D_{xwl} F_{zwl} + D_{ypl} F_{zwl} - M_{Mwl} \]

\[ N_{\text{aero}} - \dot{I}_{xwl} R_b + \dot{I}_{ywl} Q_b + I_{yw1} (P_b + P_{w1}) R_b - (I_{ywl} - I_{xwl})(P_b + P_{w1}) Q_b \]
\[ = -I_{ywl} \dot{Q}_b + I_{xwl} \dot{R}_b + D_{yw1} R_{xw1} - D_{xwl} R_{yw1} + D_{ypl} F_{zwl} - D_{ypl} F_{zwl} - M_{Nwl} - J_{Nwl} \]
\[ X_{\text{aero}_{w}} = -m_w g \sin \Theta - m_w \{2Q_b^2 (D_{sp2} - H_{sb2}) + 2R_b^2 (D_{zp2} - H_{sb2}) \} \\
+ 2P_b Q_b (H_{yb2} - D_{zp2}) + 2P_b R_b (H_{sb2} - D_{sp2}) + 2Q_b W_b - 2R_b V_b - 3P_{w2} (Q_b D_{zp} + R_b D_{zp2}) \} \\
= m_w \dot{U}_b + m_w (D_{zp2} - H_{yb2}) \dot{V}_b + m_w (D_{zp2} - H_{yb2}) \dot{W}_b - R_{zw2} - F_{zw2} \]

\[ Y_{\text{aero}_{w}} = m_w g \cos \Theta \sin \Phi - m_w \{2P_b^2 (D_{zp2} - H_{yb2}) + 2R_b^2 (D_{zp2} - H_{yb2}) \\
+ 2P_b Q_b (H_{sb2} - D_{sp2}) + 2Q_b R_b (H_{sb2} - D_{sp2}) - 2P_b W_b + 2R_b U_b - P_{w2} P_b (H_{yb2} - 4D_{zp2}) \\
+ \dot{P}_{w2} D_{zp2} + 2P_{w2} D_{zp2} - P_{w2} W_b - P_{w2} Q_b (D_{zp2} - H_{sb2}) \} \\
= m_w \dot{V}_b + m_w (D_{zp2} - H_{yb2}) \dot{P}_b + m_w (H_{sb2} - D_{sp2}) \dot{R}_b - R_{yw2} - F_{yw2} \]

\[ Z_{\text{aero}_{w}} = m_w g \cos \Theta \cos \Phi - m_w \{2P_b^2 (D_{zp2} - H_{sb2}) + 2Q_b^2 (D_{zp2} - H_{sb2}) \\
+ 2P_b R_b (H_{sb2} - D_{sp2}) + 2Q_b R_b (H_{sb2} - D_{zp2}) + 2P_b V_b - 2Q_b U_b + P_{w2} P_b (4D_{zp2} - H_{sb2}) \\
- \dot{P}_{w2} D_{yp2} + 2P_{w2} D_{zp2} + P_{w2} W_b + P_{w2} R_b (H_{sb2} - D_{sp2}) \} \\
= m_w \dot{W}_b + m_w (H_{yb2} - D_{zp2}) \dot{P}_b + m_w (D_{zp2} - H_{sb2}) \dot{Q}_b - R_{zw2} - F_{zw2} \]

\[ L_{\text{aero}_{w}} = -I_{yz2} \dot{R}_{yz2} - (I_{zx2} - I_{zyw2}) \dot{Q}_b - I_{yzw2} (R_b^2 - Q_b^2) \]

\[ = I_{zxw2} \dot{P}_b + D_{zw2} R_{zw2} - D_{yw2} R_{zw2} + D_{sp2} F_{zw2} - D_{zp2} F_{zw2} \]

\[ M_{\text{aero}_{w}} = I_{yy2} Q_b + I_{yyw2} R_b - (I_{zx2} - I_{zxw2}) (P_b + P_{w2}) R_b - I_{yzw2} (P_b + P_{w2}) Q_b \]

\[ = I_{yy2} \dot{Q}_b - I_{yzw2} \dot{R}_b - D_{zw2} R_{zw2} + D_{yx2} R_{zw2} - D_{sp2} F_{zw2} + D_{zp2} F_{zw2} - M_{Mw2} \]

\[ N_{\text{aero}_{w}} = I_{yzw2} R_b + I_{yzw2} Q_b + I_{yzw2} (P_b + P_{w2}) R_b - (I_{yy2} - I_{zx2}) (P_b + P_{w2}) Q_b \]

\[ = -I_{yzw} \dot{Q}_b + I_{yzw} \dot{R}_b + D_{zw2} R_{zw2} - D_{yw2} R_{zw2} + D_{sp2} F_{zw2} - D_{zp2} F_{zw2} - N_{Mw2} - J_{Nw2} \]

\[ X_{\text{aero}_{w}} = -m_c g \sin \Theta - m_c \{2Q_b^2 D_{zb} + 2Q_b Q_b D_{yb} + 2P_b R_b D_{zb} + 2Q_b W_b - 2R_b V_b + 3Q_b D_{zb} - 3R_b D_{yb} + \ddot{D}_{xb} \} \]

\[ = m_c \dot{U}_b + m_c D_{zb} \dot{V}_b - m_c D_{yb} \dot{W}_b + R_{xw1} + R_{xw2} \]

\[ Y_{\text{aero}_{w}} = m_c g \cos \Theta \sin \Phi - m_c \{2P_b^2 D_{yb} - 2P_b D_{yb} + 2P_b Q_b D_{xb} + 2Q_b R_b D_{zb} \}

\[ - 2P_b W_b + 2R_b U_b - 3P_b D_{zb} + 3R_b D_{xb} + \ddot{D}_{xb} \} \]

\[ = m_c \dot{V}_b - m_c D_{yb} \dot{P}_b + m_c D_{xb} \dot{R}_b + R_{yw1} + R_{yw2} \]

\[ Z_{\text{aero}_{w}} = m_c g \cos \Theta \cos \Phi - m_c \{2P_b^2 D_{zb} - 2Q_b^2 D_{zb} + 2P_b R_b D_{xb} + 2Q_b R_b D_{yb} + 2P_b V_b - 2Q_b U_b + 3P_b D_{yb} - 3Q_b D_{xb} + \ddot{D}_{zb} \} \]

\[ = m_c \dot{W}_b - m_c D_{yb} \dot{P}_b - m_c D_{xb} \dot{Q}_b + R_{xw1} + R_{xw2} \]

\[ L_{\text{aero}_{w}} = -(I_{ycz} - I_{yyw}) Q_b R_b = I_{zxw} \dot{P}_b - (b_c / 2) R_{zw2} + (b_c / 2) R_{zw2} \]

\[ M_{\text{aero}_{w}} = -(I_{ycz} - I_{zxw}) P_b R_b = I_{yyw} \dot{Q}_b + M_{Mw1} + M_{Mw2} \]

\[ N_{\text{aero}_{w}} = -(I_{yyw} - I_{zxw}) P_b Q_b = I_{zxw} \dot{R}_b + (b_c / 2) R_{zw1} - (b_c / 2) R_{zw2} + M_{Nw1} + M_{Nw2} \]
These are a set of 24 simultaneous differential equations with 24 unknowns \((U, V, W, P, Q, R, R_{zw1}, R_{zw2}, M_{zw1}, M_{zw2}, F_{zw1}, F_{zw2}, J_{Nw1}, R_{xm1}, R_{xm2}, R_{ym1}, R_{ym2}, M_{xm1}, M_{xm2}, F_{xm1}, F_{xm2}, F_{ym1}, F_{ym2}, J_{Nw2})\). Given the initial conditions, these equations were solved numerically using a Gauss-Jordan elimination routine to obtain the derivatives of the state variables. These variables were integrated using a second-order Runge-Kutta routine to obtain the desired variables.

5. LONGITUDINAL AERODYNAMIC ANALYSIS

Since the flapping is relatively slow, a quasi-steady aerodynamic model was used where unsteady wake effects are ignored. Also, linear aerodynamics was assumed in the analysis. Unlike conventional aircraft where the angles of attack used must be small to be considered in the linear range, the ornithopter can have larger angles of attack (approximately 15 degrees). This is because the flapping causes separation to occur at larger-than-usual angles of attack (Ref.9). This condition, known as dynamic-stall delay, allows higher instantaneous angles of attack to be reached without stalling.

To examine the aerodynamics of the wing, the segment method was used. This involves dividing the wing into a finite number of segments, analyzing the
forces and moments of each segment, and finally summing all the segments to obtain the total forces and moments on the wing.

The aerodynamic terms for the body and tail were analyzed separately and then summed together.

5.1 Flapping Angle

The flapping is considered to be sinusoidal.

Port wing:

The flapping angle and its derivatives for the port wing are:

\[ \gamma_1 = \gamma_0 \cos(\omega t) + \text{dihedral angle} \]
\[ P_{w1} = \dot{\gamma}_1 = -\gamma_0 \omega \cos(\omega t) \]
\[ \dot{P}_{w1} = \ddot{\gamma}_1 = -\gamma_0 \omega^2 \cos(\omega t) \]

Starboard wing:

The flapping angle and its derivatives for the starboard wing are:

\[ \gamma_2 = -\gamma_0 \cos(\omega t) - \text{dihedral angle} \]
\[ P_{w2} = \dot{\gamma}_2 = \gamma_0 \omega \cos(\omega t) \]
\[ \dot{P}_{w2} = \ddot{\gamma}_2 = \gamma_0 \omega^2 \cos(\omega t) \]
5.2 Twist Angle

The twist varies linearly along the wing span and is 90 degrees out of phase with the flapping to achieve optimum thrust.

\[ \alpha = \alpha_0 \cos(\omega t + \phi) \]
\[ \phi = 90^\circ \]

5.3 Body Contribution

5.3.1 Body Drag

The drag of the body consists of two separate effects: the drag at zero angle of attack, and drag due to a finite angle of attack. Because the body is fairly streamlined, it was assumed that the drag at zero angle of attack was due to skin friction only.

\[ C_r = \frac{0.455}{(\log_{10} Re)^{2.58}} \]
\[ C_D(\alpha = 0) = C_r \frac{S_{\text{wet}}}{S_{\text{ref}}} \]

The drag at a finite angle of attack can be obtained from USAF Stability and Control Methods Datcom (Ref. 18).

\[ C_D(\alpha) = \frac{2(k_2 - k_1)\alpha^2 S_0}{\text{Vol}^2} + \frac{2\alpha^3}{\text{Vol}^3} \int_{x_s}^{l_b} \eta r c_d \, dx \]

The drag can now be found by summing up the two drag coefficients and using the drag equation:

\[ D_b = \frac{1}{2} \rho (U_b^2 + W_b^2) S C_D \]
5.3.2 Body Lift

Due to symmetry, the lift of the body at zero angle of attack was assumed to be zero. The lift at non-zero angles of attack was obtained from USAF Datcom. The equation for lift-curve slope is given by:

\[ C_{L_a} = \frac{2(k_2 - k_1)S_0}{Vol^3} \text{ (per radian)} \]

The lift can then be found from:

\[ L_b = \frac{1}{2} \rho \left( U_b^2 + W_b^2 \right) SC_{L_a} \alpha_b \]

\[ = \frac{1}{2} \rho \left( U_b^2 + W_b^2 \right) \frac{2}{Vol_b^3} C_{L_a} \left( \frac{W_b}{U_b} \right) \]

5.3.3 Body Pitching Moment

Due to symmetry, the pitching moment of the body at zero angle of attack was assumed to be zero. The pitching moment at non-zero angles of attack was obtained from USAF Datcom. The equation for pitching-moment-curve slope is given by:

\[ C_{M_a} = \frac{2(k_2 - k_1)}{Vol} + \frac{2\alpha^3}{Vol^3} \int_0^{x_b} \frac{dS}{dx} (x_m - x) dx \]

The pitching moment is given by:

\[ M_b = \frac{1}{2} \rho \left( U_b^2 + W_b^2 \right) Vol_b C_{M_a} \alpha_b \]

\[ = \frac{1}{2} \rho \left( U_b^2 + W_b^2 \right) Vol_b C_{M_a} \left( \frac{W_b}{U_b} \right) \]
5.4 Tail Contribution

5.4.1 Total Angle of Attack of Tail

The total angle of attack of the tail is composed of three components:

1) angle of incidence of tail

2) induced angle of attack due to plunging ($W_b$) and pitch rate effects

3) induced downwash from the wing

1) Angle of incidence is given by an equilibrium angle of attack $\alpha_\text{e}$

2) Plunging:

Plunging causes an induced angle of attack which, after assuming small angles, is

$$\alpha_i = \frac{W_b}{U_b}$$

pitch rate:

The pitch rate effect consists of two parts -

i) rotation of the body about the c.g. causes an induced plunging velocity at the tail

$$\alpha_i = -Q_b \frac{x_i}{U_b}$$

ii) rotation about the aerodynamic center of the tail has an effect similar to adding chamber to the lifting surface

$$\alpha_i = Q_b \frac{c_i}{2U_b}$$

3) downwash $\epsilon$
Using an average lift coefficient the downwash was estimated from Etkin, (Ref.6, Figures B.6.1,B.6.2) which provide the ratio of downwash over lift coefficient.

The downwash was obtained by multiplying this ratio by the estimated lift coefficient. The time it takes for the downwash to reach the tail is estimated by:

$$t_d = \frac{\bar{x}_t}{U_b}$$

Thus the total angle of attack of the tail is:

$$\alpha_t = \alpha_0 + \frac{W_b}{U_b} - Q_b \frac{\bar{x}_t}{U_b} + Q_b \frac{\bar{\epsilon}_t}{2U_b} - \epsilon$$

5.4.2 Tail Lift

The lift at the tail can be found using:

$$L_t = \frac{1}{2} \rho[U_b^2 + (W_b - Q_b \bar{x}_t)^2]S_t C_{L_t} \alpha_t$$

5.4.3 Tail Drag

The drag of the tail can be estimated using the conventional lift-drag polar for a wing:

$$C_D = C_{D_t} + \frac{C_l^2}{\pi AR}$$

Thus the drag of the tail is:

$$D_t = \frac{1}{2} \rho[U_b^2 + (W_b - Q_b \bar{x}_t)^2]S_t C_D$$
5.5 Combined Body and Tail Contribution

Considering drag to be parallel to the local flow and lift to be perpendicular, the angle of attack is taken into account to transform the equations into the body-fixed axis (See Figure 14).

![Figure 14: Lift and drag transformation](image)

In the body-fixed axis system, the lift and drag can now be calculated as follows:

\[ X = L \sin \delta - D \cos \delta \]
\[ Z = -L \cos \delta - D \sin \delta \]

Applying a small angle approximation means that:

\[ \sin \delta \approx \delta \]
\[ \cos \delta \approx 1 \]

The equations simplify to:

\[ X = L \delta - D \]
\[ Z = -L - D \delta \]

Using this information and summing both the contributions from the body and tail for lift, drag, and pitching moment gives the following aerodynamic terms:
The twisting of the wing which was neglected in the dynamic analysis must be taken into consideration in the aerodynamic analysis. The twist varies linearly along the wing and is 90 degrees out of phase with the flapping angle. The thrust is assumed to be generated entirely by tilt of the lift vector(Figure 15).

The discrete-element method is used to divide up the wing into a finite number of segments of width $\Delta r$. The distance from the wing root to the segment is “$r$”. Each segment is analyzed separately and the forces and moments are summed together to obtain the total for the wing. The velocities are found for each segment. However, these velocities are parallel to the body-fixed axis.
system of the body. It is useful to obtain these velocities parallel to the wing (i.e. a body fixed axis of the wing) and the following transformation accomplishes this:

\[
\begin{align*}
\text{port wing} & \quad \begin{bmatrix} U_{wc1} \\ V_{wc1} \\ W_{wc1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_1 & \sin \gamma_1 \\ 0 & -\sin \gamma_1 & \cos \gamma_1 \end{bmatrix} \begin{bmatrix} U_{wl} \\ V_{wl} \\ W_{wl} \end{bmatrix} \\
\text{starboard wing} & \quad \begin{bmatrix} U_{wc2} \\ V_{wc2} \\ W_{wc2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_2 & -\sin \gamma_2 \\ 0 & \sin \gamma_2 & \cos \gamma_2 \end{bmatrix} \begin{bmatrix} U_{w2} \\ V_{w2} \\ W_{w2} \end{bmatrix}
\end{align*}
\]

5.6.1 Wing Lift

The angle of attack of the wing is similar to that developed for the tail except no downwash is required, pitch rate about c.g. is included in \( W_{wc} \), and a linear twist \( r\alpha \) is added.

\[
\begin{align*}
\text{port wing} & \quad \alpha_{wl1} = \alpha_{0w1} + r_1 \alpha_1 + \frac{W_{wc1}}{U_{wc1}} | r_1 + Q_{wc1} \frac{c_{wl1}}{2U_{wc1}} | r_1 \\
\text{starboard wing} & \quad \alpha_{wl2} = \alpha_{0w2} + r_2 \alpha_2 + \frac{W_{wc2}}{U_{wc2}} | r_2 + Q_{wc2} \frac{c_{wl2}}{2U_{wc2}} | r_2
\end{align*}
\]

With this, the lift of the segment of width \( \Delta r \) can be calculated.

\[
\begin{align*}
\text{port wing} & \quad L_{wc1l1} = \frac{1}{2} \rho \left( U_{wc1}^2 + W_{wc1}^2 \right) c_{wl1} | r_1 \Delta r \alpha_{wl1} \\
\text{starboard wing} & \quad L_{wc1l2} = \frac{1}{2} \rho \left( U_{wc2}^2 + W_{wc2}^2 \right) c_{wl2} | r_1 \Delta r \alpha_{wl2}
\end{align*}
\]
5.6.2 Wing Drag
The drag for a segment is given by the conventional lift-drag polar:

- Port wing
  \[ D_{wclr1} = \frac{1}{2} \rho (U_{wcl}^2 + W_{wcl}^2) c_{w1} | r1 \Delta r_1 \left( C_{D_{wcl}} + \frac{C_{L_{w1}}^2}{\pi AR} \right) \]

- Starboard wing
  \[ D_{wclr2} = \frac{1}{2} \rho (U_{wcl}^2 + W_{wcl}^2) c_{w2} | r2 \Delta r_2 \left( C_{D_{wcl}} + \frac{C_{L_{w2}}^2}{\pi AR} \right) \]

5.6.3 Wing Pitching Moment
The pitching moment at zero angle of attack has to be taken into account and was estimated using Etkin (Ref. 6, Fig B8.2)

- Port wing
  \[ M_{wclr1} = \frac{1}{2} \rho (U_{wcl}^2 + W_{wcl}^2) c_{w1} | r1 \Delta r_1 \left( C_{M_{0,\omega}} + \frac{\pi}{4} Q_{wcl} \right) \]

- Starboard wing
  \[ M_{wclr2} = \frac{1}{2} \rho (U_{wcl}^2 + W_{wcl}^2) c_{w2} | r2 \Delta r_2 \left( C_{M_{0,\omega}} + \frac{\pi}{4} Q_{wcl} \right) \]

Using the small angle approximations, the aerodynamic terms for one segment are:

Port wing:

\[
\begin{align*}
X_{\text{aero,}w1|r1} &= L_{wclr1} \left( \frac{W_{wcl}}{U_{wcl}} \right) - D_{wclr1} \\
Z_{\text{aero,}w1|r1} &= -L_{wclr1} - D_{wclr1} \left( \frac{W_{wcl}}{U_{wcl}} \right) \\
M_{\text{aero,}w1|r1} &= M_{wclr1} - Z_{\text{aero,}w1|r1} \overline{x_{wcl}}
\end{align*}
\]
Starboard wing:

\[
\begin{align*}
X_{\text{aero}_{w1}} &= X_{\text{wcl}2} - D_{\text{wcl}2} \left( \frac{W_{\text{w2}}}{U_{\text{w2}}} \right) \\
Z_{\text{aero}_{w1}} &= Z_{\text{wcl}2} - D_{\text{wcl}2} \left( \frac{W_{\text{w2}}}{U_{\text{w2}}} \right) \\
M_{\text{aero}_{w1}} &= M_{\text{wcl}2} - Z_{\text{aero}_{w1}} \bar{x}_{\text{w2}}
\end{align*}
\]

To get these aerodynamic terms back into the proper coordinate system (i.e. axis system aligned with the axis system of the body), the following transformation matrix is used:

\[
[\text{Transformation}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_1 & -\sin \gamma_1 \\ 0 & \sin \gamma_1 & \cos \gamma_1 \end{bmatrix} \quad \text{port wing}
\]

\[
[\text{Transformation}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_2 & \sin \gamma_2 \\ 0 & -\sin \gamma_2 & \cos \gamma_2 \end{bmatrix} \quad \text{starboard wing}
\]

5.7 Center Panel Contribution

The center panel is considered rigid which means that twisting is neglected. Like the wing, thrust is assumed to be generated entirely by tilt of the lift vector(Figure 15).

Since the center panel only plunges in the vertical direction, it is easier to analyze aerodynamically and the segment method is not necessary.
5.7.1 Center Panel Lift

The angle of attack of the center panel is similar to that developed for the outer wing panel except no twisting is taken into account.

\[ \alpha_{cp} = \alpha_{o} + \frac{W_{cp}}{U_{cp}} + Q_{b} \frac{c_{cp}}{2U_{cp}} \]

With this, the lift of the center panel can be calculated.

\[ L_{cp} = \frac{1}{2} \rho (U_{cp}^2 + W_{cp}^2) c_{cp} b_{cp} C_{L_{cp}} \alpha_{cp} \]

5.7.2 Center Panel Drag

The drag for a panel is given by the conventional lift-drag polar:

\[ D_{cp} = \frac{1}{2} \rho (U_{cp}^2 + W_{cp}^2) c_{cp} b_{cp} \left( C_{D_{cp}} + \frac{C_{L_{cp}}^2}{\pi A R} \right) \]

5.7.3 Center Panel Pitching Moment

The pitching moment was estimated by:

\[ M_{c} = \frac{1}{2} \rho (U_{cp}^2 + W_{cp}^2) c_{cp} b_{cp} \left( C_{M0_{c}} + \frac{\pi}{4} Q_{b} \right) \]

Using the small angle approximations, the aerodynamic terms for the center panel are:

\[
\begin{align*}
X_{sero_{cp}} &= L_{cp} \left( \frac{W_{cp}}{U_{cp}} \right) - D_{cp} \\
Z_{sero_{cp}} &= -L_{cp} - D_{cp} \left( \frac{W_{cp}}{U_{cp}} \right) \\
M_{sero_{cp}} &= M_{cp} - Z_{sero_{cp}} \bar{x}_{wc}
\end{align*}
\]
6. LATERAL AERODYNAMIC ANALYSIS

6.1 Body contribution
Stability derivatives were used to estimate the contribution from the body.

6.1.1 Body Sideslip Force
The stability derivative used to measure the sideslip force can be found by the following formula:

\[
C_{Yb} = -K_i C_{La_s} \frac{V_{\text{body}}^2}{S_w}
\]

where \(K_i = 1.5\)

The sideslip force can then be calculated:

\[
Y_b = \frac{1}{2} \rho (U_b^2 + V_b^2) S C_{Yb} \beta_b
\]

\[
= \frac{1}{2} \rho (U_b^2 + V_b^2) V_{\text{vol}} C_{Yb} \left( \frac{V_b}{U_b} \right)
\]

6.1.2 Body Yawing Moment
The equation for the stability derivative is:

\[
C_{Nlb} = \hat{x}_{cp} C_{Yb}
\]

\[
\hat{x}_{cp} = \frac{c.g.-70\% l_{\text{body}}}{b_w}
\]

The yawing moment is given by:

\[
N_b = \frac{1}{2} \rho (U_b^2 + V_b^2) V_{\text{vol}} C_{Nlb} \beta_b
\]

\[
= \frac{1}{2} \rho (U_b^2 + V_b^2) V_{\text{vol}} C_{Nlb} \left( \frac{V_b}{U_b} \right)
\]
6.1.3 Body Rolling Moment

The formula for estimating the rolling moment stability derivative is:

\[ C_{L_{\beta_b}} = -\frac{1}{2} c_p C_{Y_{\beta_b}} \]

from which the rolling moment can be calculated.

\[ L_{\text{rollb}} = \frac{1}{2} \rho (U_b^2 + V_b^2) \text{Vol}_b C_{L_{\beta_b}} \beta_b \]

\[ = \frac{1}{2} \rho (U_b^2 + V_b^2) \text{Vol}_b C_{L_{\beta_b}} \left( \frac{V_b}{U_b} \right) \]

6.2 Horizontal Tail Contribution

The horizontal tail contribution to the lateral stability is minimal, especially since there is no dihedral for the tail. The only lateral characteristic modeled was the change in angle of attack during a roll. When the aircraft rolls, the angle of attack varies linearly with the right wing tip having a value of \( p_b b/2u_b \) and the left wing tip \( -p_b b/2u_b \) (Figure 16). This change must be taken into account for the spanwise distribution of angle of attack for the tail developed in Section 5.4.1.

![Figure 16: Changes in angle of attack due to a roll \( \rho \)](image)

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6.3 Fin Contribution

6.3.1 Fin Sideslip Angle

The sideslip angle of the fin is composed of two components:

1) control-surface angle of fin

2) induced angle of attack due to sideslip\( (V_b) \) and yaw rate effects

1) control-surface angle \( \beta_a \)

2) sideslip: \( \beta_s = \frac{V_b}{U_b} \)

yaw rate

The yaw rate effect consists of two parts -

i) rotation of the body about the c.g. causes an induced velocity at

the fin

\[ \beta_i = R_b \frac{x_f}{U_b} \]

ii) rotation about the aerodynamic center of the fin

\[ \beta_i = R_b \frac{c_f}{2U_b} \]

There is also an effect from the sidewash which is analogous to the
downwash for the tail. However, due to the complexity in estimating it, the
sidewash contribution was neglected.

Thus the total sideslip of the fin is:

\[ \beta_t = \beta_a + \frac{V_b}{U_b} + R_b \frac{x_f}{U_b} + R_b \frac{c_f}{2U_b} \]
6.3.2 Fin Side Lift

The side lift can be found using:

\[ L_f = \frac{1}{2} \rho \left[ U_b^2 + (V_b + R_b \bar{x}_f)^2 \right] S_f C_{L_f} \alpha_f \]

6.3.3 Fin Drag

The drag of the fin can be estimated using the conventional lift-drag polar for a wing:

\[ C_D = C_{D_a} + \frac{C_{L_f}^2}{\pi AR} \]

Thus the drag of the fin is:

\[ D_f = \frac{1}{2} \rho \left[ U_b^2 + (V_b + R_b \bar{x}_f)^2 \right] S_f C_D \]

The fin aerodynamic forces are:

\[ X_{\text{aero}_f} = L_f \left( \frac{V_b}{U_b} + R_b \frac{\bar{x}_f}{U_b} \right) - D_f \]
\[ Y_{\text{aero}_f} = -L_f - D_f \left( \frac{V_b}{U_b} + R_b \frac{\bar{x}_f}{U_b} \right) \]
\[ N_{\text{aero}_f} = Y_{\text{aero}_f} \bar{x}_f \]
\[ L_{\text{aero}_f} = -Y_{\text{aero}_f} \bar{z}_f \]

The \( X_{\text{aero}} \) term can be added to the total body drag term \( X_{\text{aero},b} \) developed in the longitudinal aerodynamics (Section 5.5).

Summing both the contributions from the body (Section 6.1) and fin for side force, yawing moment and rolling moment gives the following aerodynamic terms:

\[
\begin{align*}
Y_{\text{aero},o} &= Y_b + Y_{\text{aero}_f} \\
N_{\text{aero},o} &= N_b + N_{\text{aero}_f} \\
L_{\text{aero},o} &= L_b + L_{\text{aero}_f}
\end{align*}
\]
6.4 Outer Wing Panel Contribution

Unlike the horizontal tail, the wing plays a major role in the lateral stability of the aircraft. As was the case with the tail, when the aircraft rolls, the angle of attack varies linearly with the right wing tip having a value of \( p_b b_w/2u_b \) and the left wing tip \(-p_b b_w/2u_b\) (See Figure 16). Also there is a component of angle of attack equivalent to \((V_d/U_b) \sin \gamma\), which arises when the plane yaws (Figure 17). These changes must be taken into account for the total angle of attack of the wing developed in Section 5.6.1.

Figure 17: Changes in angle of attack due to a yaw \( \gamma \)
Again the segment method is employed to calculate the lateral aerodynamics terms. These aerodynamic terms for one segment are:

Port wing:

\[
\begin{align*}
Y_{\text{aero},1} & = 0 \\
L_{\text{aero},1} & = -Z_{\text{aero},1} r_1 \\
N_{\text{aero},1} & = 0
\end{align*}
\]

Starboard wing:

\[
\begin{align*}
Y_{\text{aero},2} & = 0 \\
L_{\text{aero},2} & = Z_{\text{aero},2} r_2 \\
N_{\text{aero},2} & = 0
\end{align*}
\]

Although the values for \( Y_{\text{aero}} \) and \( N_{\text{aero}} \) are equal to zero in this coordinate system, this will not necessarily be the case when the aerodynamic terms are transformed back to the body-fixed axis system using the coordinate transformation in Section 5.6.3.

6.5 Center Panel Contribution

The lateral aerodynamics of the center panel is much like that of the outer wing panel with the angle of attack component, \( p_{\alpha} b_\alpha / 2 u_\alpha \), being present when the aircraft rolls.
7. COMPUTER PROGRAM

Solving the non-linear equations of motion analytically is a difficult task; and thus a numerical technique was used. This took the form of a computer program written in FORTRAN. Initial conditions of the state variables, flapping information, ornithopter geometric characteristics, and aerodynamic information was input into the computer program. This information was read from four data files: BODY.DAT, WING.DAT, TAIL.DAT, INIT.DAT. The latter file contains all the initial information required while the other files contain information for the body, wing, and tail, respectively. The program solves the equations simultaneously at time t using a Gaussian Elimination routine. This method was used because it is straightforward, easy to use, and is efficient as any other method. The major drawback is that it is computationally slower than the other routines but, for this case, computational time was not a crucial factor. The answers returned by the equation solver were the derivatives of the state variables. To obtain the state variables, a second-order Runge-Kutta technique was used to integrate the derivatives. Normally, a fourth-order Runge-Kutta is used, but in this case fast computational time and high accuracy were not required. The output from the program was written to a file (OUTPUT.DAT). A very powerful feature of the program is that any perturbations of the state variables or changes in the control systems at time t can be specified in the input.
Solving these equations by using a computer program, as opposed to finding an analytical solution, is much easier; however, the major difficulty arises when predicting how one variable depends upon another. The approach for solving the linear equations of motion for conventional aircraft (shown in Etkin) produces root locus plots which provide valuable information of how certain parameters, such as $C_{ma}$, may affect the aircraft's dynamic stability. Valuable insight is provided into the stability of the aircraft from such plots. Such insight cannot be gained from the computer analysis because of its complexity. Determining whether the value of a certain parameter will cause any instability is a trial and error approach.

8. DISCUSSION OF RESULTS

8.1 Longitudinal Stability

The longitudinal stability was determined from the computer program by setting the lateral variables $(P,R,V,\phi,\psi)$ equal to zero. The first test to determine the accuracy of the program was performed on the Canada goose to examine its longitudinal stability with the inputs being provided by Grant. The results were exactly those of Grant's, which meant no recoding flaws had been made and this instilled confidence in the veracity of the program.

The next step was to input data from Fowler's human-powered ornithopter for the 2-panel model. Unfortunately, many of the inputs, such as the aerodynamic coefficients and the geometric values, were not provided and had
to be estimated. This led to erroneous results for the longitudinal stability variables.

8.1.1 Piper Comanche PA24-250

An aircraft for which there was an abundance of information is the Piper Comanche PA24-250 (Figure 18). The longitudinal and lateral stability of the Comanche had been studied by Zingg (Ref. 20) using the linearized equations of motion. By setting the flapping frequency and flapping angle equal to zero, and using the information provided by Zingg, the 2-panel and 3-panel models were compared to Zingg's linearized model for the Comanche.

Figure 18: Three-view drawing of the Piper Comanche PA24-250

This aircraft is longitudinally stable with two modes: phugoid and short-period. The phugoid mode (Graph 1) is characterized by light damping and a long period. Graph 1 indicates that the 2-panel model and 3-panel model are
also stable with a phugoid motion closely following Zingg's linear model. The 2-panel model is more damped and has a lower period, while the 3-panel model is less damped with a higher period. However, it is clear that all three models converge to a $z'$ increase of about 5 m from the reference level.

The short-period mode (Graph 2) is characterized by heavy damping and a short period (hence the name). From Graph 2 it is evident that the 2-panel model and 3-panel model are stable and comparable to the theoretical short-period motion. Again, the 2-panel model is more damped and has a lower period, while the 3-panel model is less damped with a higher period. The convergence to about 1 m above the reference point is apparent for all three models.

The similarity of the results between the linear model with that of the 2-panel and 3-panel models confirms that the 2-panel and 3-panel models have been developed correctly for longitudinal stability calculations. However, the correctness of the lateral stability modeling will be examined later in Section 8.2.1.

8.1.2 2-Panel "Mr. Bill"

With the success of the Piper Comanche comparisons, it was time to begin inputting the values for "Mr. Bill" because all the relevant data was readily available. The initial conditions given were:
forward velocity $[U]: 13.7 \text{m/s (45 ft/s)}$

plunging velocity $[W]: 0 \text{m/s}$

pitch rate $[Q]: 0 \text{deg/s}$

pitch $[\theta]: 0 \text{deg}$

flapping frequency $[\omega]: 3 \text{Hz}$

The results indicate that after a transient stage, “Mr. Bill” reaches an equilibrium state. Due to the flapping motion, sinusoidal-type variations of the longitudinal variables occur about the equilibrium value. The results are as follows:

forward velocity $[U]: 14.5 \pm 0.1 \text{m/s (48 ft/s)}$ (Graph 3)

forward acceleration $[U^1]: 1 \pm 3 \text{m/s}^2$ (Graph 3)

plunging velocity $[W]: 0 \pm 1 \text{m/s}$ (Graph 5)

plunging acceleration $[W^1]: 0 \pm 20 \text{m/s}^2$ (2g) (Graph 6)

pitch rate $[Q]: 0 \pm 15 \text{deg/s}$ (Graph 7)

pitch $[\theta]: 0 \pm 0.5 \text{deg/s}$ (Graphs 8 & 9)

A plot of the flight-path trajectory shows that after an initial vertical decline, the ornithopter achieves level flight (Graph 10). The peak-to-peak amplitude of the vertical displacement is $10 \text{ cm}$ (Graph 11) which is higher than what was observed in the actual flights (Ref. 5). Another point of concern is that the plunging acceleration is varying between $\pm 20 \text{ m/s}^2$ (2g). Both the high vertical displacement and plunging acceleration are a result of the simple
2-panel model. The use of the 3-panel model dramatically decreases these values because it serves to balance the inertial and aerodynamic loads (Ref. 5).

8.1.3 3-Panel "Mr. Bill"

Like the 2-panel model, the results from the 3-panel model also indicate sinusoidal variations about the equilibrium value. Incorporating the 3-panel feature greatly improves the results as shown below:

- forward velocity \( U \): \( 14.7 \pm 0.05 \text{ m/s (48 ft/s)} \) (Graph 12)
- forward acceleration \( U^1 \): \( 0.5 \pm 1.25 \text{ m/s}^2 \) (Graph 13)
- plunging velocity \( W \): \( 0 \pm 0.25 \text{ m/s} \) (Graph 14)
- plunging acceleration \( W^1 \): \( 0 \pm 5 \text{ m/s}^2 (0.5g) \) (Graph 15)
- pitch rate \( Q \): \( 0 \pm 4 \text{ deg/s} \) (Graph 16)
- pitch \( \theta \): \( 0 \pm 0.25 \text{ deg/s} \) (Graphs 17 & 18)

The trajectory shows that the 3-panel ornithopter is able to achieve level flight after an initial drop (Graph 19). The two points of concern, high plunging acceleration and peak-to-peak plunging amplitude, have been rectified by the 3-panel model. The plunging acceleration variation is reduced to \( \pm 5 \text{ m/s}^2 (0.5g) \), while the peak-to-peak amplitude of the vertical displacement is 2 cm (Graph 20). These parameters indicate very tolerable levels if a pilot were seated inside.
8.1.4 2-Panel Full-Scale Ornithopter (FSO)

The initial conditions given for the FSO were the same as for "Mr. Bill" except for the following two variables:

forward velocity\( U \): 23.77 m/s (78 ft/s)
flapping frequency: 1.05 Hz

The FSO goes through an initial transient stage before reaching an equilibrium state. It also experiences sinusoidal-type variations of the longitudinal variables about the equilibrium value. The results are:

forward velocity\( U \): \( 22.75 \pm 0.375 \) m/s (75 ft/s) (Graph 21)
forward acceleration\( U' \): \( 0.5 \pm 2 \) m/s\(^2\) (Graph 22)
plunging velocity\( W \): \( 0 \pm 1.25 \) m/s (Graph 23)
plunging acceleration\( W' \): \( 0 \pm 8 \) m/s\(^2\) (1g) (Graph 24)
pitch rate\( \phi \): \( 0 \pm 8 \) deg/s (Graph 25)
pitch \( \theta \): \( 0 \pm 1 \) deg/s (Graphs 26 & 27)

A plot of the flight-path trajectory shows that after an initial vertical rise, the ornithopter achieves level flight (Graph 28). The peak-to-peak amplitude of the vertical displacement is 40 cm over a range of 10 m (Graph 29). The plunging acceleration is varying between \( \pm 8 \) m/s\(^2\) (1g). Both the high vertical displacement and plunging acceleration are, again, the result of the simple 2-panel model.
8.1.5 3-Panel Full-Scale Ornithopter (FSO)

The same trends that were noticed for "Mr. Bill" are also present for the FSO. Namely, the plunging acceleration is reduced to $\pm 2.0 \text{ m/s}^2$ (0.2g) and the peak-to-peak vertical displacement is reduced to 15 cm (Graphs 37 and 38).

The program yields the following results:

- forward velocity[$U$]: $24.6 \pm 0.1 \text{ m/s} (80 \text{ ft/s})$ (Graph 30)
- forward acceleration[$U^1$]: $0 \pm 1.25 \text{ m/s}^2$ (Graph 31)
- plunging velocity[$W$]: $-0.2 \pm 0.2 \text{ m/s}$ (Graph 32)
- plunging acceleration[$W^1$]: $0 \pm 2.0 \text{ m/s}^2$ (0.2g) (Graph 33)
- pitch rate[$Q$]: $0 \pm 1.5 \text{ deg/s}$ (Graph 34)
- pitch [$\theta$]: $-0.3 \pm 0.3 \text{ deg/s}$ (Graphs 35 & 36)

8.2 LATERAL STABILITY

The lateral stability of all four cases was examined using the complete equations of motion, with the output variables of interest being: roll angle, yaw angle, sideslip velocity, and sideslip acceleration. The longitudinal variables could not be set to zero because all the variables are coupled and thus the complete equations of motion were used. The lateral stability analysis for the Canada goose and Fowler's human-powered ornithopter was ignored because very limited information was available.
The validity of the lateral stability predictions of the 2-panel and 3-panel models was assessed by setting the flapping frequency and flapping angle equal to zero, and comparing it to Zingg's (Ref. 20) linearized model for the Comanche.

The Comanche has two lateral modes: an unstable spiral mode, and a stable dutch-roll mode. The spiral mode (Graph 39) is characterized by heavy damping and a short period. Graph 39 indicates that the 2-panel model and 3-panel model are also unstable, matching the linear model at the beginning but diverging from the theoretical results as the instability grows. The divergence is such that the developed models are spiraling at a faster rate, which means greater instability. This implies that the models are conservative in nature, showing more instability than there actually is. From a design point of view, this is preferred in comparison to a model that shows less instability.

The dutch-roll mode (Graph 40) is characterized by light damping and a long period. From Graph 40 it is evident that the 2-panel model and 3-panel model are stable and comparable to the theoretical short-period motion. The 2-panel model is more damped and has a lower period, while the 3-panel model is less damped with a higher period. All three models converge close to the reference mark of 0 m.

The similarity of the results with the linear model confirm that the 2-panel and 3-panel models have been developed correctly for lateral stability. This, combined with the successful comparison of the longitudinal stability (See
Section 8.1.1), instills a high degree of confidence in the correctness of the models.

8.2.2 2-Panel "Mr. Bill"

The first step in performing the analysis was to determine which initial conditions to use. It was decided that they would be the same as those used for the longitudinal analysis with the added initial roll of 20 degrees.

forward velocity[U]: 13.7m/s (45 ft/s)

sideslip velocity[V]: 0 m/s roll rate[P]: 0 deg/s

yaw rate[R]: 0 deg/s roll [φ]: 20 deg

yaw [ψ]: 0 deg

flapping frequency: 3 Hz

The ornithopter goes through a transient phase with oscillations, but these oscillations diminish with time to reach an equilibrium stage where the state variables, sideslip velocity[V], sideslip acceleration[V'], roll [φ], yaw [ψ] are equal to zero (See Graphs 41-44). Unlike the longitudinal case, there are no sinusoidal variations about the equilibrium value.

This motion is a characteristic of a dutch-roll mode which is stable because of the diminishing oscillations with time. The period is 6.3 seconds while the time it takes to half amplitude(t½) is 3 seconds. The simulation shows that with a positive initial roll (right wing down, left wing up), the aircraft shifts to the right by about 2.14 m after the transient stage damps out (Graph 45).
8.2.3 3-Panel "Mr. Bill"

The computer simulation for the 3-panel model shows that the equilibrium stage for the state variables \((V, V', \phi)\) is zero (See Graphs 46-49). The motion is a stable dutch-roll mode with a period of 6.3 seconds and the time-to-half amplitude of 1.4 seconds. The period is essentially the same as the 2-panel "Mr. Bill"; however, the time-to-half amplitude has decreased by a factor of 2. Because the 3-panel design is more damped, this implies that it is more stable. The trajectory of the 3-panel ornithopter also shifts to the right but by only 1.4 m (Graph 50).

8.2.4 2-Panel Full-Scale Ornithopter (FSO)

The initial conditions given for the FSO were the same as for "Mr. Bill" except for the following two variables:

- forward velocity\([U]\): 23.77 m/s (78 ft/s)
- flapping frequency: 1.05 Hz

Graphs 51 to 54 reveal that the 2-panel FSO goes through a stable dutch-roll mode with the state variables \((V, V', \phi)\) damping out to an equilibrium value of zero. The period is 5.5 seconds while the time-to-half amplitude is 1.3 seconds. The trajectory shifts to the right by 101.35 m (Graph 55).

8.2.5 3-Panel Full-Scale Ornithopter (FSO)

Once again, the state variables of the 3-panel model damp out to zero (See Graphs 56-59) having a period of 5.3 seconds and a time-to-half amplitude of 0.74 seconds. The same decrease in \(t_{1/2}\) is apparent as it was for "Mr. Bill". This indicates that the 3-panel design is inherently more stable than the 2-panel model. The trajectory shifts to the right by 70 m (Graph 60).
9. CONCLUSION

The main objective of this study was to examine the complete non-linear flight dynamics of a full-scale ornithopter. This goal has been attained with the results indicating that the full-scale ornithopter is longitudinally and laterally stable. After an initial transient phase, the state variables reach equilibrium values with small sinusoidal variations about this value. The results show that a 3-panel ornithopter model experiences lower accelerations and lower peak-to-peak displacement amplitudes than the 2-panel model. This information will prove valuable because the pilot will normally be able to tolerate a 0.5g variation and small displacements.

The completion of this stage means that all the necessary equations for a complete dynamic analysis have been developed and it would be a simple matter to refine both the dynamic and aerodynamic models. For example, the wing can be modeled with a double taper instead of as a flat plate and aeroelastic effects can be taken into account (including the twisting of the wings in the dynamic analysis). More sophisticated models which use non-linear unsteady aerodynamics can also be incorporated. Further, the data inputs such as: moment of inertias, lift-curve slopes, and downwash angles can be refined. The control systems may include feedback loops to add a dimension of piloting realism to the model. At this stage, it is possible to determine responses to different control inputs, flight conditions, and flapping frequencies. It would also
be a valuable exercise to see how the location of the center of gravity would affect the stability.

Although the simulation results are very promising, it will only be until the full-scale ornithopter is constructed and flown, with a pilot inside, that confirmation of the results can take place.
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APPENDIX A - GRAPHS
Comparison of Piper Comanche PA24–250 Phugoid Mode Using Different Models (U=76.4 m/s, Theta=4 deg)

Graph 1
Vertical Displacement vs. Horizontal Displacement

Comparison of Piper Comanche PA24-250 Short Period Mode Using Different Models (U=76.4 m/s, Theta=10deg)

Graph 2
Forward Velocity vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)
Forward Acceleration vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)
Plunging Velocity vs. Time

(2-Panel Mr. Bill, Flapping freq=3Hz)
Plunging Acceleration vs. Time
(2-Panel Mr. Bill, Flapping freq = 3 Hz)
Pitch Rate vs. Time
(2-Panel Mr. Bill, Flapping Freq=3Hz)

Graph 7
Pitch Angle vs. Time
(2-Panel Mr. Bill, Flapping freq = 3 Hz)
Vertical Displacement vs. Horizontal Displacement
(2-Panel Mr. Bill, Flapping freq=3Hz)

Graph 10
Vertical Displacement vs. Horizontal Displacement
(2-Panel Mr. Bill, Flapping freq=3Hz)
Forward Velocity vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)
Forward Acceleration vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)
Plunging Velocity vs. Time
(3-Panel, Mr. Bill, Flapping freq = 3 Hz)
Plunging Acceleration vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)

Graph 15
Pitch Rate vs. Time
(3-Panel Mr. Bill, Flapping Freq=3Hz)
Pitch Angle vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)
Vertical Displacement vs. Horizontal Displacement
(3-Panel Mr. Bill, Flapping freq = 3 Hz)
Vertical Displacement vs. Horizontal Displacement
(3-Panel Mr. Bill, Flapping freq=3Hz)
Forward Acceleration vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Plunging Velocity vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Plunging Acceleration vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Pitch Rate vs. Time
(2-Panel Full-Scale, Flapping Freq=1.05Hz)

Graph 25
Pitch Angle vs. Time

(2-Panel Full-Scale, Flapping freq = 1.05Hz)

θ (deg)

t (s)
Pitch Angle vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Vertical Displacement vs. Horizontal Displacement
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Forward Velocity vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Forward Acceleration vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Plunging Velocity vs. Time

(3-Panel Full-Scale, Flapping freq=1.05Hz)
Plunging Acceleration vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Graph 34

Pitch Rate vs. Time
(3-Panel Full-Scale, Flapping Freq=1.05Hz)
Pitch Angle vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Vertical Displacement vs. Horizontal Displacement
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Forward Displacement vs. Sideslip Displacement

Comparison of Piper Comanche PA24-250 Spiral Mode Using Different Models (U=76.4 m/s, \(\psi=-10\) deg)

Graph 39
Forward Displacement vs. Sideslip Displacement

Comparison of Piper Comanche PA24-250 Dutch Roll Mode Using Different Models (U=76.4 m/s, Phi=20 deg)

Graph 40
Sideslip Velocity vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)
Using Complete Non-Linear Equations of Motion

Graph 41
Sideslip Acceleration vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)

Using Complete Non-Linear Equations of Motion

\[(\frac{2s}{\omega}) q, \Lambda\]

Graph 42
Roll Angle vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)
Using Complete Non-Linear Equations of Motion

Graph 43
Yaw Angle vs. Time
(2-Panel Mr. Bill, Flapping freq=3Hz)
Using Complete Non-Linear Equations of Motion

Graph 44
Forward Displacement vs. Sideslip Displacement
(2-Panel Mr. Bill, Flapping freq=3Hz)

Using Complete Non-Linear Equations of Motion

Graph 45
Sideslip Velocity vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)
Using Complete Non-Linear Equations of Motion

Graph 46
Sideslip Acceleration vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)

Using Complete Non-Linear Equations of Motion

Graph 47
Roll Angle vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)
Using Complete Non-Linear Equations of Motion
Yaw Angle vs. Time
(3-Panel Mr. Bill, Flapping freq=3Hz)

Using Complete Non-Linear Equations of Motion

Graph 49
Forward Displacement vs. Sideslip Displacement
(3-Panel Mr. Bill, Flapping freq=3Hz)

Using Complete Non-Linear Equations of Motion

Graph 50
Sideslip Velocity vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion
Sideslip Acceleration vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion

Graph 52
Roll Angle vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion
Yaw Angle vs. Time
(2-Panel Full-Scale, Flapping freq=1.05Hz)

Using Complete Non-Linear Equations of Motion

Graph 54
Forward Displacement vs. Sideslip Displacement
(2-Panel Full-Scale, Flapping freq=1.05Hz)

Using Complete Non-Linear Equations of Motion
Sideslip Velocity vs. Time

(3-Panel Full-Scale, Flapping freq=1.05Hz)

Using Complete Non-Linear Equations of Motion
Sideslip Acceleration vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion

Graph 57
Roll Angle vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion
Yaw Angle vs. Time
(3-Panel Full-Scale, Flapping freq=1.05Hz)

Using Complete Non-Linear Equations of Motion

ψ_p (deg)

Graph 59
Forward Displacement vs. Sideslip Displacement
(3-Panel Full-Scale, Flapping freq=1.05Hz)
Using Complete Non-Linear Equations of Motion

Graph 60
APPENDIX B - Derivation of Equations of Motion for a Rigid Body

For a rigid body:

Newton's 2nd Law:

\[ \mathbf{F} = m \mathbf{a} \]

where

\[ \mathbf{F} = X \mathbf{n}_1 + Y \mathbf{n}_2 + Z \mathbf{n}_3 \]

\[ \mathbf{a}_{cm} = \frac{d \mathbf{v}_{cm}}{dt} \bigg|_{\text{fixed}} = \frac{d \mathbf{v}_{cm}}{dt} \bigg|_{\text{body B}} + \Omega \times \mathbf{v}_{cm} \]

where

\[ \Omega = P \mathbf{n}_1 + Q \mathbf{n}_2 + R \mathbf{n}_3 \]

\[ \mathbf{v}_{cm} = U \mathbf{n}_1 + V \mathbf{n}_2 + W \mathbf{n}_3 \]

\[ \mathbf{a}_{cm} = \dot{U} \mathbf{n}_1 + \dot{V} \mathbf{n}_2 + \dot{W} \mathbf{n}_3 + \begin{vmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 \\ P & Q & R \\ U & V & W \end{vmatrix} \]

\[ = \dot{U} \mathbf{n}_1 + \dot{V} \mathbf{n}_2 + \dot{W} \mathbf{n}_3 + \mathbf{n}_1 (QW - RV) - \mathbf{n}_2 (PW - RU) + \mathbf{n}_3 (PV - QU) \]

\[ = (\dot{U} + QW - RV) \mathbf{n}_1 + (\dot{V} - PW + RU) \mathbf{n}_2 + (\dot{W} + PV - QU) \mathbf{n}_3 \]
Newton's 2nd Law in component form is:

\[
\begin{align*}
X &= m(\ddot{U} + QW - RV) \\
Y &= m(\ddot{V} + RU - PW) \\
Z &= m(\ddot{W} + PV - QU)
\end{align*}
\]

The torque equations are:

\[
d\vec{T} = d\vec{F} = dm \cdot \vec{a}
\]

\[
d\vec{T} = \vec{r} \times (dm \cdot \vec{a})
\]

where

\[
\vec{T} = L\vec{n}_1 + M\vec{n}_2 + N\vec{n}_3
\]

\[
\int_B \vec{r} \times \vec{a} dm = \int_B \vec{r} \times [\vec{a}_{cm} + (\vec{\alpha} \times \vec{r}) + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})] dm
\]

Evaluate each term

1st term:

\[
\int_B \vec{r} \times \vec{a}_{cm} dm = -\vec{a}_{cm} \times \int_B \vec{r} dm = -\vec{a}_{cm} \times 0 = 0
\]
2nd term:

let

\[ \mathbf{\tilde{r}} = x\mathbf{n}_1 + y\mathbf{n}_2 + z\mathbf{n}_3 \]

\[ \mathbf{\tilde{\alpha}} = \frac{d\mathbf{\Omega}}{dt}\bigg|_{\text{fixed}} = \frac{d\mathbf{\Omega}}{dt}\bigg|_{\text{body}} + \mathbf{\Omega} \times \mathbf{\Omega} \]

= \dot{\mathbf{p}}\mathbf{n}_1 + \dot{\mathbf{q}}\mathbf{n}_2 + \dot{\mathbf{r}}\mathbf{n}_3

\[ \mathbf{\tilde{r}} \times (\mathbf{\tilde{\alpha}} \times \mathbf{\tilde{r}}) = x\mathbf{n}_1 + y\mathbf{n}_2 + z\mathbf{n}_3 \times \begin{vmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 \\ P & Q & R \\ x & y & z \end{vmatrix} \]

= \begin{vmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 \\ P & Q & R \\ x & y & z \end{vmatrix} = \begin{vmatrix} \dot{Q}_z - \dot{R}_y & \dot{R}_x - \dot{P}_z & \dot{P}_y - \dot{Q}_x \end{vmatrix}

= (\dot{P}y^2 - \dot{Q}xy - \dot{R}xz + \dot{P}z^2)\mathbf{n}_1 +

(-\dot{P}xy + \dot{Q}x^2 + \dot{Q}z^2 - \dot{R}zy)\mathbf{n}_2 +

(\dot{R}x^2 - \dot{P}xz - \dot{Q}yz + \dot{R}y^2)\mathbf{n}_3

= [(y^2 + z^2)\dot{P} - xy\dot{Q} - xz\dot{R}]\mathbf{n}_1 +

[-xy\dot{P} + (x^2 + z^2)\dot{Q} - yz\dot{R}]\mathbf{n}_2 +

[-xz\dot{P} - yz\dot{Q} + (x^2 + y^2)\dot{R}]\mathbf{n}_3

= [I_{xx}\dot{P} - I_{xy}\dot{Q} - I_{xz}\dot{R}]\mathbf{n}_1 +

[-I_{xy}\dot{P} + I_{yy}\dot{Q} - I_{yz}\dot{R}]\mathbf{n}_2 +

[-I_{xz}\dot{P} - I_{yz}\dot{Q} + I_{zz}\dot{R}]\mathbf{n}_3
The moment of inertia terms are defined as:

\[
I_{xx} = \int_B (y^2 + z^2) \, dm \quad I_{xy} = \int_B xy \, dm \\
I_{yy} = \int_B (x^2 + z^2) \, dm \quad I_{xz} = \int_B xz \, dm \\
I_{zz} = \int_B (x^2 + y^2) \, dm \quad I_{yz} = \int_B yz \, dm
\]

3rd term:

\[
\int_B \bar{r} \times [\Omega \times (\Omega \times \bar{r})] \, dm = \int_B \bar{r} \times \begin{bmatrix} \bar{n}_1 & \bar{n}_2 & \bar{n}_3 \\ P & Q & R \\ Qz - Ry & -Pz + Rx & Py - Qx \end{bmatrix} \, dm \\
= \int_B \begin{bmatrix} \bar{n}_1 & \bar{n}_2 & \bar{n}_3 \\ PQy - Q^2x + RPz - R^2z & -P^2y + PQx + RQz - R^2y & -P^2z + PRx - Q^2z + QRy \end{bmatrix} \, dm
\]

\[
= \int_B \left[ -P^2yz + PRxy - Q^2zy + QRy^2 + P^2yz - PQxz - RQz^2 + R^2yz \bar{n}_1 + \\
-PR^2 + Q^2zx - QRx + PQyz - Q^2xz + RP^2 - R^2xz \bar{n}_2 + \\
+P^2Qy - Q^2xy + PRy^2 + PQ^2 - R^2xy \bar{n}_3 \right] \, dm
\]

Adding all three terms yields the torque equations.
Rigid Body Dynamic Equations

Force Equations:
\[ X = m(\dot{U} + QW - RV) \]
\[ Y = m(\dot{V} + RU - PW) \]
\[ Z = m(\dot{W} + PV - QU) \]

Moment Equations:
\[ L = I_{xx} \dot{\rho} - I_{xy} \dot{Q} - I_{xz} \dot{R} + I_{xy} PR + (I_{zz} - I_{yy})QR + I_{yz} (R^2 - Q^2) - I_{xz} PQ \]
\[ M = I_{yy} \dot{\rho} - I_{xy} \dot{Q} - I_{yz} \dot{R} - I_{xy} QR + (I_{xx} - I_{zz})PR + I_{xz} (P^2 - R^2) + I_{yz} PQ \]
\[ N = I_{zz} \dot{R} - I_{xz} \dot{\rho} - I_{yz} \dot{Q} - I_{yz} PR + (I_{yy} - I_{xx})PQ - I_{xy} (P^2 - Q^2) + I_{xz} QR \]
APPENDIX C - 2-Panel Complete Non-Linear Equations of Motion

Assumptions:

1) rigid body (no aeroelastic effects)
2) mass is constant
3) negligible angular momentum of rotating machinery (engines, etc)
4) control systems fixed
5) negligible buoyancy
6) negligible twisting of wing in the dynamic analysis (however twisting is considered in the aerodynamic analysis)
7) wings are thin rectangular plates

Body

Use a body-fixed axis system at the center of gravity

\[
\bar{d}_b = d_{zb} \bar{x} + d_{yb} \bar{y} + d_{zb} \bar{z}
\]
Define

\[ \Psi: \text{ yaw angle (right)} \]
\[ \Theta: \text{ pitch angle (nose up)} \]
\[ \Phi: \text{ roll angle (right wing down)} \]

The components of weight are:

\[ X_g = -m_b g \sin \Theta_b \]
\[ Y_g = m_b g \cos \Theta_b \sin \Phi_b \]
\[ Z_g = m_b g \cos \Theta_b \cos \Phi_b \]

Add all external forces and moments and substitute into the rigid body equations from Appendix B.

\[ X_{aero} = -m_b g \sin \Theta_b + R_{xb1} + R_{xb2} = m_b (\dot{U}_b + Q_b W_b - R_b V_b) \]
\[ Y_{aero} + m_b g \cos \Theta_b \sin \Phi_b + R_{yb1} + R_{yb2} = m_b (\dot{V}_b + R_b U_b - P_b W_b) \]
\[ Z_{aero} + m_b g \cos \Theta_b \cos \Phi_b + R_{zb1} + R_{zb2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b) \]
\[ L_{aero} = -d_{xb} R_{yb1} + d_{xb} R_{yb2} + d_{yb} R_{xb1} - d_{yb} R_{xb2} + d_{yb} R_{yb2} + M_{lb1} + M_{lb2} \]
\[ = I_{xzb} \dot{P}_b - I_{xyb} \dot{Q}_b - I_{xzb} \dot{R}_b + I_{xyb} P_b R_b + (I_{xzb} - I_{yyb}) Q_b R_b + I_{xyz} (P_b^2 - Q_b^2) - I_{xzb} P_b Q_b \]
\[ M_{aero} = d_{xb} R_{xb1} + d_{xb} R_{xb2} + d_{yb} R_{xb1} - d_{yb} R_{xb2} + d_{yb} R_{xb2} + M_{mb1} + M_{mb2} \]
\[ = I_{yyb} \dot{Q}_b - I_{xyb} \dot{P}_b - I_{yzb} \dot{R}_b - I_{xyb} Q_b R_b + (I_{xzb} - I_{zzb}) P_b R_b + I_{xyz} (P_b^2 - R_b^2) + I_{yzb} P_b Q_b \]
\[ N_{aero} = -d_{yb} R_{xb1} + d_{yb} R_{xb2} - d_{yb} R_{yrb} + d_{yb} R_{yrb} + M_{nb1} + M_{nb2} \]
\[ = I_{zzb} \dot{R}_b - I_{xzb} \dot{P}_b - I_{yzb} \dot{Q}_b - I_{yzb} P_b R_b + (I_{yyb} - I_{xzb}) P_b Q_b - I_{xyz} (P_b^2 - Q_b^2) + I_{xzb} Q_b R_b \]

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Because of symmetry, $I_{xyb} \cdot I_{yrb} = 0$

$$
X_{aero} = -m_b g \sin \Theta_b + R_{xb1} + R_{xb2} = m_b (\dot{U}_b + Q_b W_b - R_b V_b)
$$

$$
Y_{aero} = m_b g \cos \Theta_b \sin \Phi_b + R_{yb1} + R_{yb2} = m_b (\dot{V}_b + R_b U_b - P_b W_b)
$$

$$
Z_{aero} = m_b g \cos \Theta_b \cos \Phi_b + R_{zb1} + R_{zb2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b)
$$

$$
L_{aero} = -d_{zb1} R_{yb1} + d_{yb1} R_{zb1} - d_{zb2} R_{yb2} + d_{yb2} R_{zb2} + M_{Lb1} + M_{Lb2} = I_{xb} \ddot{P}_b - I_{xzb} \ddot{R}_b + (I_{zxb} - I_{yzb}) Q_b R_b - I_{xzb} P_b Q_b
$$

$$
M_{aero} = d_{xb1} R_{xb1} - d_{xb1} R_{xb1} + d_{xb2} R_{xb2} - d_{xb2} R_{xb2} + M_{Mb1} + M_{Mb2}
$$

Note that with this axis system, the moments and products of inertia are changing with time.

**Wings**

**Port (Left) Wing**

Use a stability axis system at c.g. of wing

**Note:**

```
X_{aero} = -m_b g \sin \Theta_b + R_{xb1} + R_{xb2} = m_b (\dot{U}_b + Q_b W_b - R_b V_b)
Y_{aero} = m_b g \cos \Theta_b \sin \Phi_b + R_{yb1} + R_{yb2} = m_b (\dot{V}_b + R_b U_b - P_b W_b)
Z_{aero} = m_b g \cos \Theta_b \cos \Phi_b + R_{zb1} + R_{zb2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b)
L_{aero} = -d_{zb1} R_{yb1} + d_{yb1} R_{zb1} - d_{zb2} R_{yb2} + d_{yb2} R_{zb2} + M_{Lb1} + M_{Lb2}
```

```
= I_{xb} \ddot{P}_b - I_{xzb} \ddot{R}_b + (I_{zxb} - I_{yzb}) Q_b R_b - I_{xzb} P_b Q_b
M_{aero} = d_{xb1} R_{xb1} - d_{xb1} R_{xb1} + d_{xb2} R_{xb2} - d_{xb2} R_{xb2} + M_{Mb1} + M_{Mb2}
= I_{yxb} \ddot{Q}_b + (I_{zxb} - I_{yzb}) P_b R_b + I_{xzb} (P_b^2 - R_b^2)
N_{aero} = -d_{yb1} R_{yb1} + d_{yb1} R_{yb1} - d_{yb2} R_{yb2} + d_{yb2} R_{yb2} + M_{Nb1} + M_{Nb2}
= I_{zb} \ddot{R}_b - I_{xzb} \ddot{P}_b + (I_{yxb} - I_{zxb}) P_b Q_b + I_{xzb} Q_b R_b
```
Note from the wing moment of inertia analysis (Appendix E)
$I_{xywl} = I_{zzwl} = \dot{i}_{zzwl} = \dot{i}_{xywl} = \dot{i}_{zzwl} = 0$

\[\begin{align*}
X_{aero_{wl}} &= m_{wl}g\sin\Theta_{wl} + R_{xwl} = m_{wl}(\dot{U}_{wl} + Q_{wl}W_{wl} - R_{wl}V_{wl}) \\
Y_{aero_{wl}} &= m_{wl}g\cos\Theta_{wl}\sin\Phi_{wl} + R_{ywl} = m_{wl}(\dot{V}_{wl} + R_{wl}U_{wl} - P_{wingl}W_{wl}) \\
Z_{aero_{wl}} &= m_{wl}g\cos\Theta_{wl}\cos\Phi_{wl} + R_{zwl} = m_{wl}(\dot{W}_{wl} + P_{wingl}V_{wl} - Q_{wl}U_{wl}) \\
L_{aero_{wl}} &= -D_{xwl}R_{ywl} + D_{ywl}R_{xwl} + M_{Lwl} = I_{xwl}\dot{\dot{p}}_{wingl} + \dot{i}_{xwl}\dot{P}_{wingl} - I_{xywl}Q_{wl} - \dot{i}_{xywl}Q_{wl} \\
&\quad - I_{xwl}\dot{R}_{wl} - \dot{i}_{xwl}R_{wl} + I_{xywl}P_{wingl}R_{wl} + (I_{zzwl} - I_{yywl})Q_{wl}R_{wl} \\
&\quad + I_{ywl}(R_{wl}^2 - Q_{wl}^2) - I_{zzwl}P_{wingl}Q_{wl} \\
M_{aero_{wl}} &= +D_{xwl}R_{xwl} - D_{xwl}R_{xwl} + M_{Mwl} = I_{yywl}\dot{Q}_{wl} + \dot{i}_{yywl}Q_{wl} - I_{zywl}\dot{P}_{wingl} - \dot{i}_{xywl}P_{wingl} \\
&\quad - I_{yzwl}\dot{R}_{wl} - \dot{i}_{yzwl}R_{wl} - I_{xywl}Q_{wl}R_{wl} + (I_{zzwl} - I_{xwl})P_{wingl}R_{wl} \\
&\quad + I_{xwl}(P_{wingl}^2 - R_{wl}^2) + I_{yzwl}P_{wingl}Q_{wl} \\
N_{aero_{wl}} &= -D_{ywl}R_{xwl} + D_{zw}R_{ywl} + M_{Nwl} = I_{zwl}\dot{R}_{wl} + \dot{i}_{zwl}R_{wl} - I_{zzwl}\dot{P}_{wingl} - \dot{i}_{zzwl}P_{wingl} \\
&\quad - I_{yzwl}\dot{Q}_{wl} - \dot{i}_{yzwl}Q_{wl} - I_{xywl}P_{wingl}R_{wl} + (I_{yywl} - I_{xwl})P_{wingl}Q_{wl} \\
&\quad - I_{xywl}(P_{wingl}^2 - Q_{wl}^2) + I_{zzwl}Q_{wl}R_{wl}
\end{align*}\]
Starboard (Right) Wing

Use a stability axis system at c.g. of wing.

Note that with this axis system, the moments and products of inertia are changing with time.

\[
X_{\text{aero}} = -m_w g \sin \Theta_w + R_{xw} = m_w (\dot{U}_w + Q_w W_w - R_w V_w)
\]

\[
Y_{\text{aero}} = m_w g \cos \Theta_w \sin \Phi_w + R_{yw} = m_w (\dot{V}_w + R_w U_w - P_{\text{wing}} W_w)
\]

\[
Z_{\text{aero}} = m_w g \cos \Theta_w \cos \Phi_w + R_{zw} = m_w (\dot{W}_w + P_{\text{wing}} V_w - Q_w U_w)
\]

\[
L_{\text{aero}} = -D_{zw} R_{yw} + D_{yw} R_{zw} + M_{Lw} = I_{xzw} \dot{P}_{\text{wing}} + I_{xzw} P_{\text{wing}} - I_{xyw} \dot{Q}_w - I_{xyw} Q_w
\]

\[
M_{\text{aero}} = -I_{xyw} \dot{R}_w + I_{xyw} R_w - I_{xzw} Q_w + I_{xzw} P_{\text{wing}} - I_{zyw} \dot{P}_{\text{wing}} + I_{zyw} P_{\text{wing}}
\]

\[
N_{\text{aero}} = -I_{zyw} \dot{R}_w + I_{zyw} R_w - I_{xzw} Q_w + I_{xzw} P_{\text{wing}} - I_{xzw} \dot{P}_{\text{wing}} + I_{xzw} P_{\text{wing}}
\]
Note from the wing moment of inertia analysis (Appendix E)

\[ I_{xw} = I_{xz} = I_{zx} = I_{yw} = I_{xw} = 0 \]

\[
\begin{align*}
X_{aero} &= -m_w g \sin \Theta_w + R_{xw} = m_w (\dot{U}_w + Q_w W_w - R_w V_w) \\
Y_{aero} &= +m_w g \cos \Theta_w \sin \Phi_w + R_{yw} = m_w (\dot{V}_w + R_w U_w - P_{wing} V_w) \\
Z_{aero} &= +m_w g \cos \Theta_w \cos \Phi_w + R_{zw} = m_w (\dot{W}_w + P_{wing} U_w - Q_w U_w) \\
L_{aero} &= -D_{sw} R_{yw} + D_{yw} R_{zw} + M_{lw} = I_{xw} \dot{P}_{wing} + (I_{zw} - I_{yy}) Q_w R_w \\
&\quad + I_{yw} (R_w^2 - Q_w^2) \\
M_{aero} &= +D_{sw} R_{zw} - D_{xw} R_{xw} + M_{lw} = I_{yw} \dot{Q}_w + I_{yy} Q_w - I_{y} \dot{R}_w - I_{yw} R_w \\
&\quad + (I_{xw} - I_{zw}) P_{wing} R_w + I_{yw} P_{wing} Q_w \\
N_{aero} &= -D_{yw} R_{xw} + D_{xw} R_{yw} + M_{nw} = I_{zw} \dot{R}_w + I_{zw} R_w - I_{y} \dot{Q}_w - I_{zw} Q_w \\
&\quad - I_{yw} P_{wing} R_w + (I_{yw} - I_{xw}) P_{wing} Q_w
\end{align*}
\]

**Kinematic Analysis**

We need to relate the velocity and acceleration of the body and wing.
velocity:
\[
\overline{V}_c = \overline{V}_b + \overline{\Omega}_b \times (-\overline{D}_w) + \overline{V}_{\text{rel}}
\]
\[
= \overline{V}_a + \overline{\Omega}_b \times \overline{d}_b + \overline{\Omega}_b \times (-\overline{D}_w) + \overline{\Omega}_c \times (-\overline{D}_w)
\]
\[
\overline{\Omega}_b \times \overline{d}_b = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
d_{xb} & d_{yb} & d_{zb}
\end{bmatrix}
\]
\[
= (Q_b d_{zb} - R_b d_{yb}) \hat{i} - (P_b d_{zb} - R_b d_{xb}) \hat{j} + (P_b d_{yb} - Q_b d_{sb}) \hat{k}
\]
\[
\overline{\Omega}_b \times (-\overline{D}_w) = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-D_{xw} & -D_{yw} & -D_{zw}
\end{bmatrix}
\]
\[
= (-Q_b D_{xw} + R_b D_{yw}) \hat{i} - (-P_b D_{zw} + R_b D_{xw}) \hat{j} + (-P_b D_{yw} + Q_b D_{zwb}) \hat{k}
\]
\[
\overline{\Omega}_c \times (-\overline{D}_w) = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
-1 & 0 & 0
\end{bmatrix}
\]
\[
= P_w D_{zw} \hat{j} - P_w D_{yw} \hat{k}
\]
\[
\overline{V}_c = \overline{U}_b \hat{i} + \overline{V}_b \hat{j} + \overline{W}_b \hat{k} + (Q_b d_{zb} - R_b d_{yb}) \hat{i} - (P_b d_{zb} - R_b d_{xb}) \hat{j} + (P_b d_{yb} - Q_b d_{sb}) \hat{k}
\]
\[
+ (-Q_b D_{xw} + R_b D_{yw}) \hat{i} - (-P_b D_{zw} + R_b D_{xw}) \hat{j} + (-P_b D_{yw} + Q_b D_{zwb}) \hat{k}
\]
\[
+ P_w D_{zw} \hat{j} - P_w D_{yw} \hat{k}
\]
\[
= [U_b + Q_b (d_{zb} - D_{xw}) + R_b (D_{yw} - d_{yb})] \hat{i}
\]
\[
+ [V_b + P_b (D_{zw} - d_{zb}) + R_b (d_{xb} - D_{xw}) + P_w D_{zw}] \hat{j}
\]
\[
+ [W_b + P_b (d_{yb} - D_{yw}) + Q_b (D_{zw} - d_{zb}) - P_w D_{yw}] \hat{k}
\]

port wing velocities:

\[
U_{w1} = U_b + Q_b (d_{zbl} - D_{zw1}) + R_b (D_{yw1} - d_{ybl})
\]
\[
V_{w1} = V_b + P_b (D_{zw1} - d_{zbl}) + R_b (d_{xbl} - D_{xw1}) + P_w D_{zw1}
\]
\[
W_{w1} = W_b + P_b (d_{ybl} - D_{yw1}) + Q_b (D_{xw1} - d_{xbl}) - P_w D_{yw1}
\]

starboard wing velocities:

\[
U_{w2} = U_b + Q_b (d_{zbl} - D_{zwb}) + R_b (D_{yw2} - d_{ybl})
\]
\[
V_{w2} = V_b + P_b (D_{zw2} - d_{zbl}) + R_b (d_{xbl} - D_{xw2}) + P_w D_{zw2}
\]
\[
W_{w2} = W_b + P_b (d_{ybl} - D_{yw2}) + Q_b (D_{xw2} - d_{xbl}) - P_w D_{yw2}
\]

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acceleration:
\[
\overline{A}_c = \overline{A}_b + \overline{\Omega}_b \times (-\overline{D}_w) + \overline{\Omega}_b \times (\overline{\Omega}_b \times (-\overline{D}_w)) + 2\overline{\Omega}_b \times \overline{V}_{rel} + \overline{A}_{rel} \\
= \overline{A}_s + \dot{\overline{\Omega}}_b \times \overline{\omega}_b + \overline{\Omega}_b \times (\overline{\Omega}_b \times \overline{\omega}_b) + \overline{\Omega}_b \times \overline{V}_s + \dot{\overline{\omega}}_b \times (-\overline{D}_w) + \overline{\omega}_b \times (\overline{\omega}_b \times (-\overline{D}_w)) + 2\overline{\omega}_b \times (\overline{\Omega}_c \times (-\overline{D}_w)) + \dot{\overline{\omega}}_c \times (-\overline{D}_w) + \overline{\omega}_c \times (\overline{\omega}_c \times (-\overline{D}_w)) \\
\overline{A}_s = \dot{V}_b \overline{i} + \ddot{V}_b \overline{j} + \dot{\omega}_b \overline{k} \\
\dot{\overline{\omega}}_b \times \overline{\omega}_b = \\
\left| \begin{array}{ccc} \overline{i} & \overline{j} & \overline{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ d_{zb} & d_{yb} & d_{zb} \end{array} \right| \\
= (\dot{Q}_b d_{zb} - \dot{R}_b d_{yb}) \overline{i} - (\dot{P}_b d_{yb} - \dot{R}_b d_{zb}) \overline{j} + (\dot{P}_b d_{yb} - \dot{Q}_b d_{zb}) \overline{k} \\
\overline{\Omega}_b \times (\overline{\Omega}_b \times \overline{\omega}_b) = \\
\left| \begin{array}{ccc} \overline{i} & \overline{j} & \overline{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ P_b d_{zb} - R_b d_{yb} & R_b d_{zb} - P_b d_{zb} & P_b d_{yb} - Q_b d_{zb} \end{array} \right| \\
= [Q_b (P_b d_{zb} - Q_b d_{yb}) - R_b (R_b d_{zb} - P_b d_{zb})] \overline{i} - [P_b (P_b d_{zb} - Q_b d_{yb}) - R_b (Q_b d_{zb} - R_b d_{yb})] \overline{j} + [P_b (R_b d_{zb} - P_b d_{zb}) - Q_b (Q_b d_{zb} - R_b d_{yb})] \overline{k} \\
\overline{\Omega}_b \times \overline{V}_s = \\
\left| \begin{array}{ccc} \overline{i} & \overline{j} & \overline{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ U_b & V_b & W_b \end{array} \right| \\
= (Q_b W_b - R_b V_b) \overline{i} - (P_b W_b - R_b U_b) \overline{j} + (P_b V_b - Q_b U_b) \overline{k} \\
\dot{\overline{\omega}}_b \times (-\overline{D}_w) = \\
\left| \begin{array}{ccc} \overline{i} & \overline{j} & \overline{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ -D_{zw} & -D_{yw} & -D_{zw} \end{array} \right| \\
= (\dot{Q}_b D_{zw} + \dot{R}_b D_{yw}) \overline{i} - (\dot{P}_b D_{zw} + \dot{R}_b D_{zw}) \overline{j} + (\dot{P}_b D_{yw} + \dot{Q}_b D_{zw}) \overline{k} \\
\overline{\Omega}_b \times (\overline{\Omega}_b \times (-\overline{D}_w)) = \\
\left| \begin{array}{ccc} \overline{i} & \overline{j} & \overline{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ Q_b d_{zb} + R_b d_{yw} & P_b d_{zw} - R_b D_{zw} & P_b D_{yw} + Q_b D_{zw} \end{array} \right| \\
= [Q_b (P_b D_{yw} + Q_b D_{zw}) - R_b (P_b D_{zw} - R_b D_{zw})] \overline{i} - [P_b (P_b D_{yw} + Q_b D_{zw}) - R_b (Q_b D_{zw} + R_b D_{yw})] \overline{j} + [P_b (P_b D_{zw} - R_b D_{zw}) - Q_b (Q_b D_{zw} + R_b D_{yw})] \overline{k}
\[
2 \overline{\Omega}_b \times (\overline{\Omega}_c \times (-\overline{D}_w)) = \begin{vmatrix}
i & j & k \\
2P_b & 2Q_b & 2R_b \\
0 & P_w D_{zw} & -P_w D_{yw} \\
\end{vmatrix} \\
= (-2Q_b P_w D_{yw} - 2R_b P_w D_{zw}) \hat{i} + 2P_b P_w D_{yw} \hat{j} + 2P_b P_w D_{zw} \hat{k}
\]

\[
\hat{\Omega}_c \times (-\overline{D}_w) = \begin{vmatrix}
i & j & k \\
\dot{P}_w & 0 & 0 \\
-D_{zw} & -D_{yw} & -D_{zw} \\
\end{vmatrix} \\
= \dot{P}_w D_{zw} \hat{j} - \dot{P}_w D_{yw} \hat{k}
\]

\[
\overline{\Omega}_c \times (\overline{\Omega}_c \times (-\overline{D}_w)) = \begin{vmatrix}
i & j & k \\
P_w & 0 & 0 \\
0 & P_w D_{zw} & -P_w D_{yw} \\
\end{vmatrix} \\
= P_w^2 D_{yw} \hat{j} + P_w^2 D_{zw} \hat{k}
\]

\[
\overline{A}_c = [\dot{U}_b + \dot{Q}_b d_{zb} - \dot{R}_b d_{yb} + \dot{Q}_b (P_b d_{yb} - Q_b d_{zb}) - \dot{R}_b (R_b d_{zb} - P_b d_{zb}) + Q_b W_b - R_b V_b - \dot{Q}_b D_{zw} + \dot{R}_b D_{yw} + Q_b (-P_b D_{yw} + Q_b D_{zw}) - R_b (P_b D_{yw} - R_b D_{zw}) - 2Q_b P_w D_{yw} - 2R_b P_w D_{zw} \hat{j}]
\]

\[
+ [\dot{V}_b - (\dot{P}_b d_{zy} - \dot{R}_b d_{zb}) - \dot{P}_b (P_b d_{yb} - Q_b d_{zb}) + \dot{R}_b (Q_b d_{zb} - R_b d_{yb}) - (P_b W_b - R_b U_b) - (\dot{P}_b D_{zw} + \dot{R}_b D_{zw}) - P_b (-P_b D_{yw} + Q_b D_{zw}) + R_b (-Q_b D_{zw} + R_b D_{yw}) + 2P_b P_w D_{yw} + \dot{P}_w D_{zw} + P_w^2 D_{yw} \hat{j}]
\]

\[
+ [\dot{W}_b + \dot{P}_b d_{zy} - \dot{Q}_b d_{zb} + P_b (R_b d_{zb} - P_b d_{zb}) - Q_b (Q_b d_{zb} - R_b d_{yb}) + P_b V_b - Q_b U_b - \dot{P}_b D_{yw} + \dot{Q}_b D_{zw} + P_b (P_b D_{zw} - R_b D_{zw}) - Q_b (-Q_b D_{zw} + R_b D_{yw}) + 2P_b P_w D_{zw} - \dot{P}_w D_{yw} + P_w^2 D_{zw} \hat{k}]
\]

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port wing accelerations:

\[
\dot{U}_{w1} = \dot{U}_b + \dot{Q}_b (d_{zbl} - D_{zw1}) + \ddot{R}_b (D_{yw1} - d_{ybl}) + Q_b^2 (D_{zw1} - d_{zbl}) + R_b (D_{zw1} - d_{zbl}) + P_b Q_b (d_{zbl} - D_{yw1}) + P_b R_b (d_{zbl} - D_{zw1}) + Q_b W_b - R_b V_b - 2P_{w1} (Q_b D_{yw1} + R_b D_{zw1})
\]

\[
\dot{V}_{w1} = \dot{V}_b + \dot{P}_b (D_{zw1} - d_{zbl}) + \ddot{R}_b (d_{zbl} - D_{zw1}) + P_b^2 (D_{yw1} - d_{ybl}) + R_b (D_{yw1} - d_{ybl}) + P_b Q_b (d_{zbl} - D_{zw1}) + Q_b R_b (d_{zbl} - D_{zw1}) - P_b W_b + R_b U_b + 2P_b P_{w1} D_{yw1} + \dot{P}_w D_{zw1} + P_{w1}^2 D_{yw1}
\]

\[
\dot{W}_{w1} = \dot{W}_b + \dot{P}_b (d_{ybl} - D_{yw1}) + \ddot{Q}_b (D_{zw1} - d_{zbl}) + P_b^2 (D_{zw1} - d_{zbl}) + Q_b (D_{zw1} - d_{zbl}) + P_b Q_b (d_{zbl} - D_{zw1}) + Q_b R_b (d_{ybl} - D_{yw1}) + P_b V_b - Q_b U_b + 2P_b P_{w1} D_{zw1} - \dot{P}_w D_{yw1} + P_{w1}^2 D_{zw1}
\]

starboard wing accelerations:

\[
\dot{U}_{w2} = \dot{U}_b + \dot{Q}_b (d_{zbl} - D_{zw2}) + \ddot{R}_b (D_{yw2} - d_{ybl}) + Q_b^2 (D_{zw2} - d_{zbl}) + R_b (D_{zw2} - d_{zbl}) + P_b Q_b (d_{ybl} - D_{yw2}) + P_b R_b (d_{zbl} - D_{zw2}) + Q_b W_b - R_b V_b - 2P_{w2} (Q_b D_{yw2} + R_b D_{zw2})
\]

\[
\dot{V}_{w2} = \dot{V}_b + \dot{P}_b (D_{zw2} - d_{zbl}) + \ddot{R}_b (d_{zbl} - D_{zw2}) + P_b^2 (D_{yw2} - d_{ybl}) + R_b (D_{yw2} - d_{ybl}) + P_b Q_b (d_{zbl} - D_{zw2}) + Q_b R_b (d_{zbl} - D_{zw2}) - P_b W_b + R_b U_b + 2P_b P_{w2} D_{yw2} + \dot{P}_w D_{zw2} + P_{w2}^2 D_{yw2}
\]

\[
\dot{W}_{w2} = \dot{W}_b + \dot{P}_b (d_{ybl} - D_{yw2}) + \ddot{Q}_b (D_{zw2} - d_{zbl}) + P_b^2 (D_{zw2} - d_{zbl}) + Q_b (D_{zw2} - d_{zbl}) + P_b Q_b (d_{ybl} - D_{yw2}) + Q_b R_b (d_{ybl} - D_{yw2}) + P_b V_b - Q_b U_b + 2P_b P_{w2} D_{zw2} - \dot{P}_w D_{yw2} + P_{w2}^2 D_{zw2}
\]

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The following substitutions for the reaction forces and moments can be made:

\[
\begin{align*}
R_{xw1} &= -R_{xb1} & R_{xw2} &= -R_{xb2} \\
R_{yw1} &= -R_{yb1} & R_{yw2} &= -R_{yb2} \\
R_{zw1} &= -R_{zb1} & R_{zw2} &= -R_{zb2} \\
M_{Lw1} &= -M_{Lb1} & M_{Lw2} &= -M_{Lb2} \\
M_{Mw1} &= -M_{Mb1} & M_{Mw2} &= -M_{Mb2} \\
M_{Nw1} &= -M_{Nb1} & M_{Nw2} &= -M_{Nb2}
\end{align*}
\]

The geometric properties of the two wing halves are identical, which means

\[m_{w1} = m_{w2} = m_w\]

Also, since the wing is attached rigidly to the body,

\[
\begin{align*}
\Theta_{w1} &= \Theta_{w2} = \Theta_b = \Theta \\
Q_{w1} &= Q_{w2} = Q_b \\
R_{w1} &= P_{w2} = R_b \\
\Phi_{w1} &= \Phi \\
\Phi_{w2} &= \Phi \\
P_{\text{wing1}} &= P_b + P_{w1} \\
P_{\text{wing2}} &= P_b + P_{w2}
\end{align*}
\]

Substitute the above relationships along with the kinematic equations into the equations of motion for the wing.
For the port wing equations of motion

\[
X_{\text{wing}} - m_w g \sin \Theta - R_{zbl} = m_w (\ddot{U}_{w_l} + Q_{w_l} W_{w_l} - R_{w_l} V_{w_l})
\]

\[
= m_w \left( \dot{U}_b + \dot{Q}_b (d_{zbl} - D_{zw1}) + \dot{R}_b (D_{yw1} - d_{ybl}) + Q_b^2 (D_{zw1} - d_{zbl}) + R_b^2 (D_{zw1} - d_{zbl}) + P_b Q_b (d_{zbl} - D_{zw1}) + P_b R_b (d_{zbl} - D_{zw1}) + Q_b W_b - R_b V_b - 2 P_{w_l} (Q_b D_{yw1} + R_b D_{zw1}) + Q_b W_b + P_b Q_b (d_{ybl} - D_{yw1}) + Q_b^2 (D_{zw1} - d_{zbl}) - P_{w_l} Q_b D_{yw1}
\]

\[-R_b V_b - P_b R_b (D_{zw1} - d_{zbl}) - R_b^2 (d_{zbl} - D_{zw1}) - P_{w_l} R_b D_{zw1} \]

\[
Y_{\text{wing}} + m_w g \cos \Theta \sin \Phi - R_{ybl} = m_w (\dot{V}_{w_l} + R_{w_l} U_{w_l} - P_{\text{wing}} W_{w_l})
\]

\[
= m_w \left( \dot{V}_b + \dot{P}_b (D_{zw1} - d_{zbl}) + \dot{R}_b (d_{zbl} - D_{zw1}) + P_b^2 (D_{yw1} - d_{ybl}) + R_b^2 (D_{yw1} - d_{ybl}) + P_b Q_b (d_{zbl} - D_{zw1}) + Q_b R_b (d_{zbl} - D_{zw1}) - P_b W_b + R_b U_b + 2 P_b P_{w_l} D_{yw1}
\]

\[+P_{w_l} D_{zw1} + P_{w_l}^2 D_{yw1} + R_b U_b + Q_b R_b (d_{zbl} - D_{zw1}) + R_b^2 (D_{yw1} - d_{ybl}) - P_b W_b - P_b^2 (d_{ybl} - D_{yw1}) - P_b Q_b (D_{zw1} - d_{zbl}) + P_b P_{w_l} D_{yw1}
\]

\[+P_{w_l} U_b - P_{w_l} P_b (d_{ybl} - D_{yw1}) - P_{w_l} Q_b (D_{zw1} - d_{zbl}) + P_{w_l}^2 D_{yw1} \]

\[
Z_{\text{wing}} + m_w g \cos \Theta \cos \Phi - R_{zbl} = m_w (\dot{W}_{w_l} + P_{\text{wing}} V_{w_l} - Q_{w_l} U_{w_l})
\]

\[
= m_w \left( \dot{W}_b + \dot{P}_b (d_{ybl} - D_{yw1}) + \dot{Q}_b (D_{zw1} - d_{zbl}) + P_b^2 (D_{zw1} - d_{zbl}) + Q_b^2 (D_{zw1} - d_{zbl}) + P_b R_b (d_{ybl} - D_{yw1}) + Q_b V_b - Q_b U_b + 2 P_b P_{w_l} D_{zw1}
\]

\[+P_{w_l} D_{yw1} + P_{w_l} D_{zw1} + P_b V_b + P_b^2 (D_{zw1} - d_{zbl}) + P_b R_b (d_{ybl} - D_{yw1}) + P_b P_{w_l} D_{zw1}
\]

\[+P_{w_l} V_b + P_{w_l} P_b (D_{zw1} - d_{zbl}) + P_{w_l} R_b (d_{ybl} - D_{yw1}) + P_{w_l}^2 D_{zw1} - Q_b U_b - Q_b^2 (d_{zbl} - D_{zw1}) - Q_b R_b (D_{yw1} - d_{ybl}) \]
Simplifying these equations yields:

\[
X_{\text{aero}_{bl}} - m_w g \sin \Theta - R_{zb1} = m_w [\ddot{U}_b + \dot{Q}_b (d_{zb1} - D_{zw1}) + \ddot{R}_b (D_{yw1} - d_{yb1}) + 2Q_b^2 (D_{yw1} - d_{yb1}) + 2R_b^2 (D_{yw1} - d_{yb1})
+ 2P_b Q_b (d_{yb1} - D_{yw1}) + 2P_b R_b (d_{zb1} - D_{zw1}) + 2Q_b W_b - 2R_b V_b - 3P_w (Q_b D_{yw1} + R_b D_{zw1})]
\]

\[
Y_{\text{aero}_{bl}} + m_w g \cos \Theta \sin \Phi - R_{yb1} = m_w [\ddot{V}_b + \dot{P}_b (D_{zw1} - d_{zb1}) + \ddot{R}_b (d_{zb1} - D_{zw1}) + 2P_b^2 (D_{zw1} - d_{zb1}) + 2R_b^2 (D_{yw1} - d_{yb1})
+ 2P_b Q_b (d_{zb1} - D_{zw1}) + 2Q_b R_b (d_{zb1} - D_{zw1}) - 2P_b W_b + 2R_b U_b - P_w P_b (d_{yb1} - 4D_{yw1})
+ \dot{P}_w (D_{zw1} + 2P_w^2 D_{yw1} - P_w W_b - P_w Q_b (D_{zw1} - d_{zb1})]
\]

\[
Z_{\text{aero}_{bl}} + m_w g \cos \Theta \cos \Phi - R_{zb1} = m_w [\ddot{W}_b + \dot{P}_b (d_{yb1} - D_{yw1}) + \dot{Q}_b (D_{zw1} - d_{zb1}) + 2P_b^2 (D_{zw1} - d_{zb1}) + 2Q_b^2 (D_{zw1} - d_{zb1})
+ 2P_b R_b (d_{zb1} - D_{zw1}) + 2Q_b R_b (d_{yb1} - D_{yw1}) + 2P_b V_b - 2Q_b U_b + P_w P_b (4D_{zw1} - d_{zb1})
- \dot{P}_w D_{yw1} + 2P_w^2 D_{zw1} + P_w W_b + P_w R_b (d_{zb1} - D_{zw1})]
\]

For the starboard wing equations of motion:

\[
X_{\text{aero}_{s}} - m_w g \sin \Theta - R_{sb2} = m_w (\ddot{U}_{s2} + Q_{s2} W_{s2} - R_{s2} V_{s2})
= m_w [\ddot{U}_b + \dot{Q}_b (d_{sb2} - D_{zw2}) + \ddot{R}_b (D_{yw2} - d_{yb2}) + Q_b^2 (D_{sw2} - d_{sb2}) + R_b^2 (D_{zw2} - d_{zb2})
+ P_b Q_b (d_{yb2} - D_{yw2}) + P_b R_b (d_{zb2} - D_{zw2}) + Q_b W_b - R_b V_b - 2P_b (Q_b D_{yw2} + R_b D_{zw2})
+ Q_b W_b + P_b Q_b (d_{yb2} - D_{yw2}) + Q_b^2 (D_{sw2} - d_{sb2}) - P_w Q_b D_{yw2}
- R_b V_b - P_b R_b (D_{zw2} - d_{zb2}) - R_b^2 (d_{sb2} - D_{zw2}) - P_w^2 R_b D_{zw2}]
\]

\[
Y_{\text{aero}_{s}} + m_w g \cos \Theta \sin \Phi - R_{yb2} = m_w (\ddot{V}_{s2} + R_{s2} U_{s2} - P_{\text{wing}_2} W_{s2})
= m_w [\ddot{V}_b + \dot{P}_b (D_{zw2} - d_{zb2}) + \ddot{R}_b (d_{zb2} - D_{zw2}) + P_b^2 (D_{yw2} - d_{yb2}) + R_b^2 (D_{yw2} - d_{yb2})
+ P_b Q_b (d_{zb2} - D_{zw2}) + Q_b R_b (d_{zb2} - D_{zw2}) - P_b W_b + R_b U_b + 2P_b P_{s2} D_{yw2}
+ \dot{P}_s^2 D_{zw2} + P_{s2}^2 D_{yw2} + R_b U_b + Q_b R_b (d_{zb2} - D_{zw2}) + R_b^2 (D_{yw2} - d_{yb2})
- P_b W_b - P_b^2 (d_{yb2} - D_{yw2}) - P_w Q_b (D_{zw2} - d_{zb2}) + P_b P_{s2} D_{yw2}
- P_{s2} W_b - P_{w2} P_b (d_{yb2} - D_{yw2}) - P_{w2} Q_b (D_{zw2} - d_{zb2}) + P_{w2}^2 D_{yw2}]
\]

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Z_{\text{w}_{\text{zw}2}} + m_w g \cos \Theta \cos \Phi - R_{zb2} = m_w (\dot{W}_{w2} + P_{\text{wing2}} V_{w2} - Q_{w2} U_{w2})
= m_w (\dot{W}_b + \dot{P}_b (d_{yb2} - D_{yw2}) + \dot{Q}_b (D_{xw2} - d_{zb2}) + P_b^2 (D_{zw2} - d_{zb2}) + Q_b^2 (D_{zw2} - d_{zb2})
+ P_b R_b (d_{zb2} - D_{zw2}) + Q_b R_b (d_{yb2} - D_{yw2}) + P_b V_b - Q_b U_b + 2P_b P_{w2} D_{zw2}
- \dot{P}_{w2} D_{yw2} + P_{w2}^2 D_{zw2} + P_b V_b + P_b^2 (D_{xw2} - d_{zb2}) + P_b R_b (d_{zb2} - D_{zw2}) + P_b P_{w2} D_{zw2}
+ P_{w1} V_b + P_{w2} P_b (D_{zw2} - d_{zb2}) + P_{w2} R_b (d_{zb2} - D_{zw2}) + P_{w2}^2 D_{zw2}
- Q_b U_b - Q_b^2 (d_{zb2} - D_{zw2}) - Q_b R_b (D_{yw2} - d_{yb2})]

Simplifying the equations yields:

X_{\text{w}_{\text{zw}2}} = -m_w g \sin \Theta - R_{zb2}
= m_w (\dot{U}_b + \dot{Q}_b (d_{zb2} - D_{zw2}) + \ddot{Q}_b - \dot{d}_{zb2}) + 2Q_b^2 (D_{xw2} - d_{zb2}) + 2R_b^2 (D_{zw2} - d_{zb2})
+ 2P_b Q_b (d_{yb2} - D_{yw2}) + 2P_b R_b (d_{zb2} - D_{zw2}) + 2Q_b W_b - 2R_b V_b - 3P_{w2} (Q_b D_{yw2} + R_b D_{zw2})]

Y_{\text{w}_{\text{zw}2}} = m_w g \cos \Theta \sin \Phi - R_{yb2}
= m_w (\dot{V}_b + \dot{P}_b (D_{zw2} - d_{zb2}) + \ddot{P}_b - \dot{d}_{yb2}) + 2P_b^2 (D_{yw2} - d_{yb2}) + 2R_b^2 (D_{yw2} - d_{yb2})
+ 2P_b Q_b (d_{zb2} - D_{zw2}) + 2Q_b R_b (d_{zb2} - D_{zw2}) + 2P_b W_b - 2R_b V_b + P_{w2} P_b (d_{yb2} - D_{yw2})
+ 2P_{w2} D_{yw2} + 2P_{w2}^2 D_{zw2} - P_{w2} W_b - P_{w2} Q_b (D_{zw2} - d_{zb2})]

Z_{\text{w}_{\text{zw}2}} = m_w g \cos \Theta \cos \Phi - R_{ab2}
= m_w (\dot{W}_b + \dot{P}_b (d_{yb2} - D_{yw2}) + \dot{Q}_b (D_{xw2} - d_{zb2}) + 2P_b^2 (D_{zw2} - d_{zb2}) + 2Q_b^2 (D_{zw2} - d_{xb2})
+ 2P_b R_b (d_{xb2} - D_{zw2}) + 2Q_b R_b (d_{xb2} - D_{yw2}) + 2P_b V_b - 2Q_b U_b + P_{w2} P_b (4D_{zw2} - d_{zb2})
- \dot{P}_{w2} D_{yw2} + 2P_{w2}^2 D_{zw2} + P_{w2} V_b + P_{w2} R_b (d_{xb2} - D_{zw2})]
Summary - Equations of Motion

body equations:

\[ X_{\text{aero}} = m_b g \sin \Theta_b + R_{xbl} + R_{xb2} = m_b (\dot{U}_b + Q_b W_b - R_b V_b) \]
\[ Y_{\text{aero}} = m_b g \cos \Theta_b \sin \Phi_b + R_{ybl} + R_{yb2} = m_b (\dot{V}_b + R_b U_b - P_b W_b) \]
\[ Z_{\text{aero}} = m_b g \cos \Theta_b \cos \Phi_b + R_{zbl} + R_{zb2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b) \]
\[ L_{\text{aero}} = \dot{d}_{zbl} R_{ybl} + d_{ybl} R_{zbl} - d_{zb2} R_{yb2} + d_{yb2} R_{zb2} + M_{lb1} + M_{lb2} \]
\[ = I_{xxb} \dot{P}_b - I_{xzb} \dot{R}_b + (I_{xb} - I_{yyb}) Q_b R_b - I_{xzb} P_b Q_b \]
\[ M_{\text{aero}} = d_{zbl} R_{xbl} - d_{xbl} R_{zbl} + d_{zb2} R_{xb2} - d_{xb2} R_{zb2} + M_{mb1} + M_{mb2} \]
\[ = I_{yyb} \dot{Q}_b + (I_{xb} - I_{zzb}) P_b R_b + I_{xzb} (P_b^2 - R_b^2) \]
\[ N_{\text{aero}} = -d_{ybl} R_{xbl} + d_{xbl} R_{ybl} - d_{yb2} R_{xb2} + d_{xb2} R_{yb2} + M_{nb1} + M_{nb2} \]
\[ = I_{zzb} \dot{R}_b - I_{xzb} \dot{P}_b + (I_{yyb} - I_{xxb}) P_b Q_b + I_{xzb} Q_b R_b \]

port wing equations:

\[ X_{\text{aero,wi}} = m_w g \sin \Theta - R_{xbl} \]
\[ = m_w (\dot{U}_b + \dot{Q}_b (d_{zbl} - D_{zwl}) + \dot{R}_b (D_{ywl} - d_{ybl}) + 2Q_b^2 (D_{xwl} - d_{xbl}) + 2R_b^2 (D_{xwl} - d_{xbl}) + 2P_b (d_{ybl} - D_{ywl}) + 2Q_b W_b - 2R_b V_b - 3P_b Q_b (D_{xwl} - d_{xbl}) \]

\[ Y_{\text{aero,wi}} = m_w g \cos \Theta \sin \Phi - R_{ybl} \]
\[ = m_w (\dot{V}_b + \dot{P}_b (D_{zwl} - d_{zbl}) + \dot{R}_b (d_{xbl} - D_{xwl}) + 2P_b^2 (D_{xwl} - d_{xbl}) + 2R_b^2 (D_{xwl} - d_{xbl}) + 2P_b Q_b (d_{xbl} - D_{xwl}) + 2Q_b W_b + 2R_b U_b - P_w L_b P_b (d_{ybl} - 4D_{ywl}) \]
\[ + \dot{P}_w D_{zwl} + 2P_w D_{ywl} - P_w W_b - P_w Q_b (D_{xwl} - d_{xbl}) \]

\[ Z_{\text{aero,wi}} = m_w g \cos \Theta \cos \Phi - R_{zbl} \]
\[ = m_w (\dot{W}_b + \dot{P}_b (d_{ybl} - D_{ywl}) + \dot{Q}_b (D_{xwl} - d_{xbl}) + 2P_b^2 (D_{ywl} - d_{ybl}) + 2Q_b^2 (D_{zwl} - d_{zbl}) + 2P_b R_b (d_{zbl} - D_{zwl}) + 2Q_b V_b - 2Q_b U_b + P_w L_b P_b (4D_{zwl} - d_{zbl}) \]
\[ - \dot{P}_w D_{ywl} + 2P_w^2 D_{zwl} + P_w W_b + P_w R_b (d_{ybl} - D_{ywl}) \]
\[ L_{aero} + D_{zw1} R_{y1} - D_{yw1} R_{z1} - M_{lb1} = I_{xxw} \dot{P}_b + I_{xxw} \dot{P}_{w2} + (I_{zzw} - I_{yyw}) Q_b R_b + I_{yzw} (R_b^2 - Q_b^2) \]

\[ M_{aero} - D_{zw1} R_{xb1} + D_{xxw} R_{z1} - M_{mb1} = I_{yyw} \dot{Q}_b + \dot{I}_{yyw} Q_b - I_{yzw} \dot{R}_b - \dot{I}_{yzw} R_b + (I_{xxw} - I_{zzw}) (P_b + P_{w2}) R_b + I_{yzw} (P_b + P_{w2}) Q_b \]

\[ N_{aero} + D_{yw1} R_{xb1} - D_{xxw} R_{y1} - M_{nb1} = I_{zzw} \dot{R}_b + \dot{I}_{zzw} R_b - I_{yzw} \dot{Q}_b - \dot{I}_{yzw} Q_b - I_{yzw} (P_b + P_{w2}) R_b + (I_{yyw} - I_{xxw}) (P_b + P_{w2}) Q_b \]

starboard wing equations:

\[ X_{aero} - m_w g \sin \theta - R_{xb2} = m_w (\ddot{U}_b + \dot{Q}_b (d_{xb2} - D_{zw2}) + \dot{R}_b (D_{yw2} - d_{y2}) + 2 Q_b^2 (D_{xz2} - d_{z2}) + 2 R_b^2 (D_{xz2} - d_{z2}) + 2 P_b Q_b (d_{xb2} - D_{zw2}) + 2 P_b R_b (d_{xb2} - D_{zw2}) + 2 Q_b W_b - 2 R_b V_b - 3 P_{w2} (Q_b D_{yw2} + R_b D_{zw2})] \]

\[ Y_{aero} + m_w g \cos \theta \sin \phi - R_{yb2} = m_w (\ddot{V}_b + \dot{P}_b (D_{zw2} - d_{y2}) + \dot{R}_b (d_{y2} - D_{zw2}) + 2 P_b^2 (D_{yw2} - d_{y2}) + 2 R_b^2 (D_{yw2} - d_{y2}) + 2 P_b Q_b (d_{y2} - D_{zw2}) + 2 Q_b R_b (d_{y2} - D_{zw2}) - 2 P_b W_b + 2 R_b U_b - P_{w2} P_b (d_{xb2} - 4 D_{yw2}) + \dot{P}_{w2} D_{zw2} + 2 P_{w2} D_{yw2} - P_{w2} W_b - P_{w2} Q_b (D_{zw2} - d_{z2})] \]

\[ Z_{aero} + m_w g \cos \theta \cos \phi - R_{zb2} = m_w (\ddot{W}_b + \dot{P}_b (d_{y2} - D_{yw2}) + \dot{Q}_b (D_{zw2} - d_{z2}) + 2 P_b^2 (D_{zw2} - d_{z2}) + 2 Q_b^2 (D_{zw2} - d_{z2}) + 2 P_b R_b (d_{xb2} - D_{zw2}) + 2 Q_b R_b (d_{xb2} - D_{zw2}) + 2 P_b V_b - 2 Q_b U_b + P_{w2} P_b (d_{zb2} - 4 D_{zw2}) - \dot{P}_{w2} D_{yw2} + 2 P_{w2} D_{zw2} - P_{w2} V_b + P_{w2} R_b (d_{xb2} - D_{zw2})] \]

\[ L_{aero} + D_{zw2} R_{yb2} - D_{yw2} R_{z2} - M_{lb2} = I_{xxw} \dot{P}_b + I_{xxw} \dot{P}_{w2} + (I_{zzw} - I_{yyw}) Q_b R_b + I_{yzw} (R_b^2 - Q_b^2) \]

\[ M_{aero} - D_{zw2} R_{xb2} + D_{xxw} R_{z2} - M_{mb2} = I_{yyw} \dot{Q}_b + \dot{I}_{yyw} Q_b - I_{yzw} \dot{R}_b - \dot{I}_{yzw} R_b + (I_{xxw} - I_{zzw}) (P_b + P_{w2}) R_b + I_{yzw} (P_b + P_{w2}) Q_b \]

\[ N_{aero} + D_{yw2} R_{xb2} - D_{xxw} R_{y2} - M_{nb2} = I_{zzw} \dot{R}_b + \dot{I}_{zzw} R_b - I_{yzw} \dot{Q}_b - \dot{I}_{yzw} Q_b - I_{yzw} (P_b + P_{w2}) R_b + (I_{yyw} - I_{xxw}) (P_b + P_{w2}) Q_b \]

Rearranging all the unknowns to one side gives:
\[ \dot{X}_{aero} - m_b g \sin \Theta - m_b (Q_b W_b - R_b V_b) = m_b \dot{U}_b - R_{xb1} - R_{xb2} \]
\[ \dot{Y}_{aero} + m_b g \cos \Theta \sin \Phi - m_b (R_b U_b - P_b W_b) = m_b \dot{V}_b - R_{ybl} - R_{ybl} \]
\[ \dot{Z}_{aero} + m_b g \cos \Theta \cos \Phi - m_b (P_y V_b - Q_b U_b) = m_b \dot{W}_b - R_{zbl} - R_{zbl} \]
\[ L_{aero} = - (I_{xxb} - I_{yyb}) Q_b R_b + I_{xzb} P_b Q_b \]
\[ = I_{xzb} \dot{R}_b - I_{xxb} \dot{Q}_b + d_{xb1} R_{ybl} - d_{ybl} R_{zbl} + d_{xbl} R_{ybl} + d_{ybl} R_{zbl} - M_{Lb1} - M_{Lb2} \]
\[ M_{aero} = - (I_{xxb} - I_{yyb}) P_b R_b + I_{xzb} (P_b^2 - Q_b^2) \]
\[ = I_{yyb} \dot{Q}_b - d_{xb1} R_{xbl} + d_{xbl} R_{xbl} - d_{xbl} R_{xbl} + d_{ybl} R_{ybl} - M_{Mbl} - M_{Mbl} \]
\[ N_{aero} = - (I_{yyb} - I_{xxb}) P_b Q_b - I_{xzb} Q_b R_b \]
\[ = - I_{xxb} \dot{P}_b + I_{xzb} \dot{R}_b + d_{ybl} R_{xbl} - d_{xbl} R_{ybl} + d_{ybl} R_{xbl} - d_{xbl} R_{ybl} - M_{Nbl} - M_{Nbl} \]
\[ X_{aero} = - m_w g \sin \Theta - m_w [2Q_b^2 (D_{xw} - d_{xbl}) + 2R_b^2 (D_{ybl} - d_{xbl})] \]
\[ + 2P_b Q_b (d_{ybl} - D_{yw}) + 2P_b R_b (d_{xbl} - D_{zw}) + 2Q_b W_b - 2R_b V_b \]
\[ - 3P_w (Q_b D_{zw1} + R_b D_{zw1}) \]
\[ = m_w \dot{U}_b + m_w (d_{xbl} - D_{zw1}) \dot{Q}_b + m_w (D_{yw} - d_{ybl}) \dot{R}_b + R_{xbl} \]
\[ Y_{aero} = m_w g \cos \Theta \sin \Phi - m_w [2P_b^2 (D_{yw} - d_{ybl}) + 2R_b^2 (D_{yw} - d_{ybl})] \]
\[ + 2P_b Q_b (d_{xbl} - D_{yw}) + 2Q_b R_b (d_{xbl} - D_{zw}) - 2P_b W_b + 2R_b U_b \]
\[ - P_w (d_{ybl} - D_{yw}) + \dot{P}_w D_{zw1} + P_w W_b - V_b \]
\[ - P_w (Q_b (D_{yw} - d_{ybl}) \]
\[ = m_w \dot{V}_b + m_w (d_{xbl} - D_{zw1}) \dot{P}_b + m_w (d_{xbl} - D_{zw1}) \dot{R}_b + R_{xbl} \]
\[ Z_{aero} = m_w g \cos \Theta \cos \Phi - m_w [2P_b^2 (D_{zw} - d_{xbl}) + 2Q_b^2 (D_{zw} - d_{xbl})] \]
\[ + 2P_b R_b (d_{xbl} - D_{zw}) + 2Q_b R_b (d_{ybl} - D_{yw}) + 2P_b V_b - 2Q_b U_b \]
\[ + P_w (4D_{zw} - d_{xbl}) - \dot{P}_w D_{yw} + 2P_w (D_{zw} + P_w V_b + P_w R_b (d_{xbl} - D_{zw1}) \]
\[ = m_w \dot{W}_b + m_w (d_{ybl} - D_{yw}) \dot{P}_b + m_w (D_{zw} - d_{xbl}) \dot{Q}_b + R_{zbl} \]
\[ L_{aero} = - I_{xxw} \dot{P}_w - (I_{xxw} - I_{yyw}) Q_b R_w - I_{yw} (R_w^2 - Q_w^2) \]
\[ = I_{xxw} \dot{P}_w - D_{zw1} R_{ybl} + D_{yw} R_{zbl} + M_{Lw1} \]
\[ M_{aero} = - I_{yyw} Q_w + I_{yw} R_w - (I_{xxw} - I_{yyw}) (P_w + P_w) R_w - I_{yw} (P_w + P_w) Q_b \]
\[ = I_{yyw} \dot{Q}_w - I_{yw} \dot{R}_w + D_{zw1} R_{xbl} - D_{yw} R_{zbl} + M_{Mw1} \]
\[ N_{aero} = - I_{xxw} R_w + I_{yw} Q_w + I_{yw} (P_w + P_w) R_w - (I_{yyw} - I_{xxw}) (P_w + P_w) Q_b \]
\[ = I_{xxw} \dot{R}_w - I_{yw} \dot{Q}_w - D_{yw} R_{xbl} - D_{zw1} R_{ybl} + M_{Nw1} \]
\[ X_{aero_{w}} = -m_{w}g \sin \Theta - m_{w}(2Q_{b}^{2}(D_{xw2} - d_{xb2}) + 2R_{b}^{2}(D_{xw2} - d_{xb2}) + 2P_{b}Q_{b}(d_{yb2} - D_{yw2}) + 2P_{b}R_{b}(d_{zb2} - D_{zw2}) + 2Q_{b}W_{b} - 2R_{b}V_{b} - 3P_{w2}(Q_{b}D_{yw2} + R_{b}D_{zw2})] \\
= m_{w}\dot{U}_{b} + m_{w}(d_{zb2} - D_{zw2})\dot{Q}_{b} + m_{w}(D_{yw2} - d_{yb2})\dot{R}_{b} + R_{zb2} \\
\]

\[ Y_{aero_{w}} = m_{w}g \cos \Theta \sin \Phi - m_{w}[2P_{b}^{2}(D_{xw2} - d_{xb2}) + 2R_{b}^{2}(D_{yw2} - d_{yb2}) + 2P_{b}Q_{b}(d_{xb2} - D_{xw2}) + 2P_{b}R_{b} + 2Q_{b}W_{b} + 2R_{b}U_{b} - P_{w2}(d_{yb2} - 4D_{yw2}) + \dot{P}_{w2}D_{zw2} + 2P_{w2}^{2}(D_{yw2} - P_{w2}W_{b} - P_{w2}Q_{b}(D_{xw2} - d_{xb2})] \\
= m_{w}\dot{V}_{b} + m_{w}(D_{xw2} - d_{zb2})\dot{P}_{b} + m_{w}(d_{xb2} - D_{xw2})\dot{R}_{b} + R_{yb2} \\
\]

\[ Z_{aero_{w}} = m_{w}g \cos \Theta \cos \Phi - m_{w}[2P_{b}^{2}(D_{xw2} - d_{xb2}) + 2Q_{b}^{2}(D_{zw2} - d_{zb2}) + 2P_{b}Q_{b}(d_{yb2} - D_{yw2}) + 2P_{b}V_{b} + 2Q_{b}U_{b} + P_{w1}(4D_{xw2} - d_{zb2}) - \dot{P}_{w2}D_{yw2} + 2P_{w2}^{2}(D_{zw2} + P_{w2}V_{b} + P_{w2}R_{b}(d_{xb2} - D_{xw2})] \\
= m_{w}\dot{W}_{b} + m_{w}(d_{yb2} - D_{yw2})\dot{P}_{b} + m_{w}(D_{xw2} - d_{zb2})\dot{Q}_{b} + R_{zb2} \\
\]

\[ L_{aero_{w}} = I_{xxw2}\dot{P}_{w2} - (I_{xxw2} - I_{yyw2})Q_{b}R_{b} - I_{yww2}(R_{b}^{2} - Q_{b}^{2}) \\
= I_{xxw2}\dot{P}_{b} - D_{zw2}R_{yb2} + D_{yw2}R_{zb2} + M_{Lb2} \\
\]

\[ M_{aero_{w}} = I_{yyw2}\dot{Q}_{b} - I_{yyw2}(P_{b} + P_{w2})R_{b} - I_{yww2}(P_{b} + P_{w2})Q_{b} \\
= I_{yyw2}\dot{Q}_{b} - I_{yww2}\dot{R}_{b} + D_{zw2}R_{xb2} - D_{yw2}R_{zb2} + M_{Mb2} \\
\]

\[ N_{aero_{w}} = I_{zzw2}\dot{R}_{b} + I_{zzw2}Q_{b} + I_{yww2}(P_{b} + P_{w2})R_{b} - (I_{yyw2} - I_{xxw2})(P_{b} + P_{w2})Q_{b} \\
= I_{zzw2}\dot{R}_{b} - I_{yww2}\dot{Q}_{b} - D_{yw2}R_{xb2} + D_{yw2}R_{yb2} + M_{Nb2} \\
\]
In matrix form:

\[ [A][x] = [B] \]

\[
[A] = \begin{bmatrix}
  m_x & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
  0 & m_y & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
  0 & 0 & m_z & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
  0 & 0 & 0 & I_{eb} & 0 & -I_{eb} & 0 & d_{eb} & -d_{eb} & -1 & 0 & 0 & 0 & d_{eb} & -d_{eb} & -1 & 0 \\
  0 & 0 & 0 & I_{yp} & 0 & -d_{yp} & 0 & d_{yp} & 0 & -1 & 0 & -d_{yp} & 0 & d_{yp} & 0 & -1 & 0 \\
  0 & 0 & 0 & I_{eb} & 0 & I_{eb} & 0 & d_{eb} & -d_{eb} & 0 & 0 & -1 & d_{eb} & -d_{eb} & 0 & 0 & -1 & 0 \\
  m_x & 0 & 0 & 0 & m_x (d_{eb} - D_{eb}) & m_x (D_{eb} - d_{eb}) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & m_y & 0 & m_y (D_{eb} - d_{eb}) & 0 & m_y (d_{eb} - D_{eb}) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & m_z & 0 & m_z (d_{eb} - D_{eb}) & m_z (D_{eb} - d_{eb}) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & I_{r1} & 0 & -I_{r1} & 0 & d_{r1} & -d_{r1} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & -I_{r1} & 0 & I_{r1} & 0 & -d_{r1} & d_{r1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  m_x & 0 & 0 & 0 & m_x (d_{eb} - D_{eb}) & m_x (D_{eb} - d_{eb}) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & m_y & 0 & m_y (D_{eb} - d_{eb}) & 0 & m_y (d_{eb} - D_{eb}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & m_z & 0 & m_z (d_{eb} - D_{eb}) & m_z (D_{eb} - d_{eb}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & I_{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -D_{m1} & D_{m1} & 1 & 0 & 0 \\
  0 & 0 & 0 & I_{m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{m2} & 0 & -D_{m2} & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & I_{m1} & I_{m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
[x] = \begin{bmatrix}
  \dot{U}_b \\
  \dot{V}_b \\
  \dot{W}_b \\
  \dot{P}_b \\
  \dot{Q}_b \\
  \dot{R}_b \\
  \dot{R}_{zb1} \\
  \dot{R}_{yb1} \\
  \dot{R}_{zb1} \\
  M_{Lb1} \\
  M_{Mbl} \\
  M_{Nb1} \\
  R_{xb2} \\
  R_{yb2} \\
  R_{zb2} \\
  M_{Lb2} \\
  M_{Mb2} \\
  M_{Nb2}
\end{bmatrix}
\]
\[
\begin{align*}
X_{\text{aero}} &= -m_b g \sin \Theta - m_b (Q_b W_b - R_b V_b) \\
Y_{\text{aero}} &= m_b g \cos \Theta \sin \Phi - m_b (R_b U_b - P_b W_b) \\
Z_{\text{aero}} &= m_b g \cos \Theta \cos \Phi - m_b (P_b V_b - Q_b U_b) \\
L_{\text{aero}} &= -(I_{zzb} - I_{yyb}) Q_b R_b + I_{zzb} P_b Q_b \\
M_{\text{aero}} &= -(I_{zzb} - I_{zzb}) P_b R_b - I_{zzb} (P_b^2 - R_b^2) \\
N_{\text{aero}} &= -(I_{yyb} - I_{zzb}) P_b Q_b - I_{zzb} Q_b R_b \\
X_{\text{aero}} &= m_w g \sin \Theta - m_w [2Q_w^2 (D_{xw1} - d_{xbl}) + 2R_w^2 (D_{xw1} - d_{xbl}) \\
&+ 2P_b Q_b (d_{ybl} - D_{yw1}) + 2P_b R_b (d_{zbl} - D_{zw1}) + 2Q_b W_b - 2R_b V_b \\
&- 3P_w (Q_b D_{yw1} + R_b D_{zw1})] \\
Y_{\text{aero}} &= m_w g \cos \Theta \sin \Phi - m_w [2P_w^2 (D_{yw1} - d_{ybl}) + 2R_w^2 (D_{yw1} - d_{ybl}) \\
&+ 2P_b Q_b (d_{xbl} - D_{xw1}) + 2Q_b R_b (d_{ybl} - D_{yw1}) - 2P_b W_b + 2R_b U_b \\
&- P_w Q_b (d_{ybl} - 4D_{yw1}) + \dot{P}_w D_{zw1} + 2P_w^2 D_{yw1} - P_w W_b - P_w Q_b (D_{xw1} - d_{xbl})] \\
Z_{\text{aero}} &= m_w g \cos \Theta \cos \Phi - m_w [2P_w^2 (D_{xw1} - d_{zbl}) + 2Q_w^2 (D_{xw1} - d_{zbl}) \\
&+ 2P_b R_b (d_{xbl} - D_{xw1}) + 2Q_b R_b (d_{ybl} - D_{yw1}) + 2P_b V_b - 2Q_b U_b \\
&+ P_w Q_p (4D_{xw1} - d_{zbl}) + \dot{P}_w D_{yw1} + 2P_w^2 D_{zw1} + P_w V_b + P_w Q_b (d_{zbl} - D_{zw1})] \\
L_{\text{aero}} &= -I_{zzw} \dot{P}_w - (I_{zzw} - I_{yyw}) Q_b R_b - I_{zzw} (R_b^2 - Q_b^2) \\
M_{\text{aero}} &= -I_{yyw} Q_b + I_{yyw} R_b - (I_{zzw} - I_{zzw}) (P_b + P_w) R_b - I_{yyw} (P_b + P_w) Q_b \\
N_{\text{aero}} &= -I_{zzw} R_b + I_{yyw} Q_b + I_{yyw} (P_b + P_w) R_b - (I_{yyw} - I_{zzw}) (P_b + P_w) Q_b \\
X_{\text{aero}} &= m_w g \sin \Theta - m_w [2Q_w^2 (D_{xw2} - d_{xbl}) + 2R_w^2 (D_{xw2} - d_{xbl}) \\
&+ 2P_b Q_b (d_{ybl} - D_{yw2}) + 2P_b R_b (d_{zbl} - D_{zw2}) + 2Q_b W_b - 2R_b V_b \\
&- 3P_w (Q_b D_{yw2} + R_b D_{zw2})] \\
Y_{\text{aero}} &= m_w g \cos \Theta \sin \Phi - m_w [2P_w^2 (D_{yw2} - d_{ybl}) + 2R_w^2 (D_{yw2} - d_{ybl}) \\
&+ 2P_b Q_b (d_{xbl} - D_{xw2}) + 2Q_b R_b (d_{zbl} - D_{zw2}) - 2P_b W_b + 2R_b U_b \\
&- P_w Q_b (d_{ybl} - 4D_{yw2}) + \dot{P}_w D_{zw2} + 2P_w^2 D_{yw2} - P_w W_b - P_w Q_b (D_{xw2} - d_{xbl})] \\
Z_{\text{aero}} &= m_w g \cos \Theta \cos \Phi - m_w [2P_w^2 (D_{xw2} - d_{zbl}) + 2Q_w^2 (D_{xw2} - d_{zbl}) \\
&+ 2P_b R_b (d_{xbl} - D_{xw2}) + 2Q_b R_b (d_{ybl} - D_{yw2}) + 2P_b V_b - 2Q_b U_b \\
&+ P_w Q_p (4D_{xw2} - d_{zbl}) - \dot{P}_w D_{yw2} + 2P_w^2 D_{zw2} + P_w V_b + P_w Q_b (d_{zbl} - D_{zw2})] \\
L_{\text{aero}} &= -I_{zzw} \dot{P}_w - (I_{zzw} - I_{yyw}) Q_b R_b - I_{zzw} (R_b^2 - Q_b^2) \\
M_{\text{aero}} &= -I_{yyw} Q_b + I_{yyw} R_b - (I_{zzw} - I_{zzw}) (P_b + P_w) R_b - I_{yyw} (P_b + P_w) Q_b \\
N_{\text{aero}} &= -I_{zzw} R_b + I_{yyw} Q_b + I_{yyw} (P_b + P_w) R_b - (I_{yyw} - I_{zzw}) (P_b + P_w) Q_b
\end{align*}
\]
Trajectory

From page 103 Etkin,

\[
\frac{dx'}{dt} = U \cos \Theta \cos \Psi + V (\sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi) + W (\cos \Phi \sin \Theta \cos \Psi + \sin \Phi \sin \Psi)
\]

\[
\frac{dy'}{dt} = U \cos \Theta \sin \Psi + V (\sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi) + W (\cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi)
\]

\[
\frac{dz'}{dt} = -U \sin \Theta + V \sin \Phi \cos \Theta + W \cos \Phi \cos \Theta
\]
APPENDIX D - 3-Panel Complete Non-Linear Equations of Motion

Assumptions:

1) rigid body (no aeroelastic effects)
2) mass is constant
3) negligible angular momentum of rotating machinery (engines, etc)
4) control systems fixed
5) negligible buoyancy
6) negligible twisting of wing in the dynamic analysis (however twisting is considered in the aerodynamic analysis)
7) wings are thin rectangular plates

Body

Let the distance from the body c.g. to the wing pivot point be:

$$
\bar{H}_b = H_{xb}x + H_{yb}y + H_{zb}z
$$

Note that there no pivot-point moments in the x and y directions (i.e. no $J_{lb}, J_{mb}$).

The x direction moment, $J_{lb}$, is non-existent because the outer wing panel is free to turn about the x-axis with no moment resisting its motion. The same can be said for the y direction moment, $J_{mb}$, because the chordwise location of the pivot point is such that it provides no pitching constraint. This is not completely true because the moment, $M_{mb}$, at the hinge connection between the center panel and outer panel, resists the motion. However, an engineering decision was made to ignore $J_{mb}$ to reduce the number of unknowns.
Define

\[ \Psi: \text{yaw angle (+ right)} \]
\[ \Theta: \text{pitch angle (+ nose up)} \]
\[ \Phi: \text{roll angle (+ right wing down)} \]

The components of weight are:

\[ X_g = -m_b g \sin \Theta_b \]
\[ Y_g = m_b g \cos \Theta_b \sin \Phi_b \]
\[ Z_g = m_b g \cos \Theta_b \cos \Phi_b \]

Add all external forces and moments and substitute into the rigid body equations from Appendix B.

\[ X_{\text{aero}} - m_b g \sin \Theta_b + F_{xb1} + F_{xb2} = m_b (\ddot{U}_b + Q_b W_b - R_b V_b) \]
\[ Y_{\text{aero}} + m_b g \cos \Theta_b \sin \Phi_b + F_{ybl} + F_{ybl} = m_b (\ddot{V}_b + R_b U_b - P_b W_b) \]
\[ Z_{\text{aero}} + m_b g \cos \Theta_b \cos \Phi_b + F_{zbl} + F_{zbl} = m_b (\ddot{W}_b + P_b V_b - Q_b U_b) \]
\[ L_{\text{aero}} - H_{xb1} F_{xb1} + H_{ybl} F_{zbl} - H_{xb2} F_{ybl} + H_{ybl} F_{zbl} \]
\[ = I_{xb} \dot{\hat{p}}_b - I_{yrb} \dot{\hat{q}}_b - I_{zrb} \dot{\hat{r}}_b + I_{yrb} P_b R_b + (I_{zrb} - I_{yrb}) Q_b R_b + I_{zrb} (P_b^2 - Q_b^2) - I_{zrb} P_b Q_b \]
\[ M_{\text{aero}} + H_{xb1} F_{xb1} - H_{xb2} F_{xb2} - H_{zrb} F_{zrb} \]
\[ = I_{yrb} \dot{\hat{p}}_b - I_{yrb} \dot{\hat{q}}_b - I_{yrb} \dot{\hat{r}}_b - I_{yrb} Q_b R_b + (I_{yrb} - I_{zrb}) P_b R_b + I_{yrb} (P_b^2 - R_b^2) + I_{yrb} P_b Q_b \]
\[ N_{\text{aero}} - H_{ybl} F_{zbl} + H_{xb1} F_{ybl} - H_{xb2} F_{ybl} + H_{zrb} F_{ybl} + J_{nbl} + J_{nbl} \]
\[ = I_{zrb} \dot{\hat{r}}_b - I_{zrb} \dot{\hat{p}}_b - I_{yrb} \dot{\hat{q}}_b - I_{yrb} P_b R_b + (I_{yrb} - I_{zrb}) P_b Q_b - I_{yrb} (P_b^2 - Q_b^2) + I_{zrb} Q_b R_b \]
Because of symmetry \( I_{xyb}, I_{yzb} = 0 \)

\[
\begin{align*}
X_{\text{seq}} &= -m_b g \sin \Theta_b + F_{xb1} + F_{xb2} = m_b (\ddot{U}_b + Q_b W_b - R_b V_b) \\
Y_{\text{eqv}} &= m_b g \cos \Theta_b \sin \Phi_b + F_{yb1} + F_{yb2} = m_b (\ddot{V}_b + R_b U_b - P_b W_b) \\
Z_{\text{eqv}} &= m_b g \cos \Theta_b \cos \Phi_b + F_{zb1} + F_{zb2} = m_b (\ddot{W}_b + P_b V_b - Q_b U_b) \\
L_{\text{eqv}} &= -H_{xb1} F_{yb1} + H_{yb1} F_{zb1} - H_{xb2} F_{yb2} + H_{yb2} F_{zb2} \\
&= I_{zxb} \ddot{R}_b - I_{zxb} \dot{R}_b + (I_{zxb} - I_{yyb}) Q_b R_b - I_{zxb} P_b Q_b \\
M_{\text{eqv}} &= H_{xb1} F_{xb1} - H_{xb2} F_{xb2} + H_{yb1} F_{yb1} + H_{yb2} F_{yb2} - H_{zb1} F_{zb1} - H_{zb2} F_{zb2} \\
&= I_{yyb} \dot{Q}_b + (I_{zxb} - I_{yyb}) P_b R_b + I_{zxb} (P_b^2 - R_b^2) \\
N_{\text{eqv}} &= H_{yb1} F_{yb1} + H_{zb1} F_{yb1} - H_{yb2} F_{yb2} - H_{zb2} F_{yb2} + J_{Nyb1} + J_{Nyb2} \\
&= I_{zxb} \dot{R}_b - I_{zxb} \ddot{R}_b + (I_{yyb} - I_{zxb}) P_b Q_b + I_{zxb} Q_b R_b
\end{align*}
\]

**Wing**

With the 3 panel design, the wing is divided into two dynamic portions: 1) center panel 2) outer panel. The wing motion can be modeled as a superposition of the motion of both segments.

\[
\begin{align*}
\gamma &= \gamma_{max} \sin(\omega t - 180^\circ) \\
\end{align*}
\]
Outer Panel

Port (Left) Wing

Use a stability axis system at c.g. of wing

Note that only the x moment \( (M_{lw}) \) is non-existent at the interface of the outer and center panel because there is a hinge connection, which means that the outer panel is free to rotate about the x-axis. Also, with this axis system, the moments and products of inertia are changing with time.

\[
X_{aero_w} = -m_w \dot{\xi}_w \sin \Theta_w + R_{zw} + F_{zw} = m_w \dot{U}_w + Q_w \dot{W}_w - R_w \dot{V}_w
\]

\[
Y_{aero_w} = m_w \dot{\xi}_w \cos \Theta_w \sin \Phi_w + R_{yw} + F_{yw} = m_w \dot{V}_w + R_w \dot{U}_w - P_{wing} \dot{W}_w
\]

\[
Z_{aero_w} = m_w \dot{\xi}_w \cos \Theta_w \cos \Phi_w + R_{zw} + F_{zw} = m_w \dot{W}_w + P_{wing} \dot{V}_w - Q_w \dot{U}_w
\]

\[
L_{aero_w} = D_{zw} R_{zw} - D_{yw} R_{zw} - D_{zp} F_{zw} + D_{yp} F_{zw}
\]

\[
= I_{xz} \dot{P}_{wing} + \dot{I}_{xz} F_{wing} - I_{xy} \dot{Q}_{w} - \dot{I}_{xy} Q_{w} - I_{zz} \dot{R}_{w} - \dot{I}_{zz} R_{w}
\]

\[
+ I_{xy} F_{wing} R_{w} + (I_{xz} - I_{yy}) Q_{w} R_{w} + I_{xz} (P_{wing}^2 - Q_{w}^2) - I_{zz} F_{wing} Q_{w}
\]

\[
M_{aero_w} = D_{zw} R_{zw} - D_{yw} R_{zw} + D_{zp} F_{zw} - D_{yp} F_{zw} + M_{lw}
\]

\[
= I_{yy} \dot{Q}_w + \dot{I}_{yy} Q_w - I_{yx} \dot{P}_{wing} - \dot{I}_{yx} P_{wing} - I_{y} \dot{R}_w - \dot{I}_y R_w
\]

\[
- I_{xy} Q_w R_w + (I_{xx} - I_{zz}) F_{wing} R_w + I_{zz} (P_{wing}^2 - R_w^2) + I_{y} F_{wing} Q_w
\]

\[
N_{aero_w} = D_{yw} R_{uw} + D_{zw} R_{uw} - D_{yp} F_{uw} + D_{zp} F_{uw} + M_{lw} + J_{lw}
\]

\[
= I_{zz} \dot{R}_w + \dot{I}_{zz} R_w - I_{xz} \dot{P}_{wing} - \dot{I}_{xz} P_{wing} - I_{y} \dot{Q}_w - \dot{I}_y Q_w
\]

\[
- I_{yw} F_{wing} R_w + (I_{yy} - I_{zz}) F_{wing} Q_w - I_{xy} (P_{wing}^2 - Q_w^2) + I_{xz} Q_w R_w
\]
Note from the wing moment of inertia analysis \[ I_{xyw} = I_{zxw} = i_{xxw} = i_{xyw} = i_{zxw} = 0 \]

\[
\begin{align*}
X_{\text{aero}_w} &= m_w g \sin \Theta_w R_x + F_{xw} = m_w (\dot{U}_w + Q_w W_w - R_w V_w) \\
Y_{\text{aero}_w} &= m_w g \cos \Theta_w \sin \Phi_w + R_y + F_{yw} = m_w (\dot{V}_w + R_w U_w - P_{\text{wing}} W_w) \\
Z_{\text{aero}_w} &= m_w g \cos \Theta_w \cos \Phi_w + R_z + F_{zw} = m_w (\dot{W}_w + P_{\text{wing}} V_w - Q_w U_w) \\
L_{\text{aero}_w} &= -D_{zw} R_y + D_{yw} R_z - D_{spl} F_{yw} + D_{ypl} F_{zw} \\
&= I_{xxw} \dot{\phi}_{\text{wing}} + (I_{zw} - I_{yyw}) Q_w R_w + I_{yw} (R_w^2 - Q_w^2) \\
M_{\text{aero}_w} &= -D_{zw} R_x + D_{xz} R_z + D_{spl} F_{xw} - D_{xpl} F_{zw} + M_{\text{Mwl}} \\
&= I_{yyw} \dot{Q}_w + \dot{I}_{yyw} Q_w - I_{yzw} R_w + (I_{xxw} - I_{zzw}) P_{\text{wing}} R_w + I_{yzw} P_{\text{wing}} Q_w \\
N_{\text{aero}_w} &= -D_{yw} R_x + D_{xz} R_y - D_{ypl} F_{xw} + D_{xpl} F_{yw} + M_{\text{Nwl}} + J_{\text{Nwl}} \\
&= I_{zzw} \dot{R}_w + \dot{I}_{zzw} R_w + I_{yzw} \dot{Q}_w - I_{yzw} Q_w - I_{yzw} P_{\text{wing}} R_w + (I_{yyw} - I_{xxw}) P_{\text{wing}} Q_w
\end{align*}
\]

**Starboard (Right) Wing**

Use a stability axis system at c.g. of wing

![Diagram of Starboard Wing](image)

Note that with this axis system, the moments and products of inertia are changing with time.
\[
X_{\text{aero}_w} = -m_wg \sin \theta_w + R_{xw} + F_{xw} = m_w (\dot{U}_{w1} + Q_{w1} W_{w1} - R_{w1} V_{w1})
\]
\[
Y_{\text{aero}_w} = m_w g \cos \theta_w \sin \phi_w + R_{yw} + F_{yw} = m_w (\dot{V}_{w1} + R_{w2} U_{w2} - P_{w2} W_{w2})
\]
\[
Z_{\text{aero}_w} = m_w g \cos \theta_w \cos \phi_w + R_{zw} + F_{zw} = m_w (\dot{W}_{w2} + P_{w2} V_{w2} - Q_{w2} U_{w2})
\]
\[
L_{\text{aero}_w} = -D_{zw} R_{yw} + D_{yw} R_{zw} - D_{zp} F_{yw} + D_{yp} F_{zw}
\]
\[
= I_{xxw} \dot{P}_{wing2} + I_{xyw} P_{wing2} + I_{xyzw} \dot{Q}_{w2} - I_{xyw} Q_{w2} - I_{xzw} \dot{R}_{w2} - I_{xzw} R_{w2}
\]
\[
+ I_{xyw} P_{wing2} R_{w2} + (I_{xxw} - I_{yyw}) Q_{w2} R_{w2} + I_{yzw} (R_{w2}^2 - Q_{w2}^2) - I_{xzw} P_{wing2} Q_{w2}
\]
\[
M_{\text{aero}_w} = I_{yw} R_{yw} - D_{zw} R_{xxw} + D_{zp} F_{yw} - D_{yp} F_{zw} + M_{w2}
\]
\[
= I_{yyw} \dot{Q}_{w2} + I_{yyw} Q_{w2} - I_{xzw} \dot{P}_{wing2} - I_{xyw} P_{wing2} - I_{yzw} R_{w2} - I_{yzw} R_{w2}
\]
\[
- I_{xzw} Q_{w2} R_{w2} + (I_{xxw} - I_{zzw}) P_{wing2} R_{w2} + I_{xZW} (P_{wing2}^2 - R_{w2}^2) + I_{yzw} P_{wing2} Q_{w2}
\]
\[
N_{\text{aero}_w} = -D_{yw} R_{yw} + D_{zw} R_{ww} - D_{yp} F_{xw} + D_{xp} F_{yw} + M_{w2} + J_{w2}
\]
\[
= I_{xzw} \dot{R}_{w2} + I_{xzw} \dot{R}_{w2} + I_{xzw} \dot{P}_{wing2} - I_{xxw} P_{wing2} - I_{xyw} \dot{Q}_{w2} - I_{yzw} Q_{w2}
\]
\[
- I_{yzw} P_{wing2} R_{w2} + (I_{yyw} - I_{xxw}) P_{wing2} Q_{w2} - I_{xyw} (P_{wing2}^2 - Q_{w2}^2) + I_{xzw} Q_{w2} R_{w2}
\]
Center Panel

Because of symmetry \( I_{xycp}, I_{yczp} = 0 \)
Also in this case, \( I_{zcp} = 0 \) because the center panel is treated as a flat plate.

The center of gravity is located in the middle of the plate with the reaction forces being located from the center of gravity a distance of \( \pm b_{cp}/2 \) in the \( y \) direction only. There is no \( x \) and \( z \) distance components. This implies that the reaction forces only create moments in the \( x \) and \( z \) direction as can be seen from the equations below.

\[
\begin{align*}
X_{xcp} &= -m_{cp}gsin\Theta_{cp} + R_{xcp} + R_{xcp2} = m_{cp}(\dot{U}_{cp} + Q_{cp}W_{cp} - R_{cp}V_{cp}) \\
Y_{xcp} &= +m_{cp}gcos\Theta_{cp}sin\Phi_{cp} + R_{ycp} + R_{ycp2} = m_{cp}(\dot{V}_{cp} + R_{cp}U_{cp} - P_{cp}W_{cp}) \\
Z_{xcp} &= +m_{cp}gcos\Theta_{cp}cos\Phi_{cp} + R_{zcp} + R_{zcp2} = m_{cp}(\dot{W}_{cp} + P_{cp}V_{cp} - Q_{cp}U_{cp}) \\
I_{xcp} &- (b_{cp}/2)R_{xcp} + (b_{cp}/2)R_{xcp2} = I_{xcp}\dot{P}_{cp} + (I_{xcp} - I_{yycp})Q_{cp}R_{cp} \\
M_{xcp} &+ M_{ycp} + M_{zcp} = I_{yycp}\dot{Q}_{cp} + (I_{xcp} - I_{zcp})P_{cp}R_{cp} \\
N_{xcp} &+ (b_{cp}/2)R_{xcp} - (b_{cp}/2)R_{xcp2} + M_{ycp} + M_{zcp} = I_{xcp}\dot{R}_{cp} + (I_{yycp} - I_{zcp})P_{cp}Q_{cp}
\end{align*}
\]
Kinematic Analysis

Body and Center Panel

Also let the distance from the center panel c.g. to the body c.g. be:

\[ \overrightarrow{D_b} = D_{xb} \hat{i} + D_{yb} \hat{j} + D_{zb} \hat{k} \]

This is changing with so that \( \dot{\overrightarrow{D}_b} = \overrightarrow{V}_{rel} \)

velocity:

\[ \overrightarrow{V}_a = \overrightarrow{V}_b + \overrightarrow{\Omega}_b \times (\overrightarrow{D}_b) + \overrightarrow{V}_{rel} \]

\[ \overrightarrow{\Omega}_b \times (\overrightarrow{D}_b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_b & Q_b & R_b \\ D_{xb} & D_{yb} & D_{zb} \end{vmatrix} \]

\[ = (Q_b D_{zb} - R_b D_{yb}) \hat{i} - (P_b D_{xb} - R_b D_{zb}) \hat{j} + (P_b D_{yb} - Q_b D_{xb}) \hat{k} \]

\[ \overrightarrow{V}_a = U_b \hat{i} + V_b \hat{j} + W_b \hat{k} + (Q_b D_{zb} - R_b D_{yb}) \hat{i} - (P_b D_{zb} - R_b D_{xb}) \hat{j} \]

\[ + (P_b D_{yb} - Q_b D_{xb}) \hat{k} + \dot{D}_{xb} \hat{i} + \dot{D}_{yb} \hat{j} + \dot{D}_{zb} \hat{k} \]

\[ = (U_b + Q_b D_{zb} - R_b D_{yb} + \dot{D}_{xb}) \hat{i} \]

\[ + (V_b - P_b D_{yb} + R_b D_{xb} + \dot{D}_{yb}) \hat{j} \]

\[ + (W_b + P_b D_{yb} - Q_b D_{xb} + \dot{D}_{zb}) \hat{k} \]

\[ U_{cp} = U_b + Q_b D_{zb} - R_b D_{yb} + \dot{D}_{xb} \]

\[ V_{cp} = V_b - P_b D_{yb} + R_b D_{xb} + \dot{D}_{yb} \]

\[ W_{cp} = W_b + P_b D_{yb} - Q_b D_{xb} + \dot{D}_{zb} \]
acceleration:

\[ \overline{A}_a = \overline{A}_b + \dot{\overline{O}}_b \times (\overline{D}_b) + \overline{O}_b \times (\overline{A}_b \times \overline{D}_b) + \overline{O}_b \times \overline{V}_a + 2\overline{O}_b \times \overline{V}_{ml} + \overline{A}_{ml} \]

\[ \overline{A}_a = \dot{U}_b \hat{i} + \dot{V}_b \hat{j} + \dot{W}_b \hat{k} \]

\[ \overline{\Omega}_b \times \overline{D}_b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{P}_b & \dot{Q}_b & \dot{R}_b \\ D_{xb} & D_{yb} & D_{zb} \end{vmatrix} = (\dot{Q}_b D_{zb} - \dot{R}_b D_{yb}) \hat{i} - (\dot{P}_b D_{xb} - \dot{R}_b D_{zb}) \hat{j} + (\dot{P}_b D_{yb} - \dot{Q}_b D_{xb}) \hat{k} \]

\[ \overline{\Omega}_b \times (\overline{\Omega}_b \times \overline{D}_b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_b & Q_b & R_b \\ Q_b D_{xb} - R_b D_{yb} & R_b D_{xz} - P_b D_{xb} & P_b D_{yb} - Q_b D_{xb} \end{vmatrix} = [Q_b (P_b D_{yb} - Q_b D_{xb}) - R_b (R_b D_{zb} - P_b D_{zb})] \hat{i} - [P_b (P_b D_{yb} - Q_b D_{xb}) - R_b (Q_b D_{zb} - R_b D_{yb})] \hat{j} + [P_b (R_b D_{xb} - P_b D_{zb}) - Q_b (Q_b D_{zb} - R_b D_{yb})] \hat{k} \]

\[ \overline{\Omega}_b \times \overline{V}_a = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_b & Q_b & R_b \\ U_b & V_b & W_b \end{vmatrix} = (Q_b W_b - R_b V_b) \hat{i} - (P_b W_b - R_b U_b) \hat{j} + (P_b V_b - Q_b U_b) \hat{k} \]

\[ 2\overline{\Omega}_b \times \dot{\overline{D}}_b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2P_b & 2Q_b & 2R_b \\ D_{xb} & D_{yb} & D_{zb} \end{vmatrix} = (2Q_b \dot{D}_{zb} - 2R_b \dot{D}_{yb}) \hat{i} - (2P_b \dot{D}_{zb} - 2R_b \dot{D}_{xb}) \hat{j} + (2P_b \dot{D}_{yb} - 2Q_b \dot{D}_{zb}) \hat{k} \]

\[ \overline{A}_c = [\dot{U}_b + \dot{Q}_b D_{zb} - \dot{R}_b D_{yb} + Q_b (P_b D_{yb} - Q_b D_{xb}) - R_b (R_b D_{zb} - P_b D_{zb}) + Q_b W_b - R_b V_b + 2Q_b \dot{D}_{zb} - 2R_b \dot{D}_{yb} + \dot{D}_{zb} \hat{j} \]

\[ + [\dot{V}_b - \dot{P}_b D_{zb} + \dot{R}_b D_{xb} - P_b (P_b D_{yb} - Q_b D_{xb}) + R_b (Q_b D_{zb} - R_b D_{yb}) - P_b W_b + R_b U_b - 2P_b \dot{D}_{zb} + 2R_b \dot{D}_{xb} + \dot{D}_{yb} \hat{j} \]

\[ + [\dot{W}_b + \dot{P}_b D_{yb} - \dot{Q}_b D_{xb} + P_b (R_b D_{xb} - P_b D_{zb}) - Q_b (Q_b D_{zb} - R_b D_{yb}) + P_b V_b - Q_b U_b + 2P_b \dot{D}_{yb} - 2Q_b \dot{D}_{zb} + \dot{D}_{zb} \hat{k} \]

\[ \text{159} \]
\begin{align*}
\dot{U}_c &= \dot{U}_b + \dot{Q}_b D_{zb} - \dot{R}_b D_{yb} + Q_b (P_b D_{yb} - Q_b D_{zb}) - R_b (R_b D_{xb} - P_b D_{zb}) + Q_b W_b - R_b V_b \\
&\quad + 2Q_b \dot{D}_{zb} - 2R_b \dot{D}_{yb} + \ddot{D}_{zb} \\
\dot{V}_c &= \dot{V}_b - \dot{P}_b D_{zb} + \dot{R}_b D_{xb} - P_b (P_b D_{yb} - Q_b D_{zb}) + R_b (Q_b D_{zb} - R_b D_{yb}) - P_b W_b + R_b U_b \\
&\quad - 2P_b \dot{D}_{zb} + 2R_b \dot{D}_{xb} + \ddot{D}_{yb} \\
\dot{W}_c &= \dot{W}_b + \dot{P}_b D_{yb} - \dot{Q}_b D_{xb} + P_b (R_b D_{xb} - P_b D_{zb}) - Q_b (Q_b D_{zb} - R_b D_{yb}) + P_b V_b - Q_b U_b \\
&\quad + 2P_b \dot{D}_{yb} - 2Q_b \dot{D}_{xb} + \ddot{D}_{zb} \\
\end{align*}

**Body and Wing**

Since point a and b are rigidly attached:

\[ \overline{\Omega}_b = \overline{\Omega}_a = P_b \hat{i} + Q_b \hat{j} + R_b \hat{k} \]

\[ \overline{\Omega}_c = P_w \hat{i} \]

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velocity:

$$\overline{V}_c = \overline{V}_b + \overline{\Omega}_b \times (-\overline{D}_p) + \overline{V}_{rel}$$

$$= \overline{V}_a + \overline{\Omega}_b \times \overline{H}_b + \overline{\Omega}_b \times (-\overline{D}_p) + \overline{\Omega}_c \times (-\overline{D}_p)$$

$$\overline{\Omega}_b \times \overline{H}_b = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} P_b & Q_b & R_b \\ H_{xb} & H_{yb} & H_{zb} \end{bmatrix}$$

$$= (Q_b H_{zb} - R_b H_{yb}) i - (P_b H_{zb} - R_b H_{xb}) j + (P_b H_{yb} - Q_b H_{xb}) k$$

$$\overline{\Omega}_b \times (-\overline{D}_p) = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} P_b & Q_b & R_b \\ -D_{zp} & -D_{yp} & -D_{sp} \end{bmatrix}$$

$$= (-Q_b D_{zp} + R_b D_{yp}) i - (-P_b D_{zp} + R_b D_{xp}) j + (-P_b D_{yp} + Q_b D_{xp}) k$$

$$\overline{\Omega}_c \times (-\overline{D}_p) = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} P_w & 0 & 0 \\ -D_{zp} & -D_{yp} & -D_{sp} \end{bmatrix}$$

$$= P_w D_{zp} i - P_w D_{yp} k$$

$$\overline{V}_c = U_b i + V_b j + W_b k + (Q_b H_{zb} - R_b H_{yb}) i - (P_b H_{zb} - R_b H_{xb}) j + (P_b H_{yb} - Q_b H_{xb}) k$$

$$+ (-Q_b D_{zp} + R_b D_{yp}) i - (-P_b D_{zp} + R_b D_{xp}) j + (-P_b D_{yp} + Q_b D_{xp}) k + P_w D_{zp} j - P_w D_{yp} k$$

$$= [U_b + Q_b (H_{zb} - D_{zp}) + R_b (D_{yp} - H_{yb})] i$$

$$+ [V_b + P_b (D_{zp} - H_{zb}) + R_b (H_{xb} - D_{xp}) + P_w D_{zp}] j$$

$$+ [W_b + P_b (H_{yb} - D_{xp}) + Q_b (D_{xp} - H_{xb}) - P_w D_{yp}] k$$

port wing velocities:

$$U_{w1} = U_b + Q_b (H_{zbl} - D_{zpl}) + R_b (D_{ycl} - H_{ycl})$$

$$V_{w1} = V_b + P_b (D_{zpl} - H_{zbl}) + R_b (H_{xb} - D_{xpl}) + P_w D_{zpl}$$

$$W_{w1} = W_b + P_b (H_{ycl} - D_{ycl}) + Q_b (D_{xpl} - H_{zbl}) - P_w D_{ycl}$$

starboard wing velocities:

$$U_{w2} = U_b + Q_b (H_{zbl} - D_{zpl}) + R_b (D_{yp2} - H_{ycl2})$$

$$V_{w2} = V_b + P_b (D_{zpl} - H_{zbl}) + R_b (H_{xb} - D_{xpl}) + P_w D_{zpl}$$

$$W_{w2} = W_b + P_b (H_{ycl} - D_{ycl}) + Q_b (D_{xpl} - H_{zbl}) - P_w D_{ycl}$$
acceleration:
\[
\overline{A}_c = \overline{A}_b + \dot{\overline{O}}_b \times (\overline{D}_p) + \overline{\Omega}_b \times (\overline{\Omega}_b \times (\overline{D}_p)) + 2\overline{\Omega}_b \times \overline{V}_{\text{rel}} + \overline{A}_{\text{rel}}
\]
\[
= \overline{A}_a + \dot{\overline{O}}_b \times \overline{H}_b + \overline{\Omega}_b \times (\overline{\Omega}_b \times \overline{H}_b) + \overline{\Omega}_b \times \overline{V}_a + \dot{\overline{\Omega}}_b \times (-\overline{D}_p) + \overline{\Omega}_b \times (-\overline{D}_p)
\]
\[
+ 2\overline{\Omega}_b \times (\overline{\Omega}_c \times (-\overline{D}_p)) + \dot{\overline{\Omega}}_c \times (-\overline{D}_p) + \overline{\Omega}_c \times (\overline{\Omega}_c \times (-\overline{D}_p))
\]
\[2\mathbf{\Omega}_b \times (\mathbf{\Omega}_c \times (-\mathbf{D}_p)) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2P_b & 2Q_b & 2R_b \\ 0 & P_w D_{zp} & -P_w D_{yp} \end{vmatrix} = (-20bP_w D_{yp} - 2R_b P_w D_{zp})\hat{i} + 2P_b P_w D_{yp}\hat{j} + 2P_b P_w D_{zp}\hat{k}\]

\[\mathbf{\hat{\Omega}}_c \times (-\mathbf{\hat{D}}_p) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \hat{\dot{P}}_w & 0 & 0 \\ -D_{zp} & -D_{yp} & -D_{zp} \end{vmatrix} = \hat{\dot{P}}_w D_{zp}\hat{j} - \hat{\dot{P}}_w D_{yp}\hat{k}\]

\[\mathbf{\Omega}_c \times (\mathbf{\Omega}_c \times (-\mathbf{D}_p)) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_w & 0 & 0 \\ 0 & P_w D_{zp} & -P_w D_{yp} \end{vmatrix} = P_w^2 D_{yp}\hat{j} + P_w^2 D_{zp}\hat{k}\]

\[\overline{A}_c = [\dot{U}_b + \dot{Q}_b H_{zb} - \dot{R}_b H_{yb} + Q_b (P_b H_{yb} - Q_b H_{zb}) - R_b (R_b H_{zb} - P_b H_{zb}) + Q_b W_b - R_b V_b - \dot{Q}_b D_{zp} + \dot{R}_b D_{yp} + Q_b (-P_b D_{yp} + Q_b D_{zp}) - R_b (P_b D_{zp} - R_b D_{zp}) - 2Q_b P_w D_{yp} - 2R_b P_w D_{zp} \hat{j} \hat{i}]\]

\[+[(\dot{\dot{V}}_b - (\dot{\dot{P}}_b H_{zb} - \dot{\dot{R}}_b H_{yb}) - P_b (P_b H_{yb} - Q_b H_{zb}) + R_b (Q_b H_{zb} - R_b H_{yb}) - (P_b W_b - R_b U_b) - (\dot{\dot{P}}_b D_{zp} + \dot{\dot{R}}_b D_{zp}) - P_b (P_b D_{yp} + Q_b D_{zp}) + R_b (-Q_b D_{zp} + R_b D_{yp}) + 2P_b P_w D_{yp} + \dot{\dot{P}}_b D_{zp} + P_w^2 D_{yp} \hat{j}] \hat{j}]\]

\[+[(\dot{\dot{W}}_b + \dot{\dot{P}}_b H_{yb} - \dot{\dot{Q}}_b H_{zb} + P_b (R_b H_{zb} - P_b H_{zb}) - Q_b (Q_b H_{zb} - R_b H_{yb}) + P_b V_b - Q_b U_b - \dot{\dot{P}}_b D_{yp} + \dot{\dot{Q}}_b D_{zp} + P_b (P_b D_{zp} - R_b D_{zp}) - Q_b (-Q_b D_{zp} + R_b D_{yp}) + 2P_b P_w D_{zp} - \dot{\dot{P}}_b D_{yp} + P_w^2 D_{zp} \hat{k}] \hat{k}]\]
port wing accelerations:

\[
\begin{align*}
\dot{U}_{w1} &= \dot{U}_b + \dot{Q}_b (H_{zb1} - D_{zpl}) + \dot{R}_b (D_{yp1} - H_{yb1}) + Q_s^2 (D_{xpl} - H_{xb1}) \\
&\quad + R_s^2 (D_{xpl} - H_{xb1}) + P_b Q_b (H_{yb1} - D_{yp1}) + P_b R_b (H_{xb1} - D_{xpl}) \\
&\quad + Q_b W_b - R_b V_b - 2P_{w1} (Q_b D_{yp1} + R_b D_{xpl}) \\
\dot{V}_{w1} &= \dot{V}_b + \dot{P}_b (D_{zpl} - H_{zb1}) + \dot{R}_b (H_{xb1} - D_{xpl}) + P_s^2 (D_{yp1} - H_{yb1}) \\
&\quad + R_s^2 (D_{yp1} - H_{yb1}) + P_b Q_b (H_{xb1} - D_{xpl}) + Q_b R_b (H_{yb1} - D_{yp1}) \\
&\quad - P_b W_b + R_b U_b + 2P_b P_{w1} D_{yp1} + \dot{P}_{w1} D_{xpl} + P_{w1}^2 D_{ypl} \\
\dot{W}_{w1} &= \dot{W}_b + \dot{P}_b (H_{yb1} - D_{yp1}) + \dot{Q}_b (D_{xpl} - H_{xb1}) + P^2 (D_{zpl} - H_{zb1}) \\
&\quad + Q_b D_{zpl} - H_{zb1} + P_b R_b (H_{xb1} - D_{xpl}) + Q_b R_b (H_{yb1} - D_{yp1}) \\
&\quad + P_b V_b - Q_b U_b + 2P_b P_{w1} D_{xpl} - \dot{P}_{w1} D_{yp1} + P_{w1}^2 D_{zpl}
\end{align*}
\]

starboard wing accelerations:

\[
\begin{align*}
\dot{U}_{w2} &= \dot{U}_b + \dot{Q}_b (H_{zb2} - D_{xp2}) + \dot{R}_b (D_{yp2} - H_{yb2}) + Q_s^2 (D_{xp2} - H_{xb2}) \\
&\quad + R_s^2 (D_{xp2} - H_{xb2}) + P_b Q_b (H_{yb2} - D_{yp2}) + P_b R_b (H_{xb2} - D_{xp2}) \\
&\quad + Q_b W_b - R_b V_b - 2P_{w2} (Q_b D_{yp2} + R_b D_{xp2}) \\
\dot{V}_{w2} &= \dot{V}_b + \dot{P}_b (D_{xp2} - H_{xb2}) + \dot{R}_b (H_{xb2} - D_{xp2}) + P^2 (D_{yp2} - H_{yb2}) \\
&\quad + R_s^2 (D_{yp2} - H_{yb2}) + P_b Q_b (H_{xb2} - D_{xp2}) + Q_b R_b (H_{yb2} - D_{yp2}) \\
&\quad - P_b W_b + R_b U_b + 2P_b P_{w2} D_{yp2} + \dot{P}_{w2} D_{xp2} + P_{w2}^2 D_{yp2} \\
\dot{W}_{w2} &= \dot{W}_b + \dot{P}_b (H_{yb2} - D_{yp2}) + \dot{Q}_b (D_{xp2} - H_{xb2}) + P^2 (D_{xp2} - H_{xb2}) \\
&\quad + Q_b D_{xp2} - H_{xb2} + P_b R_b (H_{xb2} - D_{xp2}) + Q_b R_b (H_{yb2} - D_{yp2}) \\
&\quad + P_b V_b - Q_b U_b + 2P_b P_{w2} D_{xp2} - \dot{P}_{w2} D_{yp2} + P_{w2}^2 D_{xp2}
\end{align*}
\]
The following substitutions for the reaction forces and moments can be made:

\[
\begin{align*}
R_{xw1} &= -R_{xcp1} & R_{xw2} &= -R_{xcp2} & F_{xw1} &= -F_{xb1} & F_{xw2} &= -F_{xb2} \\
R_{yw1} &= -R_{ycp1} & R_{yw2} &= -R_{ycp2} & F_{yw1} &= -F_{yb1} & F_{yw2} &= -F_{yb2} \\
R_{zw1} &= -R_{zcp1} & R_{zw2} &= -R_{zcp2} & F_{zw1} &= -F_{zb1} & F_{zw2} &= -F_{zb2} \\
M_{Mw1} &= -M_{Mcp1} & M_{Mw2} &= -M_{Mcp2} \\
M_{Nw1} &= -M_{Ncp1} & M_{Nw2} &= -M_{Ncp2} & J_{Nw1} &= -J_{Nb1} & J_{Nw2} &= -J_{Nb2}
\end{align*}
\]

The geometric properties of the two wing halves are identical, which means

\[
m_{w1} = m_{w2} = m_w
\]

Also, since the wing is attached rigidly to the body,

\[
\begin{align*}
\Theta_{w1} &= \Theta_{w2} = \Theta_{cp} = \Theta_b = \Theta \\
Q_{w1} &= Q_{w2} = Q_{cp} = Q_b \\
R_{w1} &= R_{w2} = R_{cp} = R_b \\
\Phi_{cp} &= \Phi_b = \Phi \\
\Phi_{w1} &= \Phi_b = \Phi \\
\Phi_{w2} &= \Phi_b = \Phi \\
P_{cp} &= P_b \\
P_{wing1} &= P_b + P_{w1} \\
P_{wing2} &= P_b + P_{w2}
\end{align*}
\]

One substitutes the above relationships, along with the kinematic equations, into the equations of motion for the wing.
For the port wing equations of motion

\[\begin{align*}
X_{aero} &= -m_w g \sin \Theta + R_{zw} + F_{zw} = m_w \left( \dot{U}_{w} + Q_w W_{w} - R_w V_{w} \right) \\
&= m_w \left( \dot{U}_{b} + \dot{Q}_b (H_{zbl} - D_{zpl}) + \dot{R}_b (D_{yp} - H_{ybl}) + Q_b^2 (D_{zpl} - H_{zbl}) + R_b^2 (D_{zpl} - H_{zbl}) \right) \\
&\quad + P_b Q_b (H_{ybl} - D_{yp}) + P_b R_b (H_{zbl} - D_{zpl}) + Q_b W_b - R_b V_b - 2P_w (Q_b D_{yp} + R_b D_{zpl}) \\
&\quad + Q_b W_b + P_b Q_b (H_{ybl} - D_{yp}) + Q_b^2 (D_{zpl} - H_{zbl}) - P_w Q_b D_{yp} \\
&\quad - R_b V_b - P_b R_b (D_{zpl} - H_{zbl}) - R_b^2 (H_{zbl} - D_{zpl}) - P_w R_b D_{zpl}
\end{align*}\]

\[\begin{align*}
Y_{aero} &= m_w g \cos \Theta \sin \Phi + R_{yw} + F_{yw} = m_w \left( \dot{V}_{w} + R_w U_{w} - P_w W_{w} \right) \\
&= m_w \left( \dot{V}_{b} + \dot{P}_b (D_{zpl} - H_{zbl}) + \dot{R}_b (H_{ybl} - D_{zpl}) + \dot{Q}_b^2 (D_{yp} - H_{ybl}) + \dot{R}_b^2 (D_{yp} - H_{ybl}) \right) \\
&\quad + P_b Q_b (H_{ybl} - D_{zpl}) + Q_b R_b (H_{zbl} - D_{zpl}) - P_b U_b + R_b V_b + 2P_b P_w D_{yp} \\
&\quad + \dot{P}_w D_{zpl} + \dot{P}_w^2 D_{yp} + R_b U_b + Q_b R_b (H_{zbl} - D_{zpl}) + \dot{R}_b^2 (D_{yp} - H_{ybl}) \\
&\quad - P_b W_b - \dot{P}_b^2 (H_{ybl} - D_{yp}) - P_b Q_b (D_{zpl} - H_{zbl}) + P_b P_w D_{yp} \\
&\quad - P_w W_b - P_w P_b (H_{ybl} - D_{yp}) - P_w Q_b (D_{zpl} - H_{zbl}) + P_w^2 D_{yp}
\end{align*}\]

\[\begin{align*}
Z_{aero} &= m_w g \cos \Theta \cos \Phi + R_{zw} + F_{zw} = m_w \left( \dot{W}_{w} + P_w V_{w} - Q_w U_{w} \right) \\
&= m_w \left( \dot{W}_{b} + \dot{P}_b (H_{ybl} - D_{yp}) + \dot{Q}_b (D_{zpl} - H_{zbl}) + \dot{P}_b^2 (D_{zpl} - H_{zbl}) + \dot{Q}_b^2 (D_{zpl} - H_{zbl}) \right) \\
&\quad + P_b R_b (H_{zbl} - D_{zpl}) + Q_b R_b (H_{ybl} - D_{yp}) + P_b V_b - Q_b U_b + 2P_b P_w D_{zpl} \\
&\quad - \dot{P}_w D_{yp} + \dot{P}_w^2 D_{zp} + P_b V_b + \dot{P}_b^2 (D_{zpl} - H_{zbl}) + P_b R_b (H_{zbl} - D_{zpl}) + P_b P_w D_{zp} \\
&\quad + P_w V_b + P_w P_b (D_{zpl} - H_{zbl}) + P_w R_b (H_{zbl} - D_{zpl}) + \dot{P}_w^2 D_{zp} \\
&\quad - Q_b U_b - \dot{Q}_b (H_{zbl} - D_{zpl}) - Q_b R_b (D_{yp} - H_{ybl}) \right)
\end{align*}\]
Simplifying these equations yields:

\[ X_{\text{aero}_w} - m_w g \sin \theta + R_{xw1} + F_{xw1} \]
\[ = m_w (\dot{U}_b + \dot{Q}_b (H_{zbl} - D_{zpl}) + \ddot{R}_b (D_{ypl} - H_{ybl}) + 2Q_b^2 (D_{zpl} - H_{zbl}) + 2R_b^2 (D_{zpl} - H_{zbl}) + 2P_b Q_w (H_{ybl} - D_{ypl}) + 2P_b \dot{R}_b (H_{zbl} - D_{zpl}) + 2Q_w W_b - 2R_b V_b - 3P_{w1} (Q_b D_{ypl} + R_b D_{zpl}) ] \]

\[ Y_{\text{aero}_w} + m_w g \cos \theta \sin \Phi + R_{yw1} + F_{yw1} \]
\[ = m_w (\dot{V}_b + \dot{R}_b (H_{zbl} - D_{zpl}) + \ddot{R}_b (H_{zbl} - D_{zpl}) + 2P_b^2 (D_{ypl} - H_{ybl}) + 2R_b^2 (D_{ypl} - H_{ybl}) + 2P_b Q_w (H_{ybl} - D_{ypl}) + 2Q_w R_b (H_{zbl} - D_{zpl}) - 2P_b W_b + 2R_b U_b - P_{w1} P_b (H_{ybl} - 4D_{ypl}) + P_{w1} D_{zpl} + 2P_b^2 D_{ypl} - P_{w1} W_b - P_{w1} Q_b (D_{zpl} - H_{zbl}) ] \]

\[ Z_{\text{aero}_w} + m_w g \cos \theta \cos \Phi + R_{zw1} + F_{zw1} \]
\[ = m_w (\dot{W}_b + \dot{R}_b (H_{ybl} - D_{ypl}) + \ddot{Q}_b (D_{zpl} - H_{zbl}) + 2P_b^2 (D_{zpl} - H_{zbl}) + 2Q_b^2 (D_{zpl} - H_{zbl}) + 2P_b R_b (H_{zbl} - D_{zpl}) + 2Q_b R_b (H_{ybl} - D_{ypl}) + 2P_b W_b - 2Q_b U_b + P_{w1} P_b (4D_{zpl} - H_{zbl}) - \dot{P}_{w1} D_{ypl} + 2P_b^2 D_{zpl} + P_{w1} V_b + P_{w1} R_b (H_{zbl} - D_{zpl}) ] \]

For the starboard wing equations of motion, one has

\[ X_{\text{aero}_w} - m_w g \sin \theta + R_{xw2} + F_{xw2} = m_w (\dot{U}_w + Q_{w2} W_w - R_{w2} V_w) \]
\[ = m_w (\dot{U}_b + \dot{Q}_b (H_{zbl} - D_{zpl}) + \ddot{R}_b (D_{zpl} - H_{zbl}) + Q_b^2 (D_{zpl} - H_{zbl}) + R_b^2 (D_{zpl} - H_{zbl}) + P_b Q_w (H_{ybl} - D_{ypl}) + P_b R_b (H_{zbl} - D_{zpl}) + Q_w W_b - R_b V_b - 2P_{w2} (Q_b D_{ypl} + R_b D_{zpl}) + Q_w W_b + P_b Q_b (H_{ybl} - D_{ypl}) + Q_b^2 (D_{zpl} - H_{zbl}) - P_{w2} Q_b D_{ypl} - R_b V_b - P_b R_b (D_{zpl} - H_{zbl}) - R_b^2 (H_{zbl} - D_{zpl}) - P_{w2} R_b D_{zpl} ] \]

\[ Y_{\text{aero}_w} + m_w g \cos \theta \sin \Phi + R_{yw2} + F_{yw2} = m_w (\dot{V}_w + R_{w2} U_w - P_{w2} W_w) \]
\[ = m_w (\dot{V}_b + \dot{R}_b (H_{zbl} - D_{zpl}) + \ddot{R}_b (H_{zbl} - D_{zpl}) + P_b^2 (D_{zpl} - H_{zbl}) + R_b^2 (D_{zpl} - H_{zbl}) + P_b Q_b (H_{zbl} - D_{zpl}) + Q_b W_b - R_b U_b + 2P_b P_{w2} D_{ypl} + \dot{P}_{w2} D_{zpl} + P_b^2 D_{ypl} + R_b U_b + Q_b R_b (H_{zbl} - D_{zpl}) + R_b^2 (D_{zpl} - H_{ybl}) - P_b W_b - P_b^2 (H_{ybl} - D_{ypl}) - P_b Q_b (D_{zpl} - H_{zbl}) + P_b P_{w2} D_{ypl} - P_{w2} W_b - P_{w2} P_b (H_{ybl} - D_{ypl}) - P_{w2} Q_b (D_{zpl} - H_{zbl}) + P_{w2}^2 D_{ypl} ] \]
Simplifying these equations yields:

\[ X_{aero_{w1}} = m_w g \sin \Theta + R_{xw2} + F_{xw2} \]
\[ = m_w (\dot{U}_b + \dot{Q}_b (H_{yb2} - D_{yp2}) + \dot{R}_b (D_{xp2} - H_{xb2}) + 2Q_b^2 (D_{xp2} - H_{xb2}) + 2R_b^2 (D_{xp2} - H_{xb2}) + 2P_b Q_b (H_{yb2} - D_{yp2}) + 2P_b R_b (H_{xb2} - D_{xp2}) - 2P_b V_b - 2R_b U_b - 3P_w Q_b (D_{yp2} - R_b D_{xp2})) \]

\[ Y_{aero_{w1}} = m_w g \cos \Theta \cos \Phi + R_{yw2} + F_{yw2} \]
\[ = m_w (\dot{V}_b + \dot{P}_b (D_{xp2} - H_{xb2}) + \dot{R}_b (H_{xb2} - D_{xp2}) + 2P_b^2 (D_{xp2} - H_{xb2}) + 2R_b^2 (D_{xp2} - H_{xb2}) + 2P_b Q_b (H_{yb2} - D_{yp2}) + 2Q_b R_b (H_{xb2} - D_{xp2}) - 2P_b V_b + 2R_b U_b - P_{w2} P_b (H_{yb2} - 4D_{yp2}) + \dot{P}_w D_{xp2} + 2P_{w2} D_{xp2} + P_{w2} W_b - P_{w2} Q_b (D_{xp2} - H_{xb2})) \]

\[ Z_{aero_{w1}} = m_w g \cos \Theta \cos \Phi + R_{zw2} + F_{zw2} \]
\[ = m_w (\dot{W}_b + \dot{P}_b (H_{yb2} - D_{yp2}) + \dot{Q}_b (D_{xp2} - H_{xb2}) + 2P_b^2 (D_{xp2} - H_{xb2}) + 2Q_b^2 (D_{xp2} - H_{xb2}) + 2P_b R_b (H_{xb2} - D_{xp2}) + 2Q_b R_b (H_{yb2} - D_{yp2}) + 2P_b V_b - 2Q_b U_b + P_{w2} P_b (4D_{xp2} - H_{xb2}) - \dot{P}_w D_{yp2} + 2P_{w2} D_{xp2} + P_{w2} W_b + P_{w2} R_b (H_{xb2} - D_{xp2})) \]
Substitute the kinematic equations into the equations of motion for the center panel to obtain

\[ X_{\text{sero}} = -m_\phi \cos \theta - R_{xw1} - R_{xw2} = m_\phi \left( \ddot{U}_\phi + Q_\phi \dot{W}_\phi - R_\phi V_\phi \right) \]

\[ = m_\phi \left[ \ddot{U}_b + \dot{Q}_b D_{xb} - \dot{R}_b D_{yb} + P_b Q_b D_{yb} - Q_b^2 D_{xb} - R_b^2 D_{xb} + P_b R_b D_{zb} + Q_b W_b - R_b V_b 
+ 2Q_b \dot{D}_{xb} - 2R_b \dot{D}_{yb} + \ddot{D}_{zb} + Q_b W_b + P_b Q_b D_{yb} - Q_b^2 D_{xb} + Q_b \dot{D}_{zb} - R_b V_b 
+ P_b R_b D_{zb} - R_b^2 D_{zb} - R_b \dot{D}_{yb} \right] \]

\[ Y_{\text{sero}} = m_\phi \cos \theta \sin \phi - R_{yw1} - R_{yw2} = m_\phi \left( \ddot{V}_\phi + R_\phi U_{\phi} - P_\phi W_\phi \right) \]

\[ = m_\phi \left[ \ddot{V}_b + \dot{P}_b D_{zb} + \dot{R}_b D_{xb} - P_b^2 D_{yb} + P_b Q_b D_{xb} + Q_b R_b D_{zb} - R_b^2 D_{yb} - P_b W_b + R_b U_b 
- 2P_b \dot{D}_{zb} + 2R_b \dot{D}_{xb} + \ddot{D}_{yb} + R_b U_b + Q_b R_b D_{zb} - R_b^2 D_{yb} + R_b \dot{D}_{zb} - P_b W_b 
- P_b^2 D_{yb} + P_b Q_b D_{xb} - P_b \dot{D}_{zb} \right] \]

\[ Z_{\text{sero}} = m_\phi \cos \theta \cos \phi - R_{zw1} - R_{zw2} = m_\phi \left( \ddot{W}_\phi + P_\phi V_{\phi} - Q_\phi U_{\phi} \right) \]

\[ = m_\phi \left[ \ddot{W}_b + \dot{P}_b D_{yb} - \dot{Q}_b D_{xb} + P_b R_b D_{zb} - P_b^2 D_{zb} - Q_b^2 D_{zb} - Q_b R_b D_{yb} + P_b V_b - Q_b U_b 
+ 2P_b \dot{D}_{yb} - 2Q_b \dot{D}_{zb} + \ddot{D}_{zb} + P_b V_b - P_b^2 D_{zb} + P_b R_b D_{zb} + P_b \dot{D}_{yb} - Q_b U_b 
- Q_b^2 D_{zb} + Q_b R_b D_{yb} - Q_b \dot{D}_{zb} \right] \]

These simplify to:

\[ X_{\text{sero}} = -m_\phi \cos \theta - R_{xw1} - R_{xw2} \]

\[ = m_\phi \left[ \ddot{U}_b + \dot{Q}_b D_{xb} - \dot{R}_b D_{yb} - 2Q_b^2 D_{xb} - 2R_b^2 D_{xb} + 2P_b Q_b D_{yb} + 2P_b R_b D_{zb} 
+ 2Q_b W_b - 2R_b V_b + 3Q_b \dot{D}_{zb} - 3R_b \dot{D}_{yb} + \ddot{D}_{zb} \right] \]

\[ Y_{\text{sero}} = m_\phi \cos \theta \sin \phi - R_{yw1} - R_{yw2} \]

\[ = m_\phi \left[ \ddot{V}_b + \dot{P}_b D_{zb} + \dot{R}_b D_{xb} - 2P_b^2 D_{yb} - 2R_b^2 D_{yb} + 2P_b Q_b D_{zb} + 2Q_b R_b D_{zb} 
- 2P_b W_b + 2R_b U_b - 3P_b \dot{D}_{zb} + 3R_b \dot{D}_{xb} + \ddot{D}_{yb} \right] \]

\[ Z_{\text{sero}} = m_\phi \cos \theta \cos \phi - R_{zw1} - R_{zw2} \]

\[ = m_\phi \left[ \ddot{W}_b + \dot{P}_b D_{yb} - \dot{Q}_b D_{xb} - 2P_b^2 D_{zb} - 2Q_b^2 D_{zb} + 2P_b R_b D_{xb} + 2Q_b R_b D_{yb} 
+ 2P_b V_b - 2Q_b U_b + 3P_b \dot{D}_{yb} - 3Q_b \dot{D}_{zb} + \ddot{D}_{zb} \right] \]

\[ L_{\text{sero}} = \frac{(b_p / 2) R_{zw1} - (b_p / 2) R_{zw1}}{I_{xxcp} \dot{P}_b} + (I_{xxcp} - I_{yyxp}) Q_b R_b \]

\[ M_{\text{sero}} = M_{Mw1} - M_{Mw2} = I_{yyxp} \dot{Q}_b + (I_{xxcp} - I_{yyxp}) P_b R_b \]

\[ N_{\text{sero}} = \frac{(b_p / 2) R_{xw1} + (b_p / 2) R_{xw1} - M_{Nw1} - M_{Nw2}}{I_{xxcp} \dot{R}_b + (I_{yyxp} - I_{xxcp}) P_b Q_b} \]

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Summary - Equations of Motion

body equations:

\[ X_{\text{aero}} - m_b g \sin \Theta - F_{xw1} - F_{xw2} = m_b (\ddot{U}_b + Q_b W_b - R_b V_b) \]
\[ Y_{\text{aero}} + m_b g \cos \Theta \sin \Phi - F_{yw1} - F_{yw2} = m_b (\dot{V}_b + R_b U_b - P_b W_b) \]
\[ Z_{\text{aero}} + m_b g \cos \Theta \cos \Phi - F_{zw1} - F_{zw2} = m_b (\dot{W}_b + P_b V_b - Q_b U_b) \]
\[ L_{\text{aero}} + H_{zbl} F_{yw1} - H_{ybl} F_{xw1} + H_{zb2} F_{yw2} - H_{yb2} F_{xw2} = I_{xzb} \dot{P}_b - I_{zxb} \dot{\bar{R}}_b + (I_{zxb} - I_{yyb}) Q_b R_b - I_{xzb} P_b Q_b \]
\[ M_{\text{aero}} - H_{zbl} F_{xw1} + H_{xbl} F_{xw1} - H_{zb2} F_{xw2} + H_{xb2} F_{xw2} = I_{yyb} \dot{Q}_b + (I_{xzb} - I_{zxb}) P_b R_b + I_{xzb} (P_b^2 - R_b^2) \]
\[ N_{\text{aero}} + H_{ybl} F_{xw1} - H_{xbl} F_{yw1} + H_{yb2} F_{xw2} - H_{xb2} F_{yw2} - J_{Nw1} - J_{Nw2} = I_{zxb} \dot{\bar{R}}_b - I_{xzb} \dot{P}_b + (I_{yyb} - I_{xzb}) P_b Q_b + I_{xzb} Q_b R_b \]

port wing equations:

\[ X_{\text{aeroW}} - m_w g \sin \Theta + R_{xw1} + F_{xw1} = m_w (\ddot{U}_b + \dot{Q}_b (H_{zbl} - D_{zpl}) + \dot{\bar{R}}_b (D_{ypl} - H_{ybl})) + 2Q_b^2 (D_{xpl} - H_{xbl}) + 2R_b^2 (D_{ypl} - H_{ybl}) + 2P_b Q_b (H_{ybl} - D_{ypl}) + 2P_b R_b (H_{zbl} - D_{zpl}) + 2Q_b W_b - 2R_b V_b - 3P_{w1} (Q_b D_{ypl} + R_b D_{zpl}) \]
\[ Y_{\text{aeroW}} + m_w g \cos \Theta \sin \Phi + R_{yw1} + F_{yw1} = m_w (\ddot{V}_b + \dot{P}_b (D_{zpl} - H_{zbl}) + \dot{\bar{R}}_b (H_{xbl} - D_{xpl}) + 2P_b^2 (D_{ypl} - H_{ybl}) + 2R_b^2 (D_{ypl} - H_{ybl}) + 2R_b Q_b (H_{xbl} - D_{xpl}) + 2Q_b R_b (H_{zbl} - D_{zpl}) - 2P_b W_b + 2R_b U_b - P_{w1} P_b (H_{ybl} - 4D_{ypl}) + \dot{P}_{w1} D_{zpl} + 2P_{w1} D_{ypl} - P_{w1} W_b - P_{w1} Q_b (D_{zpl} - H_{xbl}) \]
\[ Z_{\text{aeroW}} + m_w g \cos \Theta \cos \Phi + R_{zw1} + F_{zw1} = m_w (\ddot{W}_b + \dot{P}_b (H_{ybl} - D_{ypl}) + \dot{Q}_b (D_{zpl} - H_{zbl}) + 2P_b^2 (D_{zpl} - H_{zbl}) + 2Q_b^2 (D_{zpl} - H_{zbl}) + 2P_b R_b (H_{xbl} - D_{xpl}) + 2Q_b R_b (H_{ybl} - D_{ypl}) + 2P_b V_b - 2Q_b U_b + P_{w1} P_b (4D_{zpl} - H_{zbl}) + \dot{P}_{w1} D_{ypl} + 2P_{w1} D_{zpl} + P_{w1} V_b + P_{w1} R_b (H_{xbl} - D_{xpl}) \]
starboard wing equations:

\[
X_{\text{aero}} = m_{\text{w}} g \sin \Theta + R_{\text{xw}} + F_{\text{xw}} \\
= m_{\text{w}} \dot{\bar{U}}_{\text{b}} + \dot{\bar{Q}}_{\text{b}} (H_{\text{yb2}} - D_{\text{zp2}}) + \dot{\bar{R}}_{\text{b}} (D_{\text{yp2}} - H_{\text{yb2}}) + 2Q_{\text{b}}^2 (D_{\text{zp2}} - H_{\text{zb2}}) \\
+ 2R_{\text{b}}^2 (D_{\text{zp2}} - H_{\text{zb2}}) + 2P_{\text{b}} Q_{\text{b}} (H_{\text{yb2}} - D_{\text{zp2}}) + 2P_{\text{b}} R_{\text{b}} (H_{\text{zb2}} - D_{\text{yp2}}) \\
+ 2Q_{\text{b}} W_{\text{b}} - 2R_{\text{b}} V_{\text{b}} - 3P_{\text{w2}} (Q_{\text{b}} D_{\text{zp2}} + R_{\text{b}} D_{\text{zp2}}) \\
\]

\[
Y_{\text{aero}} = m_{\text{w}} g \cos \Theta \sin \Phi + R_{\text{yw}} + F_{\text{yw}} \\
= m_{\text{w}} (\dot{\bar{V}}_{\text{b}} + \dot{\bar{P}}_{\text{b}} (D_{\text{zp2}} - H_{\text{zb2}}) + \dot{\bar{R}}_{\text{b}} (H_{\text{yb2}} - D_{\text{sp2}}) + 2P_{\text{b}}^2 (D_{\text{yp2}} - H_{\text{yb2}}) \\
+ 2R_{\text{b}}^2 (D_{\text{yp2}} - H_{\text{yb2}}) + 2P_{\text{b}} Q_{\text{b}} (H_{\text{yb2}} - D_{\text{sp2}}) + 2Q_{\text{b}} R_{\text{b}} (H_{\text{zb2}} - D_{\text{sp2}}) \\
- 2P_{\text{b}} W_{\text{b}} + 2R_{\text{b}} U_{\text{b}} - P_{\text{w2}} P_{\text{b}} (H_{\text{yb2}} - 4D_{\text{yp2}}) + \dot{\bar{P}}_{\text{w2}} D_{\text{zp2}} + 2P_{\text{w2}}^2 D_{\text{yp2}} \\
- P_{\text{w2}} W_{\text{b}} - P_{\text{w2}} Q_{\text{b}} (D_{\text{zp2}} - H_{\text{zb2}}) \\
\]

\[
Z_{\text{aero}} = m_{\text{w}} g \cos \Theta \cos \Phi + R_{\text{zw}} + F_{\text{zw}} \\
= m_{\text{w}} (\dot{\bar{W}}_{\text{b}} + \dot{\bar{P}}_{\text{b}} (H_{\text{yb2}} - D_{\text{zp2}}) + \dot{\bar{Q}}_{\text{b}} (D_{\text{xp2}} - H_{\text{xb2}}) + 2P_{\text{b}}^2 (D_{\text{xp2}} - H_{\text{xb2}}) \\
+ 2Q_{\text{b}}^2 (D_{\text{xp2}} - H_{\text{xb2}}) + 2P_{\text{b}} R_{\text{b}} (H_{\text{xb2}} - D_{\text{xp2}}) + 2Q_{\text{b}} R_{\text{b}} (H_{\text{yb2}} - D_{\text{xp2}}) \\
+ 2P_{\text{b}} V_{\text{b}} - 2Q_{\text{b}} U_{\text{b}} + P_{\text{w2}} P_{\text{b}} (4D_{\text{zp2}} - H_{\text{zb2}}) - \dot{\bar{P}}_{\text{w2}} D_{\text{xp2}} + 2P_{\text{w2}}^2 D_{\text{zp2}} \\
+ P_{\text{w2}} V_{\text{b}} + P_{\text{w2}} R_{\text{b}} (H_{\text{xb2}} - D_{\text{xp2}}) \\
\]
\[ L_{aero} = -D_{zw2} P_{yw2} + D_{yw2} R_{zw2} - D_{zp2} F_{yw2} + D_{yp2} F_{zw2} . \]

\[ = I_{xzw2} \dot{R}_b + I_{yw2} \dot{\dot{R}} - (I_{zw2} - I_{yw2}) Q_b R_b + I_{yw2}(R_b^2 - Q_b^2) \]

\[ M_{aero} = +D_{zw2} R_{zw2} - D_{zw2} R_{zw2} + D_{zp2} F_{zw2} - D_{zp2} F_{zw2} + M_{Mw2} \]

\[ = I_{yw2} \dot{Q}_b + \dot{I}_{yw2} Q_b - \dot{I}_{yw2} R_b - \dot{I}_{yw2} R_b + (I_{xzw2} - I_{yw2})(P_b + P_w) R_b \]

\[ + I_{yw2}(P_b + P_w) Q_b \]

\[ N_{aero} = -D_{yw2} R_{xw2} + D_{yw2} R_{yw2} - D_{yp2} F_{yw2} + D_{zp2} F_{zw2} + M_{Nw2} + J_{Nw2} \]

\[ = I_{xzw2} \dot{R}_b + \dot{I}_{xzw2} R_b - \dot{I}_{yw2} Q_b - \dot{I}_{yw2} Q_b - I_{yw2}(P_b + P_w) R_b \]

\[ +(I_{yw2} - I_{xzw2})(P_b + P_w) Q_b \]

**center panel equations:**

\[ X_{aero} = -m_c g \sin \Theta - R_{xw1} - R_{xw2} \]

\[ = m_c [\dot{U}_b + \dot{Q}_b D_{zb} - \dot{R}_b D_{yb} - 2Q_b^2 D_{xb} - 2R_b^2 D_{xb} + 2P_b Q_b D_{yb} + 2P_b R_b D_{zb} \]

\[ + 2Q_b W_b - 2R_b V_b + 3Q_b \ddot{D}_{zb} - 3R_b \ddot{D}_{yb} + \ddot{D}_{zb}] \]

\[ Y_{aero} = +m_c g \cos \Theta \sin \Phi - R_{ywl} - R_{ywl} \]

\[ = m_c [\dot{V}_b - \dot{P}_b D_{zb} - \dot{R}_b D_{yb} - 2P_b^2 D_{xb} - 2R_b^2 D_{yb} + 2P_b Q_b D_{xb} + 2Q_b R_b D_{zb} \]

\[ - 2P_b W_b + 2R_b U_b - 3P_b \ddot{D}_{zb} + 3R_b \ddot{D}_{yb} + \ddot{D}_{yb}] \]

\[ Z_{aero} = +m_c g \cos \Theta \cos \Phi - R_{zw1} - R_{zw2} \]

\[ = m_c [\dot{W}_b + \dot{P}_b D_{yb} - \dot{Q}_b D_{xb} - 2P_b^2 D_{zb} - 2Q_b^2 D_{zb} + 2P_b R_b D_{xb} + 2Q_b R_b D_{yb} \]

\[ + 2P_b V_b - 2Q_b U_b + 3P_b \ddot{D}_{yb} - 3Q_b \ddot{D}_{xb} + \ddot{D}_{zb}] \]

\[ L_{aero} = \left( \frac{b_c}{2} \right) R_{xw1} - \left( \frac{b_c}{2} \right) R_{xw2} = I_{xscp} \dot{P}_b + (I_{xscp} - I_{yscp}) Q_b R_b \]

\[ M_{aero} = -M_{Mwl} - M_{Mw2} = I_{yscp} \dot{Q}_b + (I_{xscp} - I_{yscp}) P_b R_b \]

\[ N_{aero} = -\left( \frac{b_c}{2} \right) R_{xw1} + \left( \frac{b_c}{2} \right) R_{xw2} - M_{Nwl} - M_{Nw2} = I_{xscp} \dot{R}_b + (I_{yscp} - I_{xscp}) P_b Q_b \]
Rearranging all the unknowns to one side gives:

\[
X_{\text{aero}} - m_b g \sin \Theta - m_b (Q_b W_b - R_b V_b) = m_b \dot{U}_b + F_{xw1} + F_{xw2} \\
Y_{\text{aero}} + m_b g \cos \Theta \sin \Phi - m_b (R_b U_b - P_b W_b) = m_b \dot{V}_b + F_{yw1} + F_{yw2} \\
Z_{\text{aero}} + m_b g \cos \Theta \cos \Phi - m_b (P_b V_b - Q_b U_b) = m_b \dot{W}_b + F_{zw1} + F_{zw2} \\
L_{\text{aero}} - (I_{zxb} - I_{yyb}) Q_b R_b + I_{zxb} P_b Q_b = I_{zxb} \dot{P}_b - I_{zxb} \dot{R}_b - H_{zbl} F_{yw1} + H_{ybl} F_{zw1} - H_{zbl} F_{yw2} + H_{ybl} F_{zw2} \\
M_{\text{aero}} - (I_{zxb} - I_{zzb}) P_b R_b - I_{zxb} (P_b^2 - R_b^2) = I_{yyb} \dot{Q}_b + H_{zbl} F_{xw1} - H_{xbl} F_{zw1} + H_{zbl} F_{xw2} - H_{xbl} F_{zw2} \\
N_{\text{aero}} - (I_{yyb} - I_{xxb}) P_b Q_b - I_{xxb} Q_b R_b = I_{zxb} \dot{P}_b + I_{zxb} \dot{R}_b - H_{ybl} F_{xw1} + H_{xbl} F_{yw1} - H_{ybl} F_{xw2} + H_{xbl} F_{yw2} + J_{Nw1} + J_{Nw2} \\
X_{\text{aero}w1} - m_w g \sin \Theta - m_w (2Q_b^2 D_{xpl} - H_{zbl}) + 2R_b^2 (D_{xpl} - H_{zbl}) \\
+ 2P_b Q_b (H_{ybl} - D_{zpl}) + 2P_b R_b (H_{zbl} - D_{xpl}) + 2Q_b W_b - 2R_b V_b - 3P_w (Q_b D_{xpl} + R_b D_{zpl}) \\
= m_w \dot{U}_w + m_w (H_{zbl} - D_{zpl}) \dot{Q}_w + m_w (D_{xpl} - H_{ybl}) \dot{R}_w - R_{xw1} - F_{xw1} \\
Y_{\text{aero}w1} + m_w g \cos \Theta \sin \Phi - m_w (2P_b^2 (D_{xpl} - H_{ybl}) + 2R_b^2 (D_{ypl} - H_{ybl}) \\
+ 2P_b Q_b (H_{ybl} - D_{xpl}) + 2Q_b R_b (H_{zbl} - D_{zpl}) + 2P_b W_b + 2R_b U_b - P_{w1} P_b (H_{b1} - 4D_{ypl}) \\
+ \dot{P}_{w1} D_{zpl} + 2P_{w1} D_{xpl} - P_{w1} W_b - P_{w1} Q_b (D_{xpl} - H_{zbl}) \\
= m_w \dot{V}_w + m_w (D_{xpl} - H_{zbl}) \dot{P}_w + m_w (H_{zbl} - D_{xpl}) \dot{R}_w - R_{yw1} - F_{yw1} \\
Z_{\text{aero}w1} + m_w g \cos \Theta \cos \Phi - m_w (2P_b^2 (D_{zpl} - H_{zbl}) + 2Q_b^2 (D_{zpl} - H_{zbl}) \\
+ 2P_b R_b (H_{zbl} - D_{xpl}) + 2Q_b R_b (H_{ybl} - D_{ypl}) + 2P_b V_b - 2Q_b U_b + P_{w1} P_b (4D_{zpl} - H_{zbl}) \\
- \dot{P}_{w1} D_{xpl} + 2P_{w1} D_{ypl} + P_{w1} V_b + P_{w1} R_b (H_{zbl} - D_{xpl}) \\
= m_w \dot{W}_w + m_w (H_{ybl} - D_{ypl}) \dot{P}_w + m_w (D_{xpl} - H_{zbl}) \dot{Q}_w - R_{zw1} - F_{zw1} \\
L_{\text{aero}w1} - I_{xwl} \dot{P}_w - (I_{xwl} - I_{yyw}) Q_b R_b - I_{xwl} (R_b^2 - Q_b^2) \\
= I_{xwl} \dot{P}_w + D_{xwl} R_{yw1} - D_{yw1} R_{zw1} + D_{zpl} F_{yw1} - D_{ypl} F_{zw1} \\
M_{\text{aero}w1} - I_{yyw} Q_b + I_{yyw} R_b - (I_{xwl} - I_{zzw}) (P_b + P_{w1}) R_b - I_{ywl} (P_b + P_{w1}) Q_b \\
= I_{yyw} \dot{Q}_w - I_{xwl} \dot{R}_w - D_{xwl} R_{yw1} + D_{xwl} R_{zw1} - D_{zpl} F_{yw1} + D_{zpl} F_{xw1} - M_{fw1} \\
N_{\text{aero}w1} - I_{zzw} R_b + I_{zzw} Q_b + I_{zyw} (P_b + P_{w1}) R_b - (I_{yyw} - I_{xwl}) (P_b + P_{w1}) Q_b \\
= -I_{yyw} \dot{Q}_w + I_{zzw} \dot{R}_w + D_{yw1} R_{xw1} - D_{xwl} R_{yw1} + D_{ypl} F_{xw1} - D_{zpl} F_{yw1} - M_{fw1} - J_{fw1} \\
\\
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\[ X_{\text{aero}} = -m_w g \sin \Theta - m_w [2Q_b^2 (D_{xp2} - H_{xb2}) + 2R_b^2 (D_{sp2} - H_{sb2})] + 2P_b Q_b (H_{yb2} - D_{yp2}) + 2P_b R_b (H_{zb2} - D_{zp2}) + 2Q_b W_b - 2R_b V_b - 3P_w (Q_b D_{yp2} + R_b D_{zp2})] \\
= m_w \dot{U}_b + m_w (H_{xb2} - D_{zp2}) \ddot{Q}_b + m_w (D_{yp2} - H_{yb2}) \dot{R}_b - R_{zw2} - F_{zw2} \]

\[ Y_{\text{aero}} = m_w g \cos \Theta \sin \Phi - m_w [2P_b^2 (D_{yp2} - H_{yb2}) + 2R_b^2 (D_{yp2} - H_{yb2})] + 2P_b Q_b (H_{xb2} - D_{xp2}) + 2Q_b R_b (H_{zb2} - D_{zp2}) - 2P_b W_b + 2R_b U_b - P_w P_b (H_{yb2} - 4D_{yp2}) \\
+ \dot{P}_{w2} D_{zp2} + 2P_{w2}^2 D_{yp2} - P_{w2} W_b - P_{w2} Q_b (D_{xp2} - H_{sb2})] \\
= m_w \dot{V}_b + m_w (H_{xb2} - D_{zp2}) \dot{P}_b + m_w (H_{yb2} - D_{xp2}) \dot{R}_b - R_{yw2} - F_{yw2} \]

\[ Z_{\text{aero}} = m_w g \cos \Theta \cos \Phi - m_w [2P_b^2 (D_{xp2} - H_{xb2}) + 2Q_b^2 (D_{sp2} - H_{sb2})] + 2P_b R_b (H_{xb2} - D_{xp2}) + 2Q_b R_b (H_{yb2} - D_{yp2}) + 2P_b V_b - 2Q_b U_b + P_w P_b (4D_{sp2} - H_{sb2}) \\
- \dot{P}_{w2} D_{yp2} + 2P_{w2}^2 D_{xp2} - P_{w2} V_b + P_{w2} R_b (H_{xb2} - D_{xp2})] \\
= m_w \dot{W}_b + m_w (H_{yb2} - D_{xp2}) \dot{P}_b + m_w (D_{xp2} - H_{sb2}) \dot{Q}_b - R_{zw2} - F_{zw2} \]

\[ L_{\text{aero}} = -I_{zzw2} \dot{P}_{w2} + (I_{zzw2} - I_{yyw2}) \dot{Q}_b R_b - I_{yzw2} (R_b^2 - Q_b^2) \\
= I_{zzw2} \dot{P}_b + D_{zw2} R_{yw2} - D_{yw2} R_{zw2} + D_{zp2} F_{yw2} - D_{yp2} F_{zw2} \]

\[ M_{\text{aero}} = -I_{yyw2} Q_b + I_{yzw2} R_b -(I_{zzw2} - I_{zzw2}) (P_b + P_{w2}) R_b - I_{yzw2} (P_b + P_{w2}) Q_b \\
= I_{yyw2} \dot{Q}_b - I_{yzw2} \dot{R}_b - D_{zw2} R_{zw2} + D_{xz2} R_{zw2} - D_{zp2} F_{zw2} + D_{xp2} F_{zw2} - M_{Mw2} \]

\[ N_{\text{aero}} = -I_{zzw2} R_b + I_{yzw2} Q_b + I_{yzw2} (P_b + P_{w2}) R_b -(I_{yyw2} - I_{zzw2}) (P_b + P_{w2}) Q_b \\
= -I_{yzw2} \dot{Q}_b + I_{zzw2} \dot{R}_b + D_{yw2} R_{zw2} - D_{xz2} R_{zw2} + D_{xp2} F_{zw2} - D_{yp2} F_{zw2} - M_{Nw2} - J_{Nw2} \]

\[ X_{\text{aero}} = -m_c g \sin \Theta - m_c [-2Q_c^2 D_{sb} - 2R_b^2 D_{sb} + 2P_b Q_b D_{yb} + 2P_b R_b D_{sb}] \\
+ 2Q_b W_b - 2R_b V_b + 3Q_b D_{sb} - 3R_b D_{yb} + D_{sb}] \\
= m_c \dot{U}_b + m_c D_{sb} \dot{Q}_b - m_c D_{sb} \dot{R}_b + R_{xw1} + R_{xw2} \]

\[ Y_{\text{aero}} = m_c g \cos \Theta \sin \Phi - m_c [-2P_c^2 D_{yb} - 2R_b^2 D_{yb} + 2P_b Q_b D_{sb} + 2Q_b R_b D_{sb}] \\
- 2P_b W_b + 2R_b U_b - 3P_b D_{sb} + 3R_b D_{sb} + D_{sb}] \\
= m_c \dot{V}_b - m_c D_{yb} \dot{P}_b - m_c D_{sb} \dot{Q}_b + R_{yw1} + R_{yw2} \]

\[ Z_{\text{aero}} = m_c g \cos \Theta \cos \Phi - m_c [-2P_c^2 D_{sb} + 2Q_c^2 D_{sb} + 2P_b R_b D_{sb} + 2Q_b R_b D_{yb}] \\
+ 2P_b V_b - 2Q_c U_b + 3P_b D_{sb} + 3Q_b D_{sb} + D_{sb}] \\
= m_c \dot{W}_b + m_c D_{yb} \dot{P}_b - m_c D_{sb} \dot{Q}_b + R_{xw1} + R_{xw2} \]

\[ L_{\text{aero}} = -(I_{xxcp} - I_{yycp}) Q_b R_b = I_{xxcp} \dot{P}_b - (b_{cp} / 2) R_{zw2} + (b_{cp} / 2) R_{zw2} \]

\[ M_{\text{aero}} = -(I_{xxcp} - I_{yycp}) P_b R_b = I_{yycp} \dot{Q}_b + M_{Mw1} + M_{Mw2} \]

\[ N_{\text{aero}} = -(I_{yycp} - I_{xxcp}) P_b Q_b = I_{xxcp} \dot{R}_b + (b_{cp} / 2) R_{xw2} + (b_{cp} / 2) R_{xw2} + M_{Nw1} + M_{Nw2} \]
In matrix form: \[ [A][x] = [B] \]

\[
\begin{bmatrix}
  m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  U_b \\
  V_c \\
  W_c \\
  R_b \\
  Q_c \\
  F_g \\
  F_{rc} \\
  F_{wr} \\
  F_{w1} \\
  F_{wr1} \\
  F_{w2} \\
  F_{wr2} \\
  F_{w3} \\
  J_{w1} \\
  J_{wr} \\
  R_{w1} \\
  R_{wr} \\
  R_{w2} \\
  R_{wr2} \\
  M_{w1} \\
  M_{wr1} \\
  M_{w2} \\
  M_{wr2} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  x \\
\end{bmatrix}
\]

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APPENDIX E - Moments and Products of Inertia of Wings

Assume the wing is a thin rectangular plate

\[ I_{xxw} = \frac{1}{12} m_w (L_w^2 + t_w^2) \]
\[ I_{yyw} = \frac{1}{12} m_w (c_w^2 + t_w^2) \]
\[ I_{zzw} = \frac{1}{12} m_w (L_w^2 + c_w^2) \]

\[ I_{xxw} = I_{xyw} = I_{yzw} = 0 \]

\[
\therefore I = \begin{bmatrix}
I_{xxw} & 0 & 0 \\
0 & I_{yyw} & 0 \\
0 & 0 & I_{zzw}
\end{bmatrix}
\]
if we rotate the wing by \(-\gamma\)

\[
\begin{aligned}
&\gamma \\
&\downarrow \\
&z
\end{aligned}
\]

this is the same as rotating the coordinate axis by \(\gamma\):

\[
\begin{aligned}
&\gamma \\
&\downarrow \\
&\bar{y} \\
&\bar{y} \\
&\downarrow \\
&\bar{z}
\end{aligned}
\]

We now find \(I\) w.r.t \(\bar{x}, \bar{y}, \bar{z}\)

direction cosine matrix

\[
\ell = 
\begin{bmatrix}
\ell_{\bar{x}\bar{x}} & \ell_{\bar{x}\bar{y}} & \ell_{\bar{x}\bar{z}} \\
\ell_{\bar{y}\bar{x}} & \ell_{\bar{y}\bar{y}} & \ell_{\bar{y}\bar{z}} \\
\ell_{\bar{z}\bar{x}} & \ell_{\bar{z}\bar{y}} & \ell_{\bar{z}\bar{z}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos 0^\circ & \cos 90^\circ & 0 \\
\cos 90^\circ & \cos \gamma & \cos(90^\circ - \gamma) \\
\cos 90^\circ & \cos(90^\circ + \gamma) & \cos \gamma
\end{bmatrix}
\]

Recall:

\[
\cos(\alpha \pm \theta) = \cos \alpha \cos \theta \mp \sin \alpha \sin \theta
\]
\( \ell = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{bmatrix} \)

\( \ell^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{bmatrix} \)

\( \mathbf{I} = \ell \mathbf{I} \ell^T \)

\[
\begin{bmatrix}
I_{xxw} & 0 & 0 \\
0 & I_{yyw} \cos \gamma & I_{zzw} \sin \gamma \\
0 & -I_{yyw} \sin \gamma & I_{zzw} \cos \gamma \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \gamma & -\sin \gamma \\
0 & \sin \gamma & \cos \gamma \\
\end{bmatrix}
\begin{bmatrix}
I_{xxw} & 0 & 0 \\
0 & I_{yyw} \cos^2 \gamma + I_{zzw} \sin^2 \gamma & -I_{yyw} \cos \gamma \sin \gamma + I_{zzw} \sin \gamma \cos \gamma \\
0 & -I_{yyw} \sin \gamma \cos \gamma + I_{zzw} \cos \gamma \sin \gamma & I_{yyw} \sin^2 \gamma + I_{zzw} \cos^2 \gamma \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_{xxw} & 0 & 0 \\
0 & I_{yyw} \cos^2 \gamma + I_{zzw} \sin^2 \gamma & -(I_{yyw} - I_{zzw}) \sin \gamma \cos \gamma \\
0 & -(I_{yyw} - I_{zzw}) \sin \gamma \cos \gamma & I_{yyw} \sin^2 \gamma + I_{zzw} \cos^2 \gamma \\
\end{bmatrix}
\]
however since $-\gamma$ and

$$\gamma = \gamma_0 \cos \omega t$$

$$\frac{d\vec{I}}{dt} = \frac{d\vec{I}}{dy} \frac{dy}{dt}$$

$$\vec{I}_{xxw} = I_{xxw}$$

$$\vec{I}_{yyw} = I_{yyw} \cos^2(-\gamma) + I_{zzw} \sin^2(-\gamma)$$

$$\vec{I}_{zzw} = I_{yyw} \sin^2(-\gamma) + I_{zzw} \cos^2(-\gamma)$$

$$\vec{I}_{yzw} = -(I_{yyw} - I_{zzw}) \sin(-\gamma) \cos(-\gamma)$$

$$\vec{I}_{xyw} = 0$$

$$\vec{I}_{xzw} = 0$$

$$\frac{d\vec{I}_{yyw}}{dt} = \gamma_0 \omega \sin \omega t (I_{zzw} - I_{yyw}) \cos(-\gamma) \sin(-\gamma)$$

$$\frac{d\vec{I}_{zzw}}{dt} = \gamma_0 \omega \sin \omega t (I_{yyw} - I_{zzw}) \cos(-\gamma) \sin(-\gamma)$$

$$\frac{d\vec{I}_{yyw}}{dt} = \gamma_0 \omega \sin \omega t (\sin^2 \gamma - \cos^2 \gamma) (I_{yyw} - I_{zzw})$$

$$\frac{d\vec{I}_{xxw}}{dt} = \frac{d\vec{I}_{xzw}}{dt} = \frac{d\vec{I}_{xyw}}{dt} = 0$$
APPENDIX F - Order of Magnitude Analysis for Twisting

This order of magnitude analysis shows why the twisting of the wing can be ignored in the dynamic analysis.

Flapping occurs about the root of the wing, and thus the moment is

\[ M_x = I_{xx} \dot{\theta}_w \]

Since the wing flaps sinusoidally, \( P_w = \gamma_0 \cos \alpha \), then

\[ M_x = I_{xx} \gamma_0 \omega^2 \cos \alpha \]

If \( b_w \) is the wing half-span then

\[ I_{xx} = \frac{1}{12} m_w (b_w^2) + m_w \left( \frac{b_w}{2} \right)^2 \]

\[ = m_w \frac{b_w^2}{4} \]

Therefore,

\[ M_x = m_w \frac{b_w^2}{4} \gamma_0 \omega^2 \cos \alpha \]

If the tip of the wing has a maximum twist angle \( \alpha_0 \), then

\[ M_y = I_{yy} \frac{\alpha_0}{2} \omega^2 \cos \alpha \]

If \( c_w \) is the mean wing chord then

\[ I_{yy} = \frac{1}{12} m_w c_w^2 \]

Therefore,

\[ M_y = \frac{1}{24} m_w c_w^2 \alpha_0 \omega^2 \cos \alpha \]

Now taking the ratio of \( M_y \) / \( M_x \) gives:

\[
\frac{M_y}{M_x} = \frac{1}{6} \frac{c_w^2 \alpha_0}{b_w^2 \gamma_0}
\]

Plugging in the numbers gives:

\[
\frac{M_y}{M_x} = \frac{1}{50}
\]
APPENDIX F - Order of Magnitude Analysis for Twisting

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\[
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\]

Plugging in the numbers gives:

\[
\frac{M_y}{M_x} = \frac{1}{50}
\]
APPENDIX G - BIRD FLIGHT

The main difference between fixed-wing aircraft and birds is the fact that birds use their wings to generate both lift and thrust. An efficient airfoil is composed to two forces: the normal force which is perpendicular to the airfoil and the leading edge suction force which is parallel. The typical airfoil for birds and bats is thin and cambered, which means no leading edge suction is created. They obtain lift and thrust by twisting their wing forward on the downstroke so that the normal force propels them forward and produces positive lift. On the upstroke, the wing tilts backwards giving positive lift but could possibly give negative thrust (See Figure G.1)

![Diagram of bird flight forces](image)

Figure G.1: Force diagram for bird flight
However, on the upstroke birds collapse their wings to produce neither lift nor thrust [Ref.3] (See Figure G.2).

This variable-span model allows flapping flight to be attained with constant bound vorticity through the flapping cycle. No transverse unsteady vortex wake is shed.

At low Reynolds numbers, the boundary layer on the wing remains turbulent and thus delays stalling. Many other complicated effects are present such as feathering, up and down movements of wing tip vortices, downwash, and contribution of the tail feathers.

The major advantage of bird flight is the bird's ability to change speed, shape (camber) and the angle of different parts of the wing to achieve an optimum mix of lift and thrust.
APPENDIX H - COMPUTER PROGRAMS
* F1.FOR
* LONGITUDINAL STABILITY (2-PANEL)
*
* This program solves the longitudinal equations of motion to
* determine the longitudinal stability of the 2-panel model. The
* program is tested using the inputs for Mr. Bill and the full-scale
* ornithopter.
*
REAL*8 A(9,9),B(9),IYYB,IXXW0,IZZW0,IYYW0,DXXW,IYYW @,IYZW,IYYDOTW,IYDOTTW,EW(5000),EWI(100),MW,MB,LW,LWC,LT,LB @,LAERO,C,LAERO,MAERO,MAERC,MAEROT,MAEROB,NAEROW @,NAEROC,XC,ZC,x(9),aa(9,9+1)
INTEGER TD,TD1,PBTIM
DATA LU4/LU2/T,FI/3.141592654/ROE/1.2256/
DATA G/9.806/,THETAT/0.0/,WBT/0.0/,UBT/0.0/

*.-Read in wing characteristics.-*
OPEN(UNIT=LU,FILE='WING1.DAT')
READ(LU,*)CLALFAW,ALFA0W,CD0W,CW,BW,GAMA0, MW,TW,EFW,W,XWC, @CMOWW
CLOSE(LU)

*.-Calculate moments of inertia of wing at flapping angle(GAMA)=0.-*
IXXW0=MW/12.0*(BW*BW+TW*TW)
IYYW0=MW/12.0*(CW*CW+TW*TW)
IZZW0=MW/12.0*(CW*CW+BW*BW)

*.-Read in tail characteristics.-*
OPEN(UNIT=LU,FILE='TAIL1.DAT')
READ(LU,*)CLALFAT,ALFA0T,CD0T,CT,BT,ECL,EFFT,XT,ZT,CM0T,GT
CLOSE(LU)

*.-Read in body characteristics.-*
OPEN(UNIT=LU,FILE='BODY1.DAT')
READ(LU,*)MB,IYYB,DXB,DYB,DZB,CLALFAB,CD0B,VOLB,CMALFAB, @SRFB
CLOSE(LU)

*.-Read in initial conditions.-*
OPEN(UNIT=LU,FILE='INITI1.DAT')
READ(LU,*)QB,UB,WB,THETA,QDOTB,TDEL,TREC,DELR,PHI,ALFA0, @XC,ZC
READ(LU,*)PBTIM,PUB,PWB,PBQ
CLOSE(LU)
do 5 i=1,100
ewi(i)=0.001
5 continue

*.-Open file for output.-*
OPEN(UNIT=LU2,FORM='FORMATTED',FILE='OUTPUT1.DAT')

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!Assign the value of the temporary variables to the unknown state variables
UBC=UB
WBC=WB
QBC=QB
THETAC=THETA

IEND=INT(TREC/TDEL+.5)  !Total number of time intervals
IRREC=INT(BW/DELR+.5)  !Total number of wing panels

DO 10 I=1,IEND

!Check if time interval is the perturbation time specified
IF(I.EQ. PBTIM) THEN
    UB=UB+PBUB  !Add perturbation values to the state variables
    WB=WB+PBWB
    THETA=THETA+PBQB
    UBC=UBC+PBUB
    WBC=WBC+PBWB
    THETAC=THETAC+PBQB
END IF

DO 25 K=1,2

T=TDEL*(I-1)  !Determine time in seconds

!Calculate flapping angle at time T and its derivatives
GAMA=GAMA0*COS(W*T)
P=W=GAMA0*W*SIN(W*T)
PDOTW=-GAMA0*W*W*COS(W*T)

!Calculate distance from wing root to wing cg
DYW=(BW/2*COS(GAMA))
DZW=(BW/2*SIN(GAMA))

!Calculate wing cg velocities
UW=UB+(DZB-DZW)*QB
VW=DZW*W
WW=WB-DXB*QB-DYW*PW

*-Block to solve for wing forces at time T by integrating over BW. *-
!Initialize total wing aerodynamic forces
XAEROC=0.0
ZAEROC=0.0
MAEROC=0.0
LAEROC=0.0

LW=0.0  !Initialize lift over entire wing
ALFA=ALFA0*COS(W*T+PHI)  !Calculate wing twist angle
DO 15 J=1,IRREC

R=J*DELR  !Calculate distance from wing root to wing panel

! Transform to convenient-axis (body-fixed)system & calculate wing panel velocities
UWC=(UB+(DZB-(R*SIN(GAMA)))*QB)
WWC=(WB+(DXB*QB)-(R*COS(GAMA)*PW))*COS(GAMA)
@ -R*(SIN(GAMA)**2)*PW)
QWC=QB*COS(GAMA)

! Calculate wing panel lift and drag
LWC=CLALFAW/2.0*(WWC/UWC+ALFA*R+ALFA0W+(CW*QWC/(2.0*UWC)))
@ *ROE*(UWC*UWC+WWC*WWC)*DELR*CW
LWC=LWC+LW  ! Record total lift over entire wing; to be used for downwash
DWC=(CD0W+(LWC*2.0/(ROE*(UWC*UWC+WWC*WWC)*DELR*CW))**2)/
@ ((BW*2.0)/(CW)**PI*EFFW)**(1/2.)*ROE*DELR*CW*(UWC*UWC+WWC
@ *WWC))

! Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=LWC-DWC*(WWC/UWC)
LAERO=ZAERO*(BW/2.-R)
MAERO=CM0W*(1/2.*ROE*(UWC*UWC+WWC*WWC)*DELR*CW*CW)-ZAERO*XWC
@ -PI/4*(1/2.*ROE*(UWC*UWC+WWC*WWC)*QWC*CW*CW*DELR)+(-.251
@ *ALFA0W+.004)*(1/2.*ROE*(UWC*UWC+WWC*WWC)*CW*CW*DELR)

! Add the panel values to the total wing aero forces
XAERO=XAERO+XAERO
ZAERO=ZAERO+ZAERO
LAERO=LAERO+LAERO
MAERO=MAERO+MAERO

CONTINUE

! Calculate remaining wing aero forces which required no panel summation
YAERO=PW/BW
NAERO=YAERO*XWC

! Transform wing aero forces back to stability-axis system
XAEROW=XAERO
YAEROW=YAERO*COS(GAMA)-ZAERO*SIN(GAMA)
ZAEROW=ZAERO*COS(GAMA)+YAERO*SIN(GAMA)
LAEROW=LAERO
MAEROW=MAERO*COS(GAMA)-NAERO*SIN(GAMA)
NAEROW=NAERO*COS(GAMA)+MAERO*SIN(GAMA)
*-Block to calculate tail forces and moments-*

!Calculate time delay for downwash to reach tail
TD=INT(XT*(-1)/TDEL)+.5

!Calculate downwash at time T=I*TDEL
EW(I+TD)=LW*2.0*(ROE*CW*BW*(UW*UW+WW*WW)*ECL+EW(I+TD)

!Check to see if this is the very first time interval
IF(I.EQ.1)THEN
TD1=TD+1 !Increment the initial time delay
END IF

!Calculate total angle of attack of tail
IF(1.GE.TD1)THEN
Ai=(WB/UB-(Q*BSXT)/UB+CT*QB/(2.0*UB)-EW(I)+ALFA0T)
ELSE
Ai=(WB/UB-(Q*BSXT)/UB+CT*QB/(2.0*UB)-EW(I)+ALFA0T) !for initial time interval
END IF

!Calculate tail lift and drag
LT=(CD0F+(LT-2.0*(ROE*UB/UB+(WB-QB*XT)*(WB-QB*XT))CT*BT
DT=(CD0F+(LT-2.0*(ROE*UB/UB+(WB-QB*XT)*(WB-QB*XT))CT*BT
(1.2.0*ROE*(UB*UB+(WB-QB*XT)*
(2.0*Q*QX)CT*BT)

!Calculate tail aero forces
XAERO=LT*(AI-(Q*CT)/(2.0*UB)-ALFA0T)-DT
ZAERO=LT-2.0*ROE*(UB*UB*(VB*WB)*CT*BT)-(ZAERO*XT)+XAERO

@ "ZT"

*-Block to calculate body forces and moments-*

!Calculate body lift and drag
LB=CLALFAB/2.0*(WB/UB)*ROE*(UB*UB+WB*WB)*(VOLB**2.3)
DB=(CD0F*SREFB+((WB/UB)**2).0137*.825*2*(1.2.0*ROE*
(UB*UB+WB*WB))

!Calculate body aero forces
XAERO=LB*(WB/UB)-DB+XAERO
ZAERO=-LB-DB*(WB/UB)*ZAERO
MAERO=-CMALFAB/2.0*ROE*(UB*UB+WB*WB)*VOLB*(WB/UB)+MAERO

!Calculate wings moments of inertia and derivatives
DXW=DXW0
IYWW=IYWW0*(COS(-GAMA)**2)+IZW0*(SIN(-GAMA)**2)
IZW=-(SIN(-GAMA)*COS(-GAMA)*(IYWW0-IZW0)
IYDODOT=(GAMA0*W*SIN(W**T))*(2*COS(-GAMA)*SIN(-GAMA)*(IZW0-
(IYWW0)
IZDODOT=(GAMA0*W*SIN(W**T))*(SIN(-GAMA)**2)-(COS(-GAMA)**2))
! Simultaneous solution of unknown variables at T + DELT

! Initialize eqn matrix to zero
DO 50 L = 1, 9
   DO 40 M = 1, 9
      A(M, L) = 0.0
   40 CONTINUE
50 CONTINUE

! Assign values to the eqn matrices
A(1, 1) = MB
A(1, 4) = -2.0
A(2, 2) = MB
A(2, 6) = -2.0
A(3, 3) = IYYB
A(3, 4) = -2.0 * DZB
A(3, 6) = 2.0 * DXB
A(3, 8) = -2.0
A(4, 1) = MW
A(4, 3) = MW * (DZB - DZW)
A(4, 4) = 1.0
A(5, 5) = 1.0
A(6, 2) = MW
A(6, 3) = -MW * DXB
A(6, 6) = 1.0
A(7, 5) = -DZW
A(7, 6) = DYW
A(7, 7) = 1.0
A(8, 3) = IYYW
A(8, 4) = DZW
A(8, 8) = 1.0
A(9, 3) = -IYZW
A(9, 4) = -DYW
A(9, 9) = 1.0
B(1) = XAEROB - (MB * G * SIN(THETA)) - (MB * QB * WB)
B(2) = ZAEROB + (MB * G * COS(THETA)) + (MB * QB * UB)
B(3) = MAEROB
B(4) = XAEROW - (MW * G * SIN(THETA)) - (2 * MW * WB * QB) + (2 * MW * DXB * (QB * QB))
   @ + (3 * MW * DYW * PW * QB)
B(5) = YAEROW - (MW * PDOTW * DZW) + (MW * PW * WB) - (MW * PW * QB * DXB)
   @ - (2 * MW * DYW * PW * PW)
B(6) = ZAEROW + (MW * G * COS(THETA)) + (2 * MW * UB * QB) - (2 * MW * DZW * QB * QB)
   @ + (2 * MW * DZB * QB * QB) + (MW * DYW * PDOTW) - (2 * MW * DZW * PW * PW)
B(7) = LAEROW - (DXW * PDOTW) + (TYZW * QB * QB)
B(8) = MAEROW - (IYDOTW * QB) - (IYZW * PW * QB)
B(9) = NAEROW + (IYZDOTW * QB) - (PW * QB * (IYYW - DXW))

CALL GAUSS(A, xx, b, x, 9, 9 + 1, 9, 9 + 1)
IF(K.EQ.1)THEN
!Perform 1st part of 2nd-order Runge-Kutta integration
UDOTB=x(1)  !f(xn,un) = _DOTB
WDOTB=x(2)  !This is not a function of xn(time), only un(UBC)
QDOTB=x(3)
XDOTB=(UB*COS(THETA))+(WB*SIN(THETA))  !Calculate trajectory
ZDOTB=(UB*SIN(THETA))+(WB*COS(THETA))
UB=UDOTB*TDEL+UBC  !Store k1+un in the temporary variables
WB=WDOTB*TDEL+WBC  !k1 = _DOTB*TDEL
QB=QDOTB*TDEL+QBC
THETA=QB*TDEL+THETAC
ELSE
!Perform 2nd part of 2nd-order Runge-Kutta integration
UDOTBC=x(1)  !f(xn+h,un+k1) = _DOTBC
WDOTBC=x(2)
QDOTBC=x(3)
RXBC=x(4)
RYBC=x(5)
RZBC=x(6)
UBC=(UDOTBC+UDOTB)/2.0*TDEL+UBC  !k2 = _DOTBC*TDEL
WBC=(WDOTBC+WDOTB)/2.0*TDEL+WBC  !un+1 = un + (k1+k2)/2
QBC=(QDOTBC+QDOTB)/2.0*TDEL+QBC
THETAC=(QBC+QB)/2.0*TDEL+THETAC
XDOTC=(UBC*COS(THETAC))+(WB*SIN(THETAC))  !Calculate trajectory
ZDOTC=(UBC*SIN(THETAC))+(WB*COS(THETAC))
XC=(XDOTC+XDOTB)/2.0*TDEL+XC
ZC=(ZDOTC+ZDOTB)/2.0*TDEL+ZC
ALFA0T=GT*QBC+ALFA0T  !Feedback of gain GT on pitch-rate
ENDIF

WRITE(6,6000)
ALFA0T=.2
ENDIF

WRITE(LU2,8500)T,UBC,UDOTBC,WBC,WDOTB,C,QBC*180/P1
@ ,THETAC*180/P1,XC,ZC,RXBC,RYBC,RZBC

ALFA0T=GT*QBC+ALFA0T  !Feedback of gain GT on pitch-rate

WRITE(6,6000)
ALFA0T=.2
ENDIF

WRITE(LU2,8500)T,UBC,UDOTBC,WBC,WDOTB,C,QBC*180/P1
@ ,THETAC*180/P1,XC,ZC,RXBC,RYBC,RZBC

!Calculate total values for the unknown state variables
UBT=UB+UBC
WBT=WBC+WBT
THETAT=THETAC+THETAT
!Assign values to the temporary variables for the next time increment
UB=UBC
WB=WBC
QB=QBC
THETA=THETAC
ENDIF

25 CONTINUE
10 CONTINUE
!Calculate average values for the unknown state variables
UBA=UBT/REAL(IEND)
WBA=WBT/REAL(IEND)
THETAA=THETAT/REAL(IEND)
WRITE(6,9000)UBA,WBA,THETAA*180/PI

CONTINUE

125  FORMAT(12F10.2)
150  FORMAT(1X,'U-VELOCITY',3X,'W-VELOCITY',3X,'PITCH RATE',3X,
       @'PITCH ANGLE',4X,'GAMA',7X,'ALFAT')
200  FORMAT(2X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4,5X,F7.4,5X,F7.4)
2000 FORMAT(2X,'SINGULAR MATRIX')
3000 FORMAT(2X,'WARNING ERROR IN IDGT DECIMAL PLACE')
5000 FORMAT(2X,'SMALL ANGLE EXCEEDED !')
6000 FORMAT(2X,'TAIL AT MAX DEFLECTION')
8000 FORMAT(2X,'LINEAR AERO EXCEEDED !')
8500 FORMAT(12(F16.8))
9000 FORMAT('AV.U-VEL=',F8.4,1X,'(m/s) AV.W-VEL=',F8.4,1X,
       @'(m/s) AV.THEATA=',F8.4,1X,'(deg)')
STOP
END
* VARIABLES (2-panel) *

A - left hand side matrix for eqn solver
AI - total angle of attack of tail including total incidence angle of tail, induced angle due to plunging and pitch rate effects, and induced downwash
ALFA - wing twist angle at time t
ALFAO - maximum wing twist angle
ALFAOT - total incidence angle of tail
ALFAOW - mean total incidence angle of wing
B - right hand side matrix for eqn solver
BT - full tail span
BW - wing half-span
CDOB - body zero-lift drag coefficient
CDOT - tail zero-lift drag coefficient
CDOW - zero-lift drag coefficient
CLALFAB - body lift-curve slope
CLALFAT - tail lift-curve slope
CLALFAW - wing lift-curve slope
CMALFAB - body pitching-moment-curve slope
CMOT - tail pitching-moment coefficient
CMOW - wing pitching-moment coefficient
CT - mean tail chord
CW - mean chord of wing
DELR - section increments along wing
DWC - drag of wing
DXB - x distance from body cg. to point on root of wing aligned with wing cg
DYB - y distance from body cg. to root of wing
DZB - z distance from body cg. to root of wing
DYW - y distance from root of wing to cg of wing panel
DZW - z distance from root of wing to cg of wing panel
ECL - downwash ratio(e/c)
EFFF - Oswald efficiency factor
EFFW - Oswald efficiency factor
ENG - energy
EW - downwash angle
EWI - initial downwash angle
G - gravitational acceleration
GAMA - maximum flapping angle wrt. the horizontal axis
GAMA - flapping angle
GT - feedback gain
IDGT - input required for eqn solver to check decimal accuracy
IEND - total number of time intervals
IRREC - total number of panels wing is sectioned into
IYQB - body moment of inertia
IYXW, IYYW, IZZW, IYZW - wing moment of inertia
IYXW0, IYYW0, IZZW0 - wing moment of inertia at flapping angle=0
IYDYOTW, IZZDOTW, IYZDOTW - wing moment of inertia derivatives
K - counter for 2nd-order Runge-Kutta integration
LAEROC - total wing rolling moment (along x) in body-fixed system
LAERO - panel value wing rolling moment (along x) in body-fixed system
LAEROW - total wing rolling moment (along x) in stability-axis system
LW - total lift over entire wing
LWC - lift over wing panel
LT - lift of tail
LB - lift of body
MAEROC - total wing pitching moment (along y) in body-fixed system
MAERO - panel value wing pitching moment (along y) in body-fixed system
MAEROW - total wing pitching moment (along y) in stability system
MAEROT,MAEROB - total tail and body pitching moment (along y)
MB - mass of body
MW - mass of wing
MLBC,MMBC,MNBC - moment reaction at wing-body interface
MLB,MMB,MNB - temp variables for moment reaction at wing-body interface
NAEROC - total wing yawing moment (along z) in body-fixed system
NAERO - panel value wing yawing moment (along z) in body-fixed system
NAEROW - total wing yawing moment (along z) in stability system
PBTIM - time to begin perturbation
PBUB - forward velocity perturbation
PBWB - plunging velocity perturbation
PBQB - pitch rate perturbation
PHI - phase angle between flapping angle and angle of attack
PI - value of pi
FW - wing angular velocity about x-axis (derivative of GAMA)
PDOTW - wing angular acceleration about x-axis
QB - temporary variable for pitch rate
QBC - pitch rate
QBT - total pitch rate
QDOTB - initial pitch acceleration
R - accumulated distance from wing root to cg of wing panel
ROE - density of air
SREFB - wetted area or surface area of body
T - time
TD - time delay
TD1 - initial time delay
TDEL - time increment
TREC - total time
THETA - temporary variable for pitch angle
THETAA - average pitch angle
THETAC - pitch angle
THETAT - total pitch angle
TW - mean wing thickness
UB - temporary variable for forward velocity
UBA - average forward velocity
UBC - forward velocity
UBT - total forward velocity
VOLB - volume of body
W - flapping frequency (rad/s)
WB - temporary variable for plunging velocity
WBA - average plunging velocity
WBC - initial plunging velocity
WBT - total plunging velocity
XWC - x distance from wing cg to wing root
XC - x point
ZC - z point
XDOTC, ZDOTC - trajectory derivatives
XT - x distance from body cg to tail cg
ZT - z distance from body cg to tail cg
* F2.FOR
* LONGITUDINAL STABILITY (3-PANEL)
*
* This program solves the longitudinal equations of motion to
* determine the longitudinal stability of the 3-panel model. The
* program is tested using the inputs for Mr. Bill and the full-scale
* ornithopter.
* A[3,3]=IYYB, A[5,5]=DZW If any of these variables is assigned a
* value of zero, the resulting matrix will be singular. If the case of
* zero flapping angle is required, then GAMAO and DZP should be made
* as small as possible ie. 1E-06
* 
* REAL*8 A(12,12),B(12),IYYB,DXW0,IZZW0,IYYW0,DXW,IFYW
* @JYZW,IYDOWTW,IYZDOTW,EW(5000),EWI(100),MW,MB,LW,LWCL,T,LB
* @LAE,OA,MEO,LAER,
* @MCERO,MAERO,MAERO,MMAERO,MMAERO,MAERO,MAEROCP,MAERO,W,AEQ,AEQ
* @X,C,IC,YC,P,D1DXB,D2DXB,DZB,D1DZB,D2DZB,D1DZB,T1,(12),aa(12,12+1)
* REAL*8 IY,IZ,W,ICP,ICP,MC,P,TXW,TC,C,DQCP
* INTEGER TD1,TD1,T1,PBTM
* DATA L4/4,L4/4,PI/3.141592654/ROE/1.2256/
* DATA G/9.806/THETAT/0.0,WBT/0.0,UBT/0.0/
*
* .-Read in wing characteristics.-
* OPEN(UNIT=LU,FILE="WING2.DAT")
* READ(LU,*),CLL,ALF,AW,CD0W,CW,BW,BW2,GAMAO,MW,TD,EFFW,W,XWC,CM0W
* READ(LU,*),BCP,CCP,ICP,IZ,W,ICP,ICP,MC,ICP,IP,ICP,IP,ICP,ICP
* CLOSE(LU)
*
* .-Calculate moments of inertia of wing at flapping angle(GAMAO)=0.-
* DXW0=MW/12.0*((BW+BW2)*(BW+BW2)*TW*TW)
* IYYW0=MW/12.0*(CW*CW+TW*TW)
* IZZW0=MW/12.0*(CW*CW+(BW+BW2)*(BW+BW2))
*
* .-Read in tail characteristics.-
* OPEN(UNIT=LU,FILE="TAIL2.DAT")
* READ(LU,*),CLL,ALF,AW,CD0T,CT,BT,CLF,XT,XT,CM,T,GT,
* @C,DE
* CLOSE(LU)
*
* .-Read in body characteristics.-
* OPEN(UNIT=LU,FILE="BODY2.DAT")
* READ(LU,*),MB,IFYB,DXBMAX,DYBMAX,DZBMAX,DXMAX,DXBEQL,DXBEQL,CLLFAB
* @CD0B,VO,CMALFAB,REFB,HXB,HZB
* CLOSE(LU)
*
* .-Read in initial conditions.-
* OPEN(UNIT=LU,FILE="INIT2.DAT")
* READ(LU,*),QB,UB,UB,UB,THETA,QTDB,TDEL,DEL,TDEL,DEL2,DEL2,PHI,ALFA0,
* @X,2C
* READ(LU,*),QBTM,PUB,PW,PW,PQ
* CLOSE(LU)
do 5 i=1,100
    ewi(i)=0.001
  5  continue

* -Open file for output-*
OPEN(UNIT=LU2,FORM='FORMATTED',FILE='OUTPUT2.DAT')

! Assign the value of the temporary variables to the unknown state variables
UBC=UB
WBC=WB
QBC=QB
THETAC=THETA

ALFAOW=IW+ZLLW "Calculate wing angle of attack"
ALFA0CP=ICP+ZLLCP "Calculate center panel angle of attack"

IEND=INT(TREC/TDEL+.5) "Total number of time intervals"

DO 10 I=1,IEND

! Check if time interval is the perturbation time specified
IF(1.EQ.PBTIM)THEN
    UB=UB+PBUB "Add perturbation values to the state variables"
    WB=WB+PBWB
    THETA=THETA+PBQ8
    UBC=UBC+PBUB
    WBC=WBC+PBWB
    THETAC=THETAC+PBQ8
END IF

DO 25 K=1,2

T=TDEL*(I-1) "Determine time in seconds"

! Calculate flapping angle at time T and its derivatives
GAMA=GAMA0*COS(W*T)
PW=-GAMA0*W*SIN(W*T)
PDOTW=GAMA0*W*W*COS(W*T)

! Calculate distance from body cg to center panel cg & derivatives
DZB=DZBEQL - DZBMAX*COS(W*T-PI)
D1DZB=W*DZBMAX*SIN(W*T-PI)
D2DZB=W*W*DZBMAX*COS(W*T-PI)
DXB=DXBEQL+DXBMAX*COS(W*T-PI) !x is not needed because any plunging motion occurs
D1DXB=W*DXBMAX*SIN(W*T-PI)
D2DXB=W*W*DXBMAX*COS(W*T-PI)
D1DYB=0

! Calculate distance from wing cg to wing root
DXW=0
DYW=((BW+BW2)/2*COS(GAMA))
DZW=((BW+BW2)/2*SIN(GAMA))

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!Calculate center panel cg velocities
UCP=UB+QB*DXB+DI1XB
VCP=DI1YB
WCP=WB-QB*DXB+DI1ZB

* Block to solve for wing forces at time T by integrating over BW *

! Initialize total wing aerodynamic forces
XAEROC=0.0
ZAEROC=0.0
MAEROC=0.0
LAEROC=0.0

LW=0.0
! Initialize lift over entire wing
ALFA=ALFA0*COS(W*T+PHI)
! Calculate wing twist angle

! Determine aero terms for the section of wing from the pivot to the wing tip
IRREC=INT(BW/DELR+.5) ! Total number of wing panels
DO 15 J=1,IRREC

R=J*DELR ! Calculate distance from wing root to wing panel

! Transform to convenient-axis(body-fixed) system & calculate wing panel velocities
UWC=(UB+((HZB-(R*SIN(GAMA))*QB))
WWC=(WB+(-HXB*QB)+(R*COS(GAMA)*PW))*COS(GAMA)
QWC=QB*COS(GAMA)

! Calculate wing panel lift and drag
LWC=CLAFAW/2.0*(WWC/UWC+ALFA*R+ALFA0W+(CW*QWC/(2.0*UWC)))
@ !ROE*(UWC/UWC+WWC*WWC)*DELR*CW
LW=LWC+LW ! Record total lift over entire wing; to be used for downwash
DWC=(CDOW+((LWC2.0*(ROE=(UWC/UWC+WWC*WWC)*DELR*CW))**2)/
@ (((BW+BW2+0.5*BCP)*2.0)*PI*EFFW)*(1/2.*ROE*DELR*CW*
@ (UWC/UWC+WWC*WWC))

! Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=-LWC-DWC*(WWC/UWC)
LAERO=ZAERO*(BW+BW2)/2-R
MAERO=(CM0W-PI4*QWC-0.251*ALFA0W+0.004)*
@ (1/2.*ROE*(UWC/UWC+WWC*WWC)*DELR*CW*CW) - ZAERO*XWC

! Add the panel values to the total wing aero forces
XAEROC=XAERO+XAEROC
ZAEROC=ZAERO+ZAEROC
LAEROC=LAERO+LAEROC
MAEROC=MAERO+MAEROC
15 CONTINUE
! Determine aero terms for the section of wing from the wing root to the pivot
! This section of the wing is considered to be rigid
IRREC=INT(BW2/DELR2+.5) ! Total number of wing panels
DO 17 J=1,IRREC

R=J*DELR2 ! Calculate distance from wing root to wing panel

! Transform to convenient-axis (body-fixed) system & calculate wing panel velocities
UWC=(UB+(HZB-(R*SIN(PI+GAMA)))*QB)
WWC=(WB+(-HXB*QB)+(-R*COS(PI+GAMA)*PW))*COS(GAMA)

! Note that twisting is neglected because this section is rigid
LWC=CLALFAW/2.0*(WWC/UWC+ALFA0W+(CW*QWC/(2.0*UWC)))

! Calculate wing panel lift and drag
DWC=(CD0W+((LWC/2.0*(ROE*(UWC/UWC+WWC*WWC)*DELR2*CW))**2)/

! Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=-LWC-DWC*(WWC/UWC)
LAERO=ZAERO*((B/W+B/2)/2+R)
MAERO=(CM0W*PI/4*QWC-0.251*ALFA0W+0.004)*

! Add the panel values to the total wing aero forces
XAERO=XAERO+XAERO
ZAERO=ZAERO+ZAERO
LAERO=LAERO+LAERO
MAERO=MAERO+MAERO

17 CONTINUE

! Calculate remaining wing aero forces which required no panel summation
YAERO=P/(B/W+B/2)
NAERO=YAERO*XWC

! Transform wing aero forces back to stability-axis system
XAERO=YAERO
YAEROW=YAERO*COS(GAMA)-ZAERO*SIN(GAMA)
ZAERO=ZAERO*COS(GAMA)+YAERO*SIN(GAMA)
LAERO=LAERO
MAEROW=MAERO*COS(GAMA)-NAERO*SIN(GAMA)
NAEROW=NAERO*COS(GAMA)+MAERO*SIN(GAMA)

! Calculate distance from wing cg to wing pivot
DXP=0
DYP=-(B/W-B/2)*COS(GAMA))
DZP=-(B/W-B/2)*SIN(GAMA))
!Calculate wing cg velocities
UW=UB+QB*(HZB-DZP)
VW=PW*DZP
WW=WB+QB*(DXP-HXB)-PW*DYP

*-Block to calculate tail forces and moments-*

!Calculate time delay for downwash to reach tail
TD = INT((XT)^(3.01)(UB+TDEL)+.5)
!Calculate downwash at time T=[*TDEL] 
EW=(1+TD)=LW*2.0/(ROE*CW*(BW+BW2)*(UB*UW+WW*WW)*ECL+ECL(I+TD)
!Check to see if this is the very first time interval
IF(1.EQ. 1)THEN
TD1=TD+1 !Increment the initial time delay
END IF

!Calculate total angle of attack of tail
IF(1.GE. TD1 )THEN
Al=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW(I)+ALFA0T)
ELSE
Al=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW(I)+ALFA0T) !for initial time interval
END IF

!Calculate tail lift and drag
LT=CLALFAT/2.0*ROE*AI*(UB*UB+(WB-QB*XT)*WB-QB*XT)*BT*CT
DT=(CD0T+(LT*2.0*(ROE*(UB*UB+(WB-QB*XT)*WB-QB*XT)))*BT@
@ *)**(2.0*(BT/CT)*PI*EFFT))/((2.0*ROE*(UB*UB+(WB-QB*XT))*)
@ *(QB-QB*XT)*CT*BT)

!Calculate lift and moment due to control system(elevator)
ZCB=CLDTE*DE*ROE/2.0*(UB*UB+(WB-QB*XT)*WB-QB*XT)*BT*CT
MCB=XT*ZCB

!Calculate tail aero forces
XAEROT=LT*(AI-(QB*CT)/(2.0*UB)-ALFA0T)-DT
ZAEROT=LT-DT*(AI-(QB*CT)/(2.0*UB)-ALFA0T)
MAEROT=CM0T*(1.0*ROE*(UB*UB+WB*WB)*CT*CT*BT)-(ZAEROT+XT)*XAEROT
@ *2T

*-Block to calculate body forces and moments-*

!Calculate body lift and drag
LB=CLALFAB/2.0*(WB/UB)*ROE*(UB*UB+WB*WB)*(VOLB**(2.3))
DB=(CD0B*SREFB+((WB/UB)**2)*0.0137*.825**2)(1/2.0*ROE*
@ *(UB*UB+WB*WB))

!Calculate body aero forces
XAEROB=LB*(WB/UB)-DB+XAEROT
ZAEROB=LB-DB*(WB/UB)+ZAEROT
MAEROB=CMALFAB/2.0*ROE*(UB*UB+WB*WB)*VOLB*(WB/UB)+MAEROT
*Block to calculate center panel forces and moments.*

!Calculate center panel lift and drag

\[ LCP = CLAF + (WCP + CCP) \times QB / (2.0 \times UCP) + ALFA0CP \times ROE / 2 \]

@ *(UCP + WCP + CCP) \times BCP + CCP \times DCP = (CD0W + (LCP \times 2.0 \times ROE + (UCP + CCP) \times BCP + CCP) \times ROE / 2) / ((BW + BW2 + 0.5 \times BCP) \times CW \times PI \times EFFW) \times (1/2.0 \times ROE \times BCP + CCP) \times (UCP + WCP + CCP) \]

!Calculate center panel aero forces

\[ XAEROCP = LCP \times (WCP + UCP) \times DCP \]

\[ ZAEROCP = -(LCP - DCP) \times (WCP + UCP) \]

\[ MAEROCP = (CM0W - PI / 4 \times QB - 0.251 \times ALFA0CP + 0.004) \times (1/2.0 \times ROE \times (UCP + WCP + CCP) \times BCP + CCP) \times (UCP + WCP + CCP) \]

!Calculate wings moments of inertia and derivatives

\[ DXXW = DXXW0 \]

\[ IYYW = IYYW0 + COS(-GAMA) \times IZZW0 \times SIN(-GAMA) \]

\[ IYZW = SIN(-GAMA) \times COS(-GAMA) \times IYYW0 \times IZZW0 \]

\[ IYYDOTW = GAMA0 \times W \times SIN(W \times T) \times (2 \times COS(-GAMA) \times SIN(-GAMA) \times IZZW0 \times IYYW0) \]

\[ IYZDOTW = GAMA0 \times W \times SIN(W \times T) \times (IYYW0 - IZZW0) \]

*Simultaneous solution of unknown variables at T+DELT - *

!Initialize eqn matrix to zero

DO 50 L = 1, 12
   DO 40 M = 1, 12
      A(M, L) = 0.0
   CONTINUE
50 CONTINUE

!Assign values to the eqn matrices

\[ A(1, 1) = MB \]

\[ A(1, 7) = 2 \]

\[ A(2, 2) = MB \]

\[ A(2, 9) = 2 \]

\[ A(3, 3) = IYYB \]

\[ A(3, 7) = 2 \times HZB \]

\[ A(3, 9) = -2 \times HXB \]

\[ A(3, 12) = 2 \]

\[ A(4, 1) = MCP \]

\[ A(4, 3) = MCP \times DZB \]

\[ A(4, 4) = 2 \]

\[ A(6, 2) = MCP \]

\[ A(6, 3) = -MCP \times DZB \]

\[ A(6, 6) = 2 \]

\[ A(10, 3) = IYYCP \]

\[ A(10, 10) = 2 \]
A(7,1) = MW
A(7,3) = MW*(HZB-DZP)
A(7,4) = -1
A(7,7) = -1
A(8,5) = -1
A(8,8) = -1
A(9,2) = MW
A(9,3) = MW*(DXP-HXB)
A(9,6) = -1
A(9,9) = -1
A(5,5) = DZW
A(5,6) = -DYW
A(5,8) = DZP
A(5,9) = -DYP
A(12,3) = IYYW
A(12,4) = -DZW
A(12,6) = DXW
A(12,7) = -DYP
A(12,9) = DXP
A(12,10) = -1
A(12,12) = -1
A(11,3) = -IYZW
A(11,4) = DYW
A(11,5) = -DXW
A(11,7) = DYP
A(11,8) = -DXP
A(11,11) = -1

B(1) = XAEROB - MB*G*SIN(THETA) - MB*QB*WB
B(2) = ZAEROB + MB*G*COS(THETA) + ZCB + MB*QB*UB
B(3) = MAEROB + MCB

B(4) = XAEROCP - MCP*G*SIN(THETA) + TXCP - MCP*(-2*QB*QB*DXB)
@ +3*QB*D1DZB + D2DXB + 2*QB*WB
B(6) = ZAEROCP + MCP*G*COS(THETA) - MCP*(-2*QB*QB*DZB)
@ -3*QB*D1DXB + D2DZB - 2*QB*UB
B(10) = MAEROCP

B(7) = XAEROW - MW*G*SIN(THETA) + TXW - MW*(2*QB*QB*(DXP-HXB))
@ +2*QB*WB - 3*QB*PW*DYP
B(8) = YAEROW - MW*(PDOTW*DXZP + 2*PW*PW*DYP - PW*WB)
@ -PW*QB*(DXP-HXB))
B(9) = ZAEROW + MW*G*COS(THETA) - MW*(2*QB*QB*(DZP-HZB))
@ -2*QB*UB - PDOTW*DYP + 2*PW*PW*DZP)
B(5) = LAEROW - DXW*PDOTW + IYZW*QB*QB
B(12) = MAEROW - IYYDOTW*QB - IYZW*PW*QB
B(11) = NAEROW + IYZDOTW*QB - (IYYW-DXXW)*PW*QB

call gauss(a,aa,b,x,12,12+1,12,12+1)
IF(K.EQ. 1) THEN
  !Perform 1st part of 2nd-order Runge-Kutta integration
  UDOTB=x(1)  !f(xn,un) = _DOTB
  WDOTB=x(2)  !This is not a function of xn(time), only un(UBC)
  QDOTB=x(3)
  XDOT=(UB*COS(THETA))+(WB*SIN(THETA))  !Calculate trajectory
  ZDOT=(-UB*SIN(THETA))+(WB*COS(THETA))
  UB=UDOTB*TDEL+UBC  !Store k1+un in the temporary variables
  WB=WDOTB*TDEL+WBC  !k1 = _DOTB*TDEL
  QB=QDOTB*TDEL+QBC
  THETA=QB*TDEL+THETAC
ELSE
  !Perform 2nd part of 2nd-order Runge-Kutta integration
  UDOTBC=x(1)  !f(xn+h,un+k1) = _DOTBC
  WDOTBC=x(2)
  QDOTBC=x(3)
  RXBC=x(4)
  RYBC=x(5)
  RZBC=x(6)
  FXBC=x(7)
  FYBC=x(8)
  FZBC=x(9)
  UBC=(UDOTB+UDOTB)/2.0*TDEL+UBC  !k2 = _DOTB*TDEL
  WBC=(WDOTB+WDOTB)/2.0*TDEL+WBC  !un+1 = un + (k1+k2)/2
  QBC=(QDOTB+QDOTB)/2.0*TDEL+QBC
  THETAC=(QBC+QBC)/2.0*TDEL+THETAC
  XDOTC=(UB*COS(THETAC))+(WB*SIN(THETAC))  !Calculate trajectory
  ZDOTC=(-UB*SIN(THETAC))+(WB*COS(THETAC))
  XC=(XDOTC+XDOTC)/2.0*TDEL+XC
  ZC=(ZDOTC+ZDOTC)/2.0*TDEL+ZC
  ALFA0T=GT*QBC+ALFA0T  !Feedback of gain GT on pitch-rate

!Check for maximum deflection of tail
IF(ALFA0T .GE. .2) THEN
  WRITE(6,6000)
  ALFA0T=.2
ELSE IF(ALFA0T .LE. -2) THEN
  WRITE(6,6000)
  ALFA0T=-.2
END IF

WRITE(LU2,8500)T,UBC,UDOTBC,WBC,WDOTBC,QBC=180/PI
@ ,THETAC=180/PI,XC,ZC,RXBC+FXBC,RYBC+FYBC,RZBC+FZBC

!Calculate total values for the unknown state variables
UBY=UBC+UBT
WBY=WBC+WBT
THETAT=THETAC+THETAT
!Assign values to the temporary variables for the next time increment
UB=UBC
WB=WBC
QB=QBC
THETA=THETAC

201
END IF
25 CONTINUE
10 CONTINUE

!Calculate average values for the unknown state variables
UBA=UBT/REAL(IEND)
WBA=WBT/REAL(IEND)
THETAA=THETAT/REAL(IEND)
WRITE(6,9000)UBA,WBA,THETAA
100 CONTINUE

125 format(12f10.3)
150 FORMAT(1X,'U-VELOCITY',3X,'W-VELOCITY',3X,'PITCH RATE',3X,
         @'PITCH ANGLE',4X,'GAMA',7X,'ALFAT')
200 FORMAT(2X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4,5X,F7.4,5X,F7.4)
2000 FORMAT(2X,'SINGULAR MATRIX')
3000 FORMAT(2X,'WARNING ERROR IN IDGT DECIMAL PLACE')
5000 FORMAT(2X,'SMALL ANGLE EXCEEDED !')
6000 FORMAT(2X,'TAIL AT MAX DEFLECTION')
8000 FORMAT(2X,'LINEAR AERO EXCEEDED !')
8500 FORMAT(12(F17.8))
9000 FORMAT(2X,'AV.U-VEL=',F9.5,2X,'AV.W-VEL=',F8.4,2X,'AV.THERETA=',
         @F8.4)
STOP
END
**VARIABLES (3-panel)**

A - left hand side matrix for eqn solver
AI - total angle of attack of tail including total incidence angle of tail,
   induced angle due to plunging and pitch rate effects, and induced downwash
ALFA - wing twist angle at time t
ALFAO - maximum wing twist angle
ALFAOT - total incidence angle of tail
ALFAOW - mean total incidence angle of wing
B - right hand side matrix for eqn solver
BT - full tail span
BW - distance of wing from wing tip to pivot
BW2 - distance of wing from pivot to wing root
CDOB - body zero-lift drag coefficient
CDOT - tail zero-lift drag coefficient
CDOW - zero-lift drag coefficient
CLALFAB - body lift-curve slope
CLALFAT - tail lift-curve slope
CLALFAW - wing lift-curve slope
CLDT - tail lift-curve slope due to elevator deflection
CMALFAB - body pitching-moment-curve slope
CMOT - tail pitching-moment coefficient
CMOW - wing pitching-moment coefficient
CT - mean tail chord
CW - mean chord of wing
DE - elevator deflection (radians)
DELR - section increments from wing tip to pivot
DELR2 - section increments from pivot to wing root
DWC - drag of wing
DCP - drag of center panel
DXB - x distance from body cg. to center panel cg
DYP - y distance from body cg. to center panel cg
DZB - z distance from body cg. to center panel cg
DXBEQL - equilibrium (gamma=0) x distance from body cg. to center panel cg
DYBEQL - equilibrium (gamma=0) y distance from body cg. to center panel cg
DZBEQL - equilibrium (gamma=0) z distance from body cg. to center panel cg
DXBMAX - x distance from equilibrium position(DXBEQL) to maximum position (magnitude only)
DYBMAX - y distance from equilibrium position(DXBEQL) to maximum position (magnitude only)
DZBMAX - z distance from equilibrium position(DXBEQL) to maximum position (magnitude only)
D1DXB - x component of center panel velocity wrt body (derivative of DXB)
D1DYB - y component of center panel velocity wrt body (derivative of DYB)
D1DZB - z component of center panel velocity wrt body (derivative of DZB)
D2DXB - x component of center panel acceleration wrt body (derivative of D1DXB)
D2DYB - y component of center panel acceleration wrt body (derivative of D1DYB)
D2DZB - z component of center panel acceleration wrt body (derivative of D1DZB)
DXP - x distance from wing cg to pivot point
DYP - y distance from wing cg to pivot point
DZP - z distance from wing cg to pivot point
DYW - y distance from root of wing to cg of wing panel (magnitude only)
DZW - z distance from root of wing to cg of wing panel (magnitude only)
ECL - downwash ratio(e/c1)
EFFT - Oswald efficiency factor
EFFW - Oswald efficiency factor
ENG - energy
EW - downwash angle
EWI - initial downwash angle
G - gravitational acceleration
GAMAO - maximum flapping angle wrt. the horizontal axis
GAMA - flapping angle
GT - feedback gain
HXB - x distance from body cg to pivot point on wing
HYB - y distance from body cg to pivot point on wing
HZB - z distance from body cg to pivot point on wing
IDGT - input required for eqn solver to check decimal accuracy
IEND - total number of time intervals
IRREC - total number of panels wing is sectioned into
ICP - incidence angle of center panel
IW - incidence angle of wing
IYYB - body moment of inertia
IYYCP - center panel moment of inertia (about its cg)
DXXW, IYYW, IZZW, IYZW - wing moment of inertia
DXXW0, IYYW0, IZZW0 - wing moment of inertia (about its cg) at flapping angle=0
IYYDOTW, IZZDOTW, IYZDOTW - wing moment of inertia derivatives
K - counter for 2nd-order Runge-Kutta integration
LAEROC - total wing rolling moment (along x) in body-fixed system
LAERO - panel value wing rolling moment (along x) in body-fixed system
LAEROW - total wing rolling moment (along x) in stability-axis system
LW - total lift over entire wing
LWC - lift over wing panel
LT - lift of tail
LB - lift of body
LCP - lift of center panel
MAEROC - total wing pitching moment (along y) in body-fixed system
MAERO - panel value wing pitching moment (along y) in body-fixed system
MAEROW - total wing pitching moment (along y) in stability system
MAEROT, MAEROB - total tail and body pitching moment (along y)
MB - mass of body
MCP - mass of center panel
MW - mass of wing
MCB - moment of body resulting from elevator deflection
MLBC, MMBC, MB - moment reaction at wing-body interface
MLB, MMB, MB - temp variables for moment reaction at wing-body interface
NAEROC - total wing yawing moment (along z) in body-fixed system
NAERO - panel value wing yawing moment (along z) in body-fixed system
NAEROW - total wing yawing moment (along z) in stability system
PBTIM - time to begin perturbation
PBUB - forward velocity perturbation
PBWB - plunging velocity perturbation
PBQB - pitch rate perturbation
PHI - phase angle between flapping angle and angle of attack
PI - value of pi
PW - wing angular velocity about x-axis (derivative of GAMA)
PDOTW - wing angular acceleration about x-axis
QB - temporary variable for pitch rate
QBC - pitch rate
QBT - total pitch rate
QDOTB - initial pitch acceleration
R - accumulated distance from wing root to cg of wing panel
ROE - density of air
SREFB - wetted area or surface area of body
T - time
TD - time delay
TD1 - initial time delay
TDEL - time increment
TREC - total time
THETA - temporary variable for pitch angle
THETAA - average pitch angle
THETAC - pitch angle
THETAT - total pitch angle
TXCP - thrust in the x direction for center panel
TXW - thrust in the x direction for wing
TW - mean wing thickness
UB - temporary variable for forward velocity
UBA - average forward velocity
UBC - forward velocity
UBT - total forward velocity
UCP - center panel forward velocity
VOLB - volume of body
W - flapping frequency (rad/s)
WB - temporary variable for plunging velocity
WBA - average plunging velocity
WBC - initial plunging velocity
WBT - total plunging velocity
WCP - center panel plunging velocity
XWC - x distance from wing cg to wing root
XC - x point
ZC - z point
XDOTC, ZDOTC - trajectory derivatives
XT - x distance from body cg to tail cg
ZT - z distance from body cg to tail cg
ZCB - lift of tail due to elevator deflection
ZLCP - zero lift line angle of center panel
ZLLW - zero lift line angle of wing
* F3.FUR
* LATERAL AND LONGITUDINAL STABILITY (2-PANEL)

* This program solves the complete equations of motion to determine
* the longitudinal and lateral stability of the 2-panel model. The
* program is tested using the inputs for Mr. Bill and the full-scale
* ornithopter.

REAL*8 A(18,18),B(18),DCXB,YYB,IZZB,DCXW,DCYW0,DCYW1,IZD0TW1
@,DCW1,JYYW1,IZZW1,IZD0TW1,JYYW0,JYYW1,IZD0TW1,JYYD0TW1
@,DCX2,JYYW2,IZZW2,IZD0TW2,JYYW2,JYYW0,JYY2TW2,JYYZOTW2
@,EW1(100000),EW2(100000),EW1(100000)
@,MW,MW,MW1,MW2,MW3,LW1,LW2,LW3,L2,LB,LAEROCLAEERO1,LAERO2,LAERO
@,MAERO,MAERO1,MAERO2,MAERO3,MAERO4,MAERO5,MAERO6,MAERO7
@,NAERO3
@,XC,YC,ZC,X(18),AA(18,18+1),PB,TIM,PUB,PB,BP,BP,BP,BPB,PBQP,PBRC
INTEGER TD,TD1
DATA L4/4,LU27,PI/3.14159265358979328/ROE/1.2256/
DATA G/9.806/PHIT1/0.0/PSIT1/0.0/VT/0.0/,THET1/0.0/,WBT/0.0/
@,UBT/0.0/

* Read in wing characteristics.
OPEN(UNIT=LU,FILE=’WING3.DAT’)
READ(LU,*)CLALF,A,ALFA0W,CD0W,CW,BW,GAMA0,MW,TW,EFFW,W,XWC,
@,CM0W,DHDL,W,CN3BC,L2W
CLOSE(LU)

* Calculate moments of inertia of wing at flapping angle(GAMA)=0.*
DCXW0=MW/12.0*(BW**BW+TW**TW)
DCYW0=MW/12.0*(CW**CW+TW**TW)
IZZW0=MW/12.0*(CW**CW+BW**BW)

* Read in tail characteristics.
OPEN(UNIT=LU,FILE=’TAIL3.DAT’)
READ(LU,*)CLALFAT,ALFA0T,CDOT,CT,BT,EC,EF,XT,XT,CMOT,DHDLT
READ(LU,*)CLALFAT,ALFA0F,CD0F,CF,BF,EF,XF,ZC,ZC,XF,ZF
CLOSE(LU)

* Read in body characteristics.
OPEN(UNIT=LU,FILE=’BODY3.DAT’)
READ(LU,*)MB,DCX,BYB,IZZB,DCX,B1,DYB1,DZB,DXB1,DXB2,DYB2,DZB2
@,CLALF,B,CD0B,VB,CMALF,B,SREFB
CLOSE(LU)

* Read in initial conditions.
OPEN(UNIT=LU,FILE=’INIT3.DAT’)
READ(LU,*)UB,V,W,B,PB,QB,RB,THETA,PHI,PSI,BETA,TDEL,TREC,DEL
@,PHASE,ALF0,XC,YC,ZC
READ(LU,*)PB,UB,PB,PB,PB,PB,PB,PB,PB,PB,PB,PBR,PEL,PEL,PEL
CLOSE(LU)
do 5 i=1,100
   ew(i)=0.001
5 continue

CYBB=1.5*CLALFAB*(VOLB**2.0)/(2.0*(BW*CW))
CNBB=XCP*CYBB
CLBB=-ZCP*CYBB

*Open file for output*
OPEN(UNIT=LU2,FORM=FORMATTED,FILE=OUTPUT3.DAT)

!Assign the value of the temporary variables to the unknown state variables
UBC=UB
VBC=VB
WBC=WB
PBC=PB
QBC=QB
RBC=RB
THETAC=THETA
PHIC=PHI
PSIC=PSI

IEND=INT(TREC/TDEL+.5) !Total number of time intervals
IRREC=INT(BW/DELR+.5) !Total number of wing panels

DO 10 I=1,IEND

!Check if time interval is the perturbation time specified
IF(I.EQ.PBTIM)THEN
   UB=UB+PUBUB !Add perturbation values to the state variables
   VB=VB+PVVB
   WB=WB+PBWB
   THETA=THETA+PBQB
   PHI=PHI+PBPB
   PSI=PSI+PBRR
   UBC=UBC+PUBUB
   VBC=VBC+PVVB
   WBC=WBC+PBWB
   THETAC=THETAC+PBQB
   PHIC=PHIC+PBPB
   PSIC=PSIC+PBRR
   ALFA0F=ALFA0F+PBRUDDER
   ALFA0T=ALFA0T+PBELEVATOR
END IF

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DO 25 K=1,2

T=TDELT*(I-1)  !Determine time in seconds

*.Calculate aerodynamic forces for the portside wing
!Calculate flapping angle at time T and its derivatives
GAMA1=GAMAO*COS(W*T)+DHDLW+PHI
PW1=-GAMAO*W*SIN(W*T)
PDOTW1=-GAMAO*W*W*COS(W*T)

!Calculate distance from wing root to wing cg
DXW1=0
DYW1=(BW/2*COS(GAMA1))
DZW1=(BW/2*SIN(GAMA1))

!Calculate wing cg velocities
UW1=UB+QB*(DZB1-DZW1)+RB*(DYW1-DYB1)
VW1=VB+PB*(DZW1-DZB1)+RB*(DXB1-DXW1)+PW1*DZW1
WW1=WB+PB*(DYB1-DYW1)+QB*(DXW1-DXB1)-PW1*DYW1

*.Block to solve for wing forces at time T by integrating over BW-
!Initialize total wing aerodynamic forces
XAERO=0.0
YAERO=0.0
ZAERO=0.0
LAERO=0.0
MAERO=0.0
NAERO=0.0

LW1=0.0   !Initialize lift over entire port wing
ALFA=ALFA0*COS(W*T+PHASE)  !Calculate wing twist angle

DO 15 J=1,IRREC

R=J*DELR  !Calculate distance from wing root to wing panel

!Transform to convenient-axis(body-fixed)system &calculate wing panel velocities
UWC=UB+QB*(DZB1+R*SIN(GAMA1))+RB*(-R*COS(GAMA1)-DYB1)
VWC=(VB+PB*(-R*SIN(GAMA1)-DZB1)+RB*(DXB1-DXW1)
@ +PW1*R*SIN(GAMA1))*COS(GAMA1)
@ +WB+PB*(DYB1+R*COS(GAMA1))+QB*(DXW1-DXB1)
@ +PW1*R*COS(GAMA1))*SIN(GAMA1)
@ WWC=(VB+PB*(-R*SIN(GAMA1)-DZB1)+RB*(DXB1-DXW1)
@ -PW1*R*SIN(GAMA1))*SIN(GAMA1)
@ +WB+PB*(DYB1+R*COS(GAMA1))+QB*(DXW1-DXB1)
@ +PW1*R*COS(GAMA1))*COS(GAMA1)
QWC=QB*COS(GAMA1)+RB*SIN(GAMA1)
PWC=PB+PW1
!Calculate wing panel lift and drag
CLW=CLALFAW*(WUC/UWC + ALFA*R + ALFA0W + CW*QWC/(2.0*UWC))

LWC=CLW/2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR*CW
LW1=LWC+LW1

!Record total lift over entire wing; to be used for downwash
DWC=(CD0W+CLW**2/(BW*2.0/CW*PI*EFFW))

!Calculate aero forces for the panel
XAERO=LWC*(WUC/UWC)-DWC
ZAERO=LWC-DWC*(WUC/UWC)
MAERO=(CM0W-PI/4*QWC)

! /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR*CW*CW

! - ZAERO*XWC

YAERO=0
LAERO=ZAERO*R
NAERO=0

!Add the panel values to the total wing aero forces
XAEROC=XAERO+XAEROC
ZAEROC=ZAERO+ZAEROC
LAEROC=LAERO+LAEROC
MAEROC=MAERO+MAEROC
YAEROC=YAERO+YAEROC
LAEROC=LAERO+LAEROC
NAEROC=NAERO+NAEROC

15 CONTINUE

!Transform wing aero forces back to stability-axis system
XAEROW1=XAERO
YAEROW1=YAEROC*COS(GAMA1)-ZAEROC*SIN(GAMA1)
ZAEROW1=ZAEROC*COS(GAMA1)+YAEROC*SIN(GAMA1)
LAEROW1=LAERO
MAEROW1=MAEROC*COS(GAMA1)-NAEROC*SIN(GAMA1)
NAEROW1=NAEROC*COS(GAMA1)+MAEROC*SIN(GAMA1)+YAEROW1*XWC

*-Calculate aerodynamic forces for the starboard wing

!Calculate flapping angle at time T and its derivatives
GAMA2=GAMA0*COS(W*T)-DHD'LW+PHI
PW2=GAMA0*W*SIN(W*T)
PDOTW2=GAMA0*W*W*COS(W*T)

!Calculate distance from wing root to wing cg
DXW2=0
DYW2=(BW/2*COS(GAMA2))
DZW2=(BW/2*SIN(GAMA2))

!Calculate wing cg velocities
UW2=UB+QB*(DZB2-DZW2)+RB*(DYW2-DYB2)
VW2=VB+PB*(DZB2-DZW2)+RB*(DXB2-DXW2)+PW2*DZW2
WW2=WB+PB*(DYB2-DYW2)+QB*(DXW2-DXB2)-PW2*DYW2
!!Initialize total wing aerodynamic forces
XAEROC=0.0
YAERO=0.0
ZAERO=0.0
LAERO=0.0
MAERO=0.0
NAERO=0.0
LW2=0.0 !!Initialize lift over entire starboard wing

DO 16 J=1,IRREC

R=DELR !!Calculate distance from wing root to wing panel

!!Transform to convenient-axis(body-fixed)system & calculate wing panel velocities
UWC=UB+QB*(DZB2+R*SIN(GAMA2))+RB*(-R*COS(GAMA2)-DYB2)
VWC=(VB+PB*(-R*SIN(GAMA2)-DZB2)+RB*(DXB2-DXW2) -PW2*R*SIN(GAMA2))*COS(GAMA2)
-WB+PB*(DZB2+R*COS(GAMA2))+Q*B*(DXW2-DXB2)
+PW2*R*COS(GAMA2))*SIN(GAMA2)
WWC=(VB+PB*(-R*SIN(GAMA2)-DZB2)+RB*(DXB2-DXW2) -PW2*R*SIN(GAMA2))*SIN(GAMA2)
+(WB+PB*(DZB2+R*COS(GAMA2))+QB*(DXW2-DXB2)
+PW2*R*COS(GAMA2))*COS(GAMA2)
QWC=QB*COS(GAMA2)-RB*SIN(GAMA2)

PWC=PB+PW2

!!Calculate wing panel lift and drag
CLW=CLALFAW*(WWC/UWC + ALFA*R + ALFA0W + CW*QWC/(2.0*UWC) - R*PWC/UWC)
LWC=CLW/2.0*ROE*(UWC*UWC+VWC/VWC+WWC*WWC)*DELR*CW
LW2=LWC+LW2 !!Record total lift over entire wing; to be used for downwash
DWC=(CD0W+CLW)*2/(BW+2.0/CW*PI*EFF)+(BC0W)*DWC)

!!Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=LWC-DWC*(WWC/UWC)
MAERO=(CM0W-P14*QWC)
-2.0*ROE*(UWC/UWC+VWC/VWC+WWC*WWC)*DELR*CW*CW
- ZAERO*XWC

YAERO=0
LAERO=ZAERO*R
NAERO=0

!!Add the panel values to the total wing aero forces
XAERO=XAERO+XAERO
ZAERO=ZAERO+ZAERO
LAERO=LAERO+LAERO
MAERO=MAERO+MAERO
YAERO=YAERO+YAERO
LAERO=LAERO+LAERO
NAERO=NAERO+NAERO

16 CONTINUE
! Transform wing aero forces back to stability-axis system
XAEROW2=XAEROC
YAEROW2=YAEROC*COS(GAMA2)+ZAEROC*SIN(GAMA2)
ZAEROW2=ZAEROC*COS(GAMA2)-YAEROC*SIN(GAMA2)
LAEROW2=LAEROC
MAEROW2=MAEROC*COS(GAMA2)+NAEROC*SIN(GAMA2)
NAEROW2=NAEROC*COS(GAMA2)-MAEROC*SIN(GAMA2) + YAEROW2*XWC

*Block to calculate tail forces and moments.*

!Calculate time delay for downwash to reach tail
TD=INT(XT^(-1)*(UB*TDEL)++.5)
!Calculate downwash at time T=I*TDEL
EW1(I+TD)=LW1*2.0*(ROE*CW*BW*(UW1*UW1+VW1*VW1+WW1*WW1))
@  *ECL+EW1(I+TD)
EW2(I+TD)=LW2*2.0*(ROE*CW*BW*(UW2*UW2+VW2*VW2+WW2*WW2))
@  *ECL+EW1(I+TD)
! Check to see if this is the very first time interval
IF(I.EQ.1)THEN
  TD1=TD+1  ! Increment the initial time delay
END IF

!Calculate total angle of attack of tail
IF(I.GE. TD1 )THEN
  AI1=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW1(I)+ALFA0T
  +BT*PB/(2.0*UB))
@  AI2=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW2(I)+ALFA0T
  -BT*PB/(2.0*UB))
ELSE
  AI1=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW1(I)+ALFA0T
  +BT*PB/(2.0*UB))  ! for initial time interval
  AI2=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW1(I)+ALFA0T
  -BT*PB/(2.0*UB))  ! for initial time interval
END IF

! Calculate tail lift and drag
LT1=CLALFAT*AI1/2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*BT/2.0*CT
CLT1=LT1*2.0*(ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0)
DT1=(CDOT+CLT1**2/(BT/CT*PI*EFFT))
@  /2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0
! Calculate tail aero forces
XAEROT1=LT1*(AI1-QB*CT/(2.0*UB)-ALFA0T)-DT1
ZAEROT1=LT1-CLT1*(AI1-QB*CT/(2.0*UB)-ALFA0T)

! Calculate tail lift and drag
LT2=CLALFAT*AI2/2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*BT/2.0*CT
CLT2=LT2*2.0*(ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0)
DT2=(CDOT+CLT2**2/(BT/CT*PI*EFFT))
@  /2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0
! Calculate tail aero forces
XAEROT2=LT2*(AI2-QB*CT/(2.0*UB)-ALFA0T)-DT2
ZAEROT2=LT2-CLT2*(AI2-QB*CT/(2.0*UB)-ALFA0T)
XAEROT=XAEROT1+XAEROT2
ZAEROT=ZAEROT1+ZAEROT2
MAEROT=CM0T/2.0*ROE*(UB*UB+VB*VB+WB*WB)*CT*CT*BT
@   -ZAEROT*XT + XAEROT*ZT

-BLOCK to calculate fin forces and moments-

Calculate fin lift and drag
LF=CLALFAF*(VB/UB+ALFAOF+RB*XF/UB+CF*RB/(2.0*UB))
@  /2.0*ROE*(UB*UB+(VB+RB*XF)*(VB+RB*XF))*BF*CF
CF=LF^2.0/(ROE*(UB*UB+(VB+RB*XF)*(VB+RB*XF))*BF*CF)
DF=(CD0F+CLF)**2/(BF*CF*PI*EFFP))
@  /2.0*ROE*(UB*UB+(VB+RB*XF)*(VB+RB*XF))*BF*CF

Calculate fin aero forces
XAEROF=LF*(VB/UB+RB*XF)-DF
YAEROF=LF-DF*(VB/UB+RB*XF)
NAEROF=YAEROF*XF
LAEROF=-YAEROF*ZF

-BLOCK to calculate body forces and moments-

Calculate body lift and drag
LB=CLALFAB*(WB/UB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB**2.0/(3.0)
DB=(CD0B*SREFB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)

Calculate body aero forces
XAEROB=LB*(WB/UB)-DB+XAEROT+XAEROF
ZAEROB=LB-DB*(WB/UB)+ZAEROT
MAEROB=CMALFAB*(WB/UB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB+MAEROT

YAEROB=CYBB*(VB/UB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB**2.0/(3.0)
@   + YAEROF
NAEROB=CNBB*(VB/UB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB + NAEROF
LAEROB=CLBB*(VB/UB)/2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB + LAEROF

Calculate portside wings moments of inertia and derivatives
IXXW1=IXXW0
IYYW1=IYYW0*(COS(-GAMA1)**2)+IZZW0*(SIN(-GAMA1)**2)
IZZW1=-SIN(-GAMA1)*COS(-GAMA1)*(IYYW0-IZZW0)
IYYDOTW1=(GAMA0*W*SIN(W*T))*(2*COS(-GAMA1)*SIN(-GAMA1)*(IZZW0-
@   IYYW0))
IZZDOTW1=(GAMA0*W*SIN(W*T))*(SIN(-GAMA1)**2)-(COS(-GAMA1)**2))
@  *IYYW0-IZZW0)
IZZW1=IYYW0*(SIN(-GAMA1)**2)+IZZW0*(COS(-GAMA1)**2)
IZZDOTW1=(GAMA0*W*SIN(W*T))*(2*COS(-GAMA1)*SIN(-GAMA1)*(IYYW0-
@   IZZW0))

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!Calculate starboard wings moments of inertia and derivatives

\[ DXW2=DXW0 \]
\[ IYYW2=IYYW0*(\cos(GAMA2)\times2)+IZZW0*(\sin(GAMA2)\times2) \]
\[ IYZ2=\sin(GAMA2)\times\cos(GAMA2)*(IYYW0-IZZW0) \]
\[ IYYDOTW2=(GAMA0\times W*\sin(W*st))\times(2*\cos(GAMA2)\times\sin(GAMA2)\times(IYYW0-\]
\[ @ IYYW0)) \]
\[ IYDOTW2=-(GAMA0\times W*\sin(W*st))*((\sin(GAMA2)\times2)-(\cos(GAMA2)\times2)) \]
\[ @ *(IYYW0-IZZW0) \]
\[ IZZW2=IYYW0*(\sin(GAMA2)\times2)+IZZW0*(\cos(GAMA2)\times2) \]
\[ IZZDOTW2=(GAMA0\times W*\sin(W*st))*((2*\cos(GAMA2)\times\sin(GAMA2)\times(IYYW0- \]
\[ @ IZZW0)) \]

\*Simultaneous solution of unknown variables at T+DELT -*

!Initialize eqn matrix to zero

DO 50 L=1,18
DO 40 M=1,18
A(M,L)=0.0

40 CONTINUE
50 CONTINUE

!Assign values to the eqn matrices

A(1,1)=MB
A(1,7)=-1.0
A(1,13)=-1.0
A(2,2)=MB
A(2,8)=-1.0
A(2,14)=-1.0
A(3,3)=MB
A(3,9)=-1.0
A(3,15)=-1.0
A(4,4)=DXB
A(4,6)=DZB
A(4,8)=DZB1
A(4,9)=DYB1
A(4,10)=-1.0
A(4,14)=DZB2
A(4,15)=DYB2
A(4,16)=-1.0
A(5,5)=IYYB
A(5,7)=DZB1
A(5,9)=DXB1
A(5,11)=-1.0
A(5,13)=DZB2
A(5,15)=DXB2
A(5,17)=-1.0
A(6,4)=DZB
A(6,6)=IZZB
A(6,7)=DYB1
A(6,8)=DXB1
A(6,12)=-1.0
A(6,13)=DYB2
A(6,14)=DXB2
A(6,18)=-1.0
A(7,1)=MW
A(7,5)=MW*(DZB1-DZW1)
A(7,6)=MW*(DYW1-DYB1)
A(7,7)=1.0
A(8,2)=MW
A(8,4)=MW*(DZW1-DZB1)
A(8,6)=MW*(DXB1-DXW1)
A(8,8)=1.0
A(9,3)=MW
A(9,4)=MW*(DYB1-DYW1)
A(9,5)=MW*(DXW1-DXB1)
A(9,9)=1.0
A(10,4)=DXW1
A(10,8)=DZW1
A(10,9)=DYW1
A(10,10)=1.0
A(11,5)=IYYW1
A(11,6)=IYZW1
A(11,7)=DZW1
A(11,9)=DXW1
A(11,11)=1.0
A(12,5)=IYZW1
A(12,6)=IWW1
A(12,7)=DYW1
A(12,8)=DXW1
A(12,12)=1.0
A(13,1)=MW
A(13,5)=MW*(DZB2-DZW2)
A(13,6)=MW*(DYW2-DYB2)
A(13,13)=1.0
A(14,2)=MW
A(14,4)=MW*(DZW2-DZB2)
A(14,6)=MW*(DXB2-DXW2)
A(14,14)=1.0
A(15,3)=MW
A(15,4)=MW*(DYB2-DYW2)
A(15,5)=MW*(DXW2-DXB2)
A(15,15)=1.0
A(16,4)=DXW2
A(16,14)=DZW2
A(16,15)=DYW2
A(16,16)=1.0
A(17,5)=IYYW2
A(17,6)=IYZW2
A(17,13)=DZW2
A(17,15)=DXW2
A(17,17)=1.0
A(18,5)=IYZW2
A(18,6)=IWW2
A(18,13)=DYW2
A(18,14)=DXW2
A(18,18)=1.0
CALL GAUSS(AA,BX,18,18+1,18,18+1)
IF(K.EQ. 1)THEN  
  !Perform 1st part of 2nd-order Runge-Kutta integration  
  UDOTB=X(1)        !f(xn,un) = _DOTB  
  VDOTB=X(2)        !This is not a function of xn(time), only un(UBC)  
  WDOTB=X(3)  
  PDOTB=X(4)  
  QDOTB=X(5)  
  RDOTB=X(6)  
  XDOT=UB*COS(THE)A*COS(PSI)+VB*(SIN(PHI)*SIN(THE)*COS(PSI))  
  @ -COS(PHI)*SIN(PSI)+WB*(COS(PHI)*SIN(THE)*COS(PSI))  
  @ +SIN(PHI)*SIN(PSI)) !Calculate trajectory  
  YDOT=UB*COS(THE)*SIN(PSI)+VB*(SIN(PHI)*SIN(THE)*SIN(PSI))  
  @ +COS(PHI)*SIN(PSI)+WB*(COS(PHI)*SIN(THE)*SIN(PSI))  
  @ -SIN(PHI)*SIN(PSI))  
  ZDOT=-UB*SIN(THE)+VB*SIN(PHI)*COS(THE)+WB*COS(PHI)  
  @ *COS(THE)  
  UB=UDOTB*TDEL+UBC  !Store k1+un in the temporary variables  
  VB=VDOTB*TDEL+VBC  !k1 = _DOTB*TDEL  
  WB=WDOTB*TDEL+WBC  
  PB=PDOTB*TDEL+PBC  
  QB=QDOTB*TDEL+QBC  
  RB=RDOTB*TDEL+RBC  
  TEMPTHE=THE  
  TEMPPHI=PHI  
  THE=QB*COS(TEMPPHI)-RB*SIN(TEMPPHI))*TDEL+THEAC  
  PHI=(PB+QB*SIN(TEMPPHI)*TAN(TEMPPHEA))  
  @ +RB*COS(TEMPPHI)*TAN(TEMPPTHETA)))*TDEL+PHIC  
  PSIC=((QB*SIN(TEMPPHI)+RB*COS(TEMPPHI))*(1.0/COS(TEMPPTHETA)))  
  @ *TDEL+PSIC  
ELSE  
  !Perform 2nd part of 2nd-order Runge-Kutta integration  
  UDOTBC=X(1)       !f(xn+h,un+k1) = _DOTBC  
  VDOTBC=X(2)       !This is not a function of xn(time), only un(UBC)  
  WDOTBC=X(3)  
  PDOTBC=X(4)  
  QDOTBC=X(5)  
  RDOTBC=X(6)  
  UBC=(UDOTBC+UDOTB)/2.0*TDEL+UBC  !k2=_DOTBC*TDEL  
  VBC=(VDOTBC+VDOTB)/2.0*TDEL+VBC  !un+1 = un + (k1+k2)/2  
  WBC=(WDOTBC+WDOTB)/2.0*TDEL+WBC  
  PBC=(PDOTBC+PDOTB)/2.0*TDEL+PBC  
  QBC=(QDOTBC+QDOTB)/2.0*TDEL+QBC  
  RBC=(RDOTBC+RDOTB)/2.0*TDEL+RBC  
  TEMPTHE=THEAC  
  TEMPPHI=PHIC  
  THETAC=((QB*COS(TEMPPHI)-RBC*SIN(TEMPPHI))  
  @ +(QB*COS(PHI)-RB*SIN(PHI)))/2.0*TDEL+THETAC  
  PHIC=((PBC+QB*COS(TEMPPHI)*TAN(TEMPPTHETA)+RBC*COS(TEMPPHI))  
  @ *TAN(TEMPPTHETA)+(PB+QB*SIN(PHI)*TAN(THETA)+RB*COS(PHI))  
  @ +THETA)))/2.0*TDEL+PHIC  
  PSIC=((QB*SIN(TEMPPHI)+RBC*COS(TEMPPHI))*(1.0/COS(TEMPPTHETA)))  
  @ +(QB*SIN(PHI)+RB*COS(PHI))*(1.0/COS(THETA)))/2.0*TDEL  
  @ +PSIC
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XDOTC=UBC*COS(THETAC)*COS(PSCC)+VBC*(SINVPP)*SINV(THETAC)
@  +WBC*COS(PSSC)*COS(THETAC)
YDOTC=UBC*COS(THETAC)*SINV(PSSC)+VBC*(SINV(PP))*SINV(THETAC)
@  +WBC*COS(PSSC)*SINV(THETAC)
ZDOTC=UBC*SINV(THETAC)+VBC*SINV(PSSC)*COS(THETAC)
@  +WBC*COS(PSSC)*COS(THETAC)

XC=(XDOTC+XDOT)/2.0*TDEL+XC
YC=(YDOTC+YDOT)/2.0*TDEL+YC
ZC=(ZDOTC+ZDOT)/2.0*TDEL+ZC

WRITE(LU2,8500)T,UBC,YDOTC,VBC,SINDOTC,WBC,YDOTBC,PBC*180/PI
@  ,SINDOTC*180/PI,QBC*180/PI,THETAC*180/PI,RBC*180/PI
@  ,SINDOTC*180/PI,XC,YC,ZC

!Calculate total values for the unknown state variables
UBT=UBC+UBT
VBT=VBC+VBT
WBT=WBC+WBT
THETAT=THETAC+THETAT
PHIT=PHIC+PHIT
PSIT=PSIC+PSIT

!Assign values to the temporary variables for the next time increment
UB=UBC
VB=VBC
WB=WBC
PB=PBC
QB=QBC
RB=RBC
THETA=THETAC
PHI=PHIC
PSI=PSIC

END IF
25 CONTINUE
10 CONTINUE

!Calculate average values for the unknown state variables
UBA=UBT/REAL(IEND)
VBA=VBT/REAL(IEND)
WBA=WBT/REAL(IEND)
THETA=THETAC/REAL(IEND)
PHIA=PHIT/REAL(IEND)
PSIA=PSIT/REAL(IEND)
WRITE(6,9000)UBA,VBA,WBA,THETA*180/PI,PHIA*180/PI,PSIA*180/PI
100 CONTINUE
CLOSE(LU2)
125  format(12f10.2)
150  format(1x,'U-VELOCITY',3x,'W-VELOCITY',3x,'PITCH RATE',3x,
     @'PITCH ANGLE',4x,'GAMA',7x,'ALFAT')
200  format(2x,f8.4,5x,f8.4,5x,f8.4,5x,f8.4,5x,f7.4,5x,f7.4)
8500  format(16(f16.8))
9000  format('AV.U-VEL=',f8.4,1x,'(m/s)  AV.V-VEL=',f8.4,1x,
     @'(m/s)  AV.W-VEL=',f8.4,1x,'(m/s)  AV.THETA=',f8.4,1x,
     @'(deg)  AV.PHI=',f8.4,1x,'(deg)  AV.PSI=',f8.4,1x,'(deg)')
STO
END
**F4.FOR**

**LATERAL AND LONGITUDINAL STABILITY** (3-PANEL)

*This program solves the complete equations of motion to determine*

*the longitudinal and lateral stability of the 3-panel model. The*

*program is tested using the inputs for Mr. Bill and the full-scale*

*ornithopter.*

REAL*8 A(24,24),B(24),DXXB,IXYX,IZZB,DXZB,DXW0,IXYW0,IZZW0
@,DXW1,IXYW1,IZZW1,IZZDOTW1,IXYW1,IXYDOTW1,IXZDOTW1
@,DXW2,IXYW2,IZZW2,IZZDOTW2,IXYW2,IXYDOTW2,IXZDOTW2
@,EW1(10000),EW2(10000),EW1(10000),LW1,LW2,LWC,LAEROC,LAERO
@,LAEROW1,LAEROW2,MAERO,MAEROCC,MAEROW1,MAEROW2,MAEROT,MAEROB
@,MAEROCP,NAEROW1,NAEROW2,NAEROCC,NC,YC,ZC,X(24),AA(24,24+1)
@,D1DXB,D2DXB,D2ZB,D1DXZB,D2DXZB,D1DYY,L1,L2,LB
@,PB,TIM,PBUB,PBV,W,PBWB,PBQB,PBQB,PBRUDDER,PBELEVATOR
REAL*8 CLALFAW,CD0W,CW,BW,BW2,GAMA0,MW,TW,EFFW,W,XWC,CM0W
REAL*8 DHDWL,CNBCL2W,CNBCL2CP,BCP,CP,IW,ZZLW,ICP,ZZLCMCP
@,DXCP,IXYCP,IZZCP,ICP,DCP,ALFAOW,ALFA0C
REAL*8 CLALFAT,ALFA0T,CD0T,CT,BT,ECL,EFFT,XT,XT,CM0T,CLDT,DE
@,DHDLT,CLALFA,ALFA0F,CD0F,CF,EF,EF,EF,CF,CP,CPZ,ZP,ZF
REAL*8 MB,DXBMX,MYBMX,DZBMX,DXBEQL
@,DZBEQ,CLALFB,CD0B,VOLB,CMALFAB,SREFB,HX01,HY01,Hz01
@,HXB2,HYB2,HZB2
REAL*8 UB,VB,WB,PL,QB,RB,THETA,PHI,PSI,BETA
@,TDEL,TRC,DEL,DEL2,PHASE,ALFA0

INTEGER TD,TD1
DATA L4,/L4/7/PI/3.141592654/,ROE/1.2256/
DATA G/9.806/,PHIT/0.0/,PSIT/0.0/,VBT/0.0/,THETAT/0.0/,WBT/0.0/
@,UBT/0.0/

*Read in wing characteristics.*

OPEN(UNIT=L4,FILE="WING4.DAT")
READ(L4,*)CLALFAW,CD0W,CW,BW,BW2,GAMA0,MW,TW,EFFW,W,XWC
@,CM0W
READ(L4,*)DHDWL,CNBCL2W,BCP,CP,IW,ZZLW,ICP,ZZLCMCP
READ(L4,*)IDXCP,IXYCP,IZZCP
CLOSE(L4)

*Calculate moments of inertia of wing at flapping angle(GAMA)=0.*

DXW0=MW/12.0*((BW+BW2)*(BW+BW2)+TW*TW)
IXYW0=MW/12.0*(CW*CW+TW*TW)
IZZW0=MW/12.0*(CW*CW+(BW+BW2)*(BW+BW2))

*Read in tail characteristics.*

OPEN(UNIT=L4,FILE="TAIL4.DAT")
READ(L4,*)CLALFAT,ALFA0T,CD0T,CT,BT,ECL,EFFT,XT,XT,CM0T,CLDT,DE
@,DHDLT,CLALFA,ALFA0F,CD0F,CF,EF,EF,EF,CF,CP,CPZ,ZP,ZF
CLOSE(L4)
*-Read in body characteristics-*

OPEN(UNIT=LU,FILE='BODY4.DAT')
READ(LU,*)MB,DXB,IXYB,IZZB,DXBMAX,DYBMAX,DZBMAX,DXBREAL
@,DZBREAL,CLALFAB,CD0,B,VOLB,CMALFAB,SREFB,HXB1,HYB1,HZB1
@,HXB2,HYB2,HZB2
CLOSE(LU)

*-Read in initial conditions-*

OPEN(UNIT=LU,FILE='INIT4.DAT')
READ(LU,*)UB,VB,WB,PB,QB,RB,THETA,PHI,PSI,BETA
@,TDEL,TREC,DELZ,DELZ2,PHASE,ALFA0,XC,YC,ZC
READ(LU,*)PBTIM,PUB,PVB,PBWB,PBPB,PBQP,PBRB,PBRUDDER,PBELEVATOR
CLOSE(LU)

do 5 i=1,100
   ewi(i)=0.001
5  continue

    CYBB=-1.5*CLALFAB*(VOLB**((2/3))/(2*(BW+BW2)**(1/3))
    CNBB=XC*CYBB
    CLBB=-ZC*CYBB

*-Open file for output-*

OPEN(UNIT=LU2,FORM='FORMATTED',FILE='OUTPUT4.DAT')

!Assign the value of the temporary variables to the unknown state variables

UBC=UB
VBC=VB
WBC=WB
PBC=PB
QBC=QB
RBC=RB
THETAC=THETA
PHIC=PHI
PSIC=PSI

ALFA0W=1W+ZLLW  !Calculate wing angle of attack
ALFA0CP=ICP+ZLLCP  !Calculate center panel angle of attack
IEND=INT(TREC/TDEL+.5)  !Total number of time intervals

220
DO 10 I=1,10
!
!Check if time interval is the perturbation time specified
IF(1.EQ.PBTIM)THEN
  UB=UB+PBU
  VB=VB+PBV
  WB=WB+PBWB
  THETA=THETA+PBQB
  PHI=PHI+PBPH
  PSI=PSI+PBRB
  UBC=UBC+PBU
  VBC=VBC+PBV
  WBC=WBC+PBWB
  THETAC=THETAC+PBQB
  PHIC=PHIC+PBPH
  PSIC=PSIC+PBRB
  ALFAO=ALFA0+PBRUDDER
  ALFAO=T=ALFA0+PBELEVATOR
END IF
!
DO 25 K=1,2
!
T=TDEL*(I-1)  !Determine time in seconds
!
!Calculate distance from body cg to center panel cg & derivatives
DZB=DZBEXL - DZBM*X*COS(W*T-PI)
D1DZB=W*DZBM*X*SIN(W*T-PI)
D2DZB=W*X*DZBM*X*COS(W*T-PI)
DXB=DXBEQ+DXBM*X*COS(W*T-PI)  !x is not needed because only plunging motion occurs
D1DXB=-W*DXBM*X*SIN(W*T-PI)
D2DXB=W*X*DXBM*X*COS(W*T-PI)
DYB=0
D1DYB=0
D2DYB=0
!
!Calculate center panel cg velocities
UCP=UB+QB*DZB-RB*DYB+D1DXB
VCP=VB+PB*DZB+RB*DXB+D1DYB
WCP=WB+PB*DYB-QB*DXB+D1DZB
!
!*- Calculate aerodynamic forces for the portside wing
!Calculate flapping angle at time T and its derivatives
GAMA1=GAMA0*COS(W*T)+DHDLW+PHI
PW1=GAMA0*W*SIN(W*T)
PDOTW1=GAMA0*W*X*COS(W*T)
!
!Calculate distance from wing cg to wing root
DXW1=0
DYW1=((BW+BW2)/2*COS(GAMA1))
DZW1=((BW+BW2)/2*SIN(GAMA1))
!Calculate distance from wing cg to wing pivot
DXP1=0
DYP1=\((BW-BW2)/2*COS(GAMA1)\)
DZP1=\((BW-BW2)/2*SIN(GAMA1)\)

*-Block to solve for wing forces at time T by integrating over BW-*

!Initialize total wing aerodynamic forces
XAERO=0.0
YAERO=0.0
ZAERO=0.0
LAERO=0.0
NAERO=0.0

LW1=0.0 !Initialize lift over entire wing
ALFA=ALFA0*COS(W*T+PHASE) !Calculate wing twist angle

!Determine zero terms for the section of wing from the pivot to the wing tip
IRREC=INT(BW/DELR+.5) !Total number of wing panels
DO 15 J=1,IRREC

R=J*DELR !Calculate distance from wing root to wing panel

!Transform to convenient-axis(body-fixed)system & calculate wing panel velocities
UWC=UB+QB*(HZB1+R*SIN(GAMA1))+RB*(-R*COS(GAMA1)-HYB1)
VWC=(VB+PB*(-R*SIN(GAMA1)-HZB1)+RB*(HXB1-DXP1)
  +WB+PB*(HYB1-R*COS(GAMA1))+QB*(DXP1-HXB1)
  -PW1*R*COS(GAMA1)))*COS(GAMA1)
  +(WB+PB*(HYB1-R*COS(GAMA1))+QB*(DXP1-HXB1)
  -PW1*R*COS(GAMA1)))*SIN(GAMA1)
WWC=(VB+PB*(-R*SIN(GAMA1)-HZB1)+RB*(HXB1-DXP1)
  -PW1*R*COS(GAMA1)))*SIN(GAMA1)
  +(WB+PB*(HYB1-R*COS(GAMA1))+QB*(DXP1-HXB1)
  -PW1*R*COS(GAMA1)))*COS(GAMA1)
QWC=QB*COS(GAMA1)+RB*SIN(GAMA1)
PWC=PB+PW1

!Calculate wing panel lift and drag
CLW=CLALFAW*(UWC*UWC + ALFA*R + ALFA0W + CW*QWC/(2.0*UWC)
  + R*PWC/UWC)
LWC=CLW/2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR*CW
LW1=LWC+LW1 !Record total lift over entire wing; to be used for downwash
DWC=(CDOW+CLW*2/(BW+BW2+0.5*BCP)*2.0*CW*PI*EFFW))

!Calculate zero forces for the panel
XAERO=LWC*(UWC/UWC)-DW
ZAERO=LWC-DWC*(UWC/UWC)
MAERO=(CM0W-PI/4*QWC)

@ /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR*CW*CW
 @ - ZAERO*XWC

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YAERO=0
LAERO=-ZAERO*R
NAERO=0

! Add the panel values to the total wing aero forces
XAERO=XAERO+XAERO
YAERO=YAERO+YAERO
ZAERO=ZAERO+ZAERO
LAERO=LAERO+LAERO
MAERO=MAERO+MAERO
NAERO=NAERO+NAERO

CONTINUE

! Determine aero terms for the section of wing from the wing root to the pivot
! This section of the wing is considered to be rigid
IRREC=INT(BW2/DELR2+.5) ! Total number of wing panels
DO 16 1=1,IRREC
R=I*DELR2 ! Calculate distance from wing root to wing panel

! Transform to convenient-axis (body-fixed) system & calculate wing panel velocities
UWC=UB+QB*(HZZ1+R*SIN(GAMA1))+RB*(-R*COS(GAMA1)-HYB1)
VWC=(VB+PB*-(R*SIN(GAMA1)-HZZ1)+RB*(HXY1-DXP1)
     @ -PW1*R*SIN(GAMA1))*COS(GAMA1)
     @ +(WB+PB*(HYB1+R*COS(GAMA1))+QB*(DXP1-HXY1)
     @ -PW1*R*COS(GAMA1))*SIN(GAMA1)
WWC=(VB+PB*(-R*SIN(GAMA1)-HZZ1)+RB*(HXY1-DXP1)
     @ -PW1*R*SIN(GAMA1))*SIN(GAMA1)
     @ +(WB+PB*(HYB1+R*COS(GAMA1))+QB*(DXP1-HXY1)
     @ -PW1*R*COS(GAMA1))*COS(GAMA1)
QWC=QB*COS(GAMA1)+RB*SIN(GAMA1)
PWC=PB+PW1

! Calculate wing panel lift and drag
CLW=CLALFAW*(WWC/UWC+ALFA*R+ALFA0W+CQWC/(2.0*UWC)
     @ +R*PWC/UWC)
LWC=CLW/2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW
LW1=LWC+LW1 ! Record total lift over entire wing; to be used for downwash
DWC=(CD0W+CLW**2/(BW+BW2+0.5*BCP)*2.0/(CW*PI*EFFW))
     @ /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW

! Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=LWC-DWC*(WWC/UWC)
MAERO=(CM0W*PI/4*QWC)
     @ /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW
     @ - ZAERO*XWC

YAERO=0
LAERO=-ZAERO*R
NAERO=0
!Add the panel values to the total wing aero forces
XAEROC=XAERO+XAEROC
YAEROC=YAERO+YAEROC
ZAEROC=ZAERO+ZAEROC
LAEROC=LAERO+LAEROC
MAEROC=MAERO+MAEROC
NAEROC=NAERO+NAEROC

!Transform wing aero forces back to stability-axis system
XAEROW1=XAEROC
YAEROW1=YAEROC*COS(GAMA1)-ZAEROC*SIN(GAMA1)
ZAEROW1=ZAEROC*COS(GAMA1)+YAEROC*SIN(GAMA1)
LAEROW1=LAEROC
MAEROW1=MAEROC*COS(GAMA1)-NAEROC*SIN(GAMA1)
NAEROW1=NAEROC*COS(GAMA1)+MAEROC*SIN(GAMA1)+YAEROW1*XWC

!Calculate wing cg velocities
UW1=UB+QB*(HZB1-DZP1)+RB*(DYP1-HYB1)
VW1=VB+PB*(DZP1-HZB1)+RB*(HXB1-DXP1)+PW1*DZP1
WW1=WB+PB*(HYB1-DYP1)+Q*WB*(DXP1-HXB1)-PW1*DYP1

*- Calculate aerodynamic forces for the starboard wing
!Calculate flapping angle at time T and its derivatives
GAMA2=-GAMA0*COS(W*T)+DHDLW+PHI
PW2=GAMA0*W*SIN(W*T)
PDOTW2=GAMA0*W*W*COS(W*T)

!Calculate distance from wing cg to wing root
DXW2=0
DYW2=-(BW+BW2)/2*COS(GAMA2)
DZW2=-(BW+BW2)/2*SIN(GAMA2)

!Calculate distance from wing cg to wing pivot
DXP2=0
DYP2=-(BW-BW2)/2*COS(GAMA2)
DZP2=-(BW-BW2)/2*SIN(GAMA2)

*-Block to solve for wing forces at time T by integrating over BW-*

!Initialize total wing aerodynamic forces
XAEROC=0.0
YAEROC=0.0
ZAEROC=0.0
LAEROC=0.0
MAEROC=0.0
NAEROC=0.0
LW2=0.0 !Initialize lift over entire wing

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!Determine aero terms for the section of wing from the pivot to the wing tip
IRREC=INT((BW/DELR+.5)  !Total number of wing panels
DO 17 J=1,IRREC

R=J*DELR  !Calculate distance from wing root to wing panel

!Transform to convenient-axis(body-fixed)system & calculate wing panel velocities
UWC=UB+QB*(HZB2+R*SIN(GAMA2))+RB*(-R*COS(GAMA2)-HYB2)
VWC=(VB+PB*(-R*SIN(GAMA2)-HZB2)+RB*(HXB2-DXP2)
+PW2*R*SIN(GAMA2))*COS(GAMA2)
+WB+PB*(HYB2+R*COS(GAMA2))+QB*(DXP2-HXB2)
+PW2*R*COS(GAMA2))*SIN(GAMA2)
WWC=(WB+PB*(-R*SIN(GAMA2)-HZB2)+RB*(HXB2-DXP2)
+PW2*R*SIN(GAMA2))*SIN(GAMA2)
+(WB+PB*(HYB2+R*COS(GAMA2))+QB*(DXP2-HXB2)
+PW2*R*COS(GAMA2))*COS(GAMA2)
QWC=QB*COS(GAMA2)-RB*SIN(GAMA2)
PWC=PB+PW2

!Calculate wing panel lift and drag
CLW=CLALFAW*(WWC/UWC + ALFA*R + ALFA0W + CW*QWC/(2.0*UWC)
-LW2=LWC+LW2  !Record total lift over entire wing; to be used for downwash
DWC=(CDW0+CLW**2/(BW+BW2+0.5*BCP)**2.0/CW**PI*EFFW))/

!Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=LWC-DWC*(WWC/UWC)
MAERO=(CM0W-PI/4)*QWC

!Add the panel values to the total wing aero forces
XAERO=XAERO+XAEROC
YAERO=YAERO+YAEROC
ZAERO=ZAERO+ZAEROC
LAERO=LAERO+LAEROC
MAEROC=MAERO+MAEROC
NAEROC=NAERO+NAEROC

17 CONTINUE
! Determine aero terms for the section of wing from the wing root to the pivot
! This section of the wing is considered to be rigid
IRREC=INT(BW2/DELR2+.5) ! Total number of wing panels
DO 18 J=1,IRREC

R=J*DELR2 ! Calculate distance from wing root to wing panel

! Transform to convenient-axis (body-fixed) system & calculate wing panel velocities
UWC=UB+QB*(HZB2+R*SIN(GAMA2))+RB*(-R*COS(GAMA2)-HYB2)

VWC=(VB+PB*(-R*SIN(GAMA2)-HZB2)+RB*(HXB2-DXP2)
@ -PW2*R*SIN(GAMA2))*COS(GAMA2)
@ -(WB+PB*(HXB2+R*COS(GAMA2))+QB*(DXP2-HXB2)
@ -PW2*R*COS(GAMA2))*SIN(GAMA2)
WWC=(VB+PB*(-R*SIN(GAMA2)-HZB2)+RB*(HXB2-DXP2)
@ -PW2*R*SIN(GAMA2))*SIN(GAMA2)
@ +(WB+PB*(HXB2+R*COS(GAMA2))+QB*(DXP2-HXB2)
@ -PW2*R*COS(GAMA2))*COS(GAMA2)
QWC=QB*COS(GAMA2)-RB*SIN(GAMA2)
PWC=PB+PW2

! Calculate wing panel lift and drag
CLW=CLALFAW*(WWC/UWC + ALFA*R + ALFA0W + CW*QWC/(2.0*UWC)
@ - R*PWC/UWC)
LWC=CLW/2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW
LW2=LWC+LW2 ! Record total lift over entire wing; to be used for downwash
DWC=(CDOW+CLW**2/((BW+BW2+0.5*BCP)**2.0/CW*PI*EFFW))
@ /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW

! Calculate aero forces for the panel
XAERO=LWC*(WWC/UWC)-DWC
ZAERO=LWC-DWC*(WWC/UWC)
MAERO=(CMW-PI/4*QWC)
@ /2.0*ROE*(UWC*UWC+VWC*VWC+WWC*WWC)*DELR2*CW*CW
@ - ZAERO*XWC

YAERO=0
LAERO=ZAERO*R
NAERO=0

! Add the panel values to the total wing aero forces
XAEROC=XAERO+XAEROC
YAEROC=YAERO+YAEROC
ZAEROC=ZAERO+ZAEROC
LAEROC=LAERO+LAEROC
MAEROC=MAERO+MAEROC
NAEROC=NAERO+NAEROC
18 CONTINUE
! Transform wing aero forces back to stability-axis system
XAEROW2=XAERO
YAEROW2=YAERO*COS(GAMA1)-XAERO*SIN(GAMA1)
ZAEROW2=ZAERO*COS(GAMA1)+YAERO*SIN(GAMA1)
LAEROW2=LAERO
MAEROW2=MAERO*COS(GAMA1)-NAERO*SIN(GAMA1)
NAEROW2=NAERO*COS(GAMA1)+MAERO*SIN(GAMA1)+YAEROW2*XWC

! Calculate wing cg velocities
UW2=UB+QB*(HZB1-DZP1)+RB*(DYP1-HYB1)
VW2=VB+PB*(DZP1-HZB1)+RB*(HXB1-DXP1)+PW1*DZP1
WW2=WB+PB*(HYB1-DYP1)+QB*(DXP1-HXB1)-PW1*DYP1

*.-Block to calculate tail forces and moments.-*

! Calculate time delay for downwash to reach tail
TD=INT(XT*(-1)/(UB*TDEL)+.5)
! Calculate downwash at time T = INT(TDEL
EW1(I+TD)=L*W1*2.0((ROE*CW*BW*(UW1*UW1+VW1*VW1+WW1*WW1))
@ *ECL+EW1(I+TD)
EW2(I+TD)=L*W2*2.0((ROE*CW*BW*(UW2*UW2+VW2*VW2+WW2*WW2))
@ *ECL+EW1(I+TD)
! Check to see if this is the very first time interval
IF(I.EQ.1)THEN
TD1=TD+1! Increment the initial time delay
END IF

! Calculate total angle of attack of tail
IF(I.GE.TD1)THEN
A11=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW1(I)+ALFA0T
@ +BT*PB/(2.0*UB))
A12=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW2(I)+ALFA0T
@ -BT*PB/(2.0*UB))
ELSE
A11=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW1(I)+ALFA0T
@ +BT*PB/(2.0*UB))! for initial time interval
A12=(WB/UB-(QB*XT)/UB+CT*QB/(2.0*UB)-EW2(I)+ALFA0T
@ -BT*PB/(2.0*UB))! for initial time interval
END IF

! Calculate tail lift and drag
LT1=CLALFAT*A11/2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*BT/2.0*CT
CLT1=LT1*2.0*(ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0
DT1=(CD0T+CLT1)**2/(BT/CT**2*EFFT))
@ /2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT*BT/2.0
! Calculate tail aero forces
XAEROT1=LT1*(A11-QB*CT/(2.0*UB)-ALFA0T)-DT1
ZAEROT1=-LT1-DT1*(A11-QB*CT/(2.0*UB)-ALFA0T)
!Calculate tail lift and drag
LT2=CLLAFAT*AL2/2.0*ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*BT/2.0*CT
CTLT2=LT2^2.0/(ROE*(UB*UB+(WB-QB*XT)*(WB-QB*XT))*CT/2.0)
DT2=(CDOT+CLT2**2/(BT/CT*PI**2))/2.0*CT*CT*BT

!Calculate tail aero forces
XAEROT2=LT2*(AL2-QB*CT/(2.0*UB)-ALFA0T)-DT2
ZAEROT2=-LT2-DT2*(AL2-QB*CT/(2.0*UB)-ALFA0T)

XAEROT=XAEROT1+XAEROT2
ZAEROT=ZAEROT1+ZAEROT2
MAEROT=CMOT/2.0*ROE*(UB*UB+VB*VB+WB*WB)*CT*CT*BT

!-Block to calculate fin forces and moments-

!Calculate fin lift and drag
LF=CLLAFB*(VB/UB+ALFA0F+RB*XF/UB+CF*RB/(2.0*UB))

!Calculate fin aero forces
XAEROF=LF*(VB/UB+RB*XF)-DF
YAEROF=-DF*(VB/UB+RB*XF)
NAEROF=YAEROF*XF
LAEROF=YAEROF*ZF

!-Block to calculate body forces and moments-

!Calculate body lift and drag
LB=CLLAFB*(WB/UB)^2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB**(2.0/3.0)
DB=(CD0B*SREFB/2.0*ROE*(UB*UB+VB*VB+WB*WB)

!Calculate body aero forces
ZAEROB=LB*(WB/UB)-DB+XAEROT+XAEROF
ZAEROB=LB-DB*(WB/UB)-ZAOB
MAEROB=CMLAFB*(WB/UB)^2.0*ROE*(UB*UB+VB*VB+WB*WB)*VOLB+MAEROT
YAEROB=CYBB*(VB/UB)^2.0*ROE*(VB*UB+VB*VB+WB*WB)*VOLB**(2.0/3.0)

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*Block to calculate center panel forces and moments.*
  !Calculate center panel lift and drag
  LCP=CLALFAW*(WCP/UCP+CCP*QB/(2.0*UCP)+ALFA0CP)
  @ *ROE/2*(UCP*VCP+VCP+VCP+WCP*WCP)*BCP*CCP
  DCP=(CD0W*WCP*2.0*(ROE*(UCP*VCP+VCP+VCP+WCP*WCP)*BCP*CCP))
  @ **2.0*((BW+BW+0.5*BCP*2.0*(CW)*PI*EFFW))/2.0*ROE*BCP*CCP*
  @ (UCP*VCP+VCP+VCP+WCP*WCP)

  !Calculate center panel aero forces
  XAEROCP=LCP*(WCP/UCP)-DCP
  ZAEROCP=LCP-DCP*(WCP/UCP)
  MAEROCP=(CM00-P/4*QB)
  @ /2.0*ROE*(UCP*VCP+VCP+VCP+WCP*WCP)*BCP*CCP*CCP-ZAEROCP*XWC

  YAEROCP=0
  LAEROCP=0
  NAEROCP=0

  !Calculate portside wings moments of inertia and derivatives
  DXW2=DXW0
  IYYW1=IYYW0*(COS(-GAMA1)**2)+IZZW0*(SIN(-GAMA1)**2)
  IYYZW1=-(SIN(-GAMA1))*COS(-GAMA1)*(IYYW0-IZZW0)
  IYYDOW1=(GAMA0**W*SIN(W*T))*(2*COS(-GAMA1)*SIN(-GAMA1)*(IZZW0-@ IYYW0))
  IYYDOW1=(GAMA0**W*SIN(W*T))*(SIN(-GAMA1)**2)-(COS(-GAMA1)**2))
  @ *(IYYW0-IZZW0)
  IZZW1=IZZW0*(SIN(-GAMA1)**2)+IZZW0*(COS(-GAMA1)**2)
  IZZDOW1=(GAMA0**W*SIN(W*T))*(2*COS(-GAMA1)*SIN(-GAMA1)*(IYYW0-@ IZZW0))

  !Calculate starboard wings moments of inertia and derivatives
  DXW2=DXW0
  IYYW2=IYYW0*(COS(GAMA2)**2)+IZZW0*(SIN(GAMA2)**2)
  IYYZW2=-(SIN(GAMA2))*COS(GAMA2)*(IYYW0-IZZW0)
  IYYDOW2=(GAMA0**W*SIN(W*T))*(2*COS(GAMA2)*SIN(GAMA2)*(IZZW0-@ IYYW0))
  IYYDOW2=(GAMA0**W*SIN(W*T))*(SIN(GAMA2)**2)-(COS(GAMA2)**2))
  @ *(IYYW0-IZZW0)
  IZZW2=IZZW0*(SIN(GAMA2)**2)+IZZW0*(COS(GAMA2)**2)
  IZZDOW2=(GAMA0**W*SIN(W*T))*(2*COS(GAMA2)*SIN(GAMA2)*(IYYW0-@ IZZW0))

*Simultaneous solution of unknown variables at T+DELT.*

  !Initialize eqn matrix to zero
  DO 50 L=1,124
  DO 40 M=1,124
  A(M,L)=0.0
  40 CONTINUE
  50 CONTINUE

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!Assign values to the eqn matrices
A(1,1)=MB
A(1,7)=1.0
A(1,16)=1.0
A(2,2)=MB
A(2,8)=1.0
A(2,17)=1.0
A(3,3)=MB
A(3,9)=1.0
A(3,18)=1.0
A(4,4)=IXZB
A(4,6)=DXB
A(4,8)=HZB1
A(4,9)=HYB1
A(4,17)=HZB2
A(4,18)=HYB2
A(5,5)=IY
A(5,7)=HZB1
A(5,9)=HXB1
A(5,16)=HZB2
A(5,18)=HXB2
A(6,6)=DXW2
A(6,17)=DZP2
A(6,18)=DYP2
A(6,21)=DZW2
A(6,22)=DYP2
A(7,1)=MW
A(7,4)=MW*(DYP1-HYB1)
A(7,5)=MW*(HZB1-DZP1)
A(7,7)=1.0
A(7,11)=1.0
A(8,2)=MW
A(8,4)=MW*(HXB1-DXP1)
A(8,6)=MW*(DZP1-HZB1)
A(8,8)=1.0
A(8,12)=1.0
A(9,3)=MW
A(9,5)=MW*(DXP1-HXB1)
A(9,6)=MW*(HYB1-DYP1)
A(9,9)=1.0
A(9,13)=1.0
A(10,4)=IZW1
A(10,5)=IYZW1
A(10,7)=DYP1
A(10,8)=DYP1
A(10,10)=1.0
A(10,11)=DYP1
A(10,12)=DXW1
A(10,15)=1.0
A(11,4)=IYZW1
A(11,5)=IYZW1
A(11,7)=DZP1
A(11,9)=DYP1
A(11,11)=DZW1
A(11,13)=DXW1
A(11,14)=1.0
A(12,6)=DXW1
A(12,8)=DZP1
A(12,9)=DYP1
A(12,12)=DZW1
A(12,13)=DYW1
A(13,6)=IXXCP
A(13,13)=BCP/2.0
A(13,22)=BCP/2.0
A(14,5)=IYYCP
A(14,14)=1.0
A(14,23)=1.0
A(15,4)=IZZCP
A(15,11)=BCP/2.0
A(15,15)=1.0
A(15,20)=BCP/2.0
A(15,24)=1.0
A(16,1)=MW
A(16,4)=MW*(DYP2-HYB2)
A(16,5)=MW*(HZB2-DZP2)
A(16,16)=1.0
A(16,20)=1.0
A(17,2)=MW
A(17,4)=MW*(HXB2-DEXP2)
A(17,6)=MW*(DZP2-HZB2)
A(17,17)=1.0
A(17,21)=1.0
A(18,3)=MW
A(18,5)=MW*(DEXP2-HXB2)
A(18,6)=MW*(HYB2-DYP2)
A(18,18)=1.0
A(18,22)=1.0
A(19,4)=IZZB
A(19,6)=IXZB
A(19,7)=HYB1
A(19,8)=HXB1
A(19,10)=1.0
A(19,16)=HYB2
A(19,17)=HXB2
A(19,19)=1.0
A(20,1)=MCP
A(20,4)=MCP*DYB
A(20,5)=MCP*DZB
A(20,11)=1.0
A(20,20)=1.0
A(21,2)=MCP
A(21,4)=MCP*DXB
A(21,6)=MCP*DZB
A(21,12)=1.0
A(21,21)=1.0
A(22,3)=MCP
A(22,5)=MCP*DXB
A(22,4)=MCP*DYB
A(22,13)=1.0
A(22,22)=1.0
A(23,4)=IYZW2
A(23,5)=IYYW2
A(23,16)=DZP2
A(23,18)=DXP2
A(23,20)=DZW2
A(23,22)=DXW2
A(23,23)=1.0
A(24,4)=IZZW2
A(24,5)=IYZW2
A(24,16)=DYP2
A(24,17)=DXP2
A(24,19)=1.0
A(24,20)=DYW2
A(24,21)=DXW2
A(24,24)=1.0

B(1)=XAEROB-MB*G*SIN(THETA)-MB*(QB*WB-RB*VB)
B(2)=YAEROB+MB*G*COS(THETA)*SIN(PHI)+MB*(RB*UB-PB*WB)
B(3)=ZAEROB+MB*G*COS(THETA)*COS(PHI)-MB*(PB*VB-QB*UB)
B(4)=LAEROB-(IZZB-IYYB)*QB*RB+IZZB*PB*QB
B(5)=MAEROB-(IZXW-IZZW)*PB*RB-DZB*PB*PB*RB
B(6)=LAEROW2-DXXW*PDOW2-(IZZW2-IYYW2)*QB*RB

@ -IYZW2*(RB*RB-QB*QB)

B(7)=XAEROW1-MW*G*SIN(THETA)-MW*(2*(QB*QB+RB*RB)
  *(DXP1-HXB1)+2*PB*QB*(HYB1-DYP1)+2*PB*RB*(HZB1-DZP1)
  @ +2*QB*WB-2*RB*VB-3*PW1*(QB*DYP1+RB*ZP1))
B(8)=YAERO1+MW*G*COS(THETA)*SIN(PHI+GAMA1)-MW*((2*PB*PB
  +2*RB*RB)*(DYP1-HYB1)+2*PB*QB*(HXB1-DXP1)
  @ +2*QB*RB*(HZB1-DZP1)-2*PB*WB+2*RB*UB-PW1*PB*(HYB1-4*DYP1)
  @ +PDOW1*DZP1+2*PW1*PB-DP1*PB-PW1*QB*(DXP1-HXB1))
B(9)=ZAERO1+MW*G*COS(THETA)*COS(PHI+GAMA1)-MW*((2*PB*PB
  +2*QB*QB)*(DZP1-HZB1)+2*PB*RB*(HXB1-DXP1)
  @ +2*QB*RB*(HYB1-DYP1)+2*PB*VB-2*QB*UB+PW1*PB*(4*DZP1-HZB1)
  @ -PDOW1*DYP1+2*PW1*PB-DP1*PB-PW1*RB*(HXB1-DXP1))
B(10)=NAERO1-IZZDOTW1*RB+IYZDOTW1*QB+IYYW1*(PB+PW1)*RB
  @ -(IYYW1-DXW1)*(PB+PW1)*QB
B(11)=MAERO1-IYYDOTW1*QB+IYZDOTW1*RB-(DXXW1-IZZW1)*(PB+PW1)*RB
  @ -IYYW1*(PB+PW1)*QB
B(12)=LAEROW1-DXXW*PDOW1-(IZZW1-IYYW1)*QB*RB
  @ -IYZW1*(RB*RB-QB*QB)
B(13)=LAEROCP-(IZZCP-IYSCP)*QB*RB
B(14)=MAEROCP-(DXXCP-IZZCP)*PB*RB
B(15)=NAEROCP-(IYSCP-DXXCP)*PB*QB
B(16)=XAERO2-MW*G*SIN(THETA)-MW*(2*(QB*QB+RB*RB)
  *(DXP2-HXB2)+2*PB*QB*(HYB2-DYP2)+2*PB*RB*(HZB2-DZP2)
  @ +2*QB*WB-2*RB*VB-3*PW2*(QB*DYP2+RB*ZP2))
B(17)=YAERO2+MW*G*COS(THETA)*SIN(PHI+GAMA2)-MW*((2*PB*PB
  +2*RB*RB)*(DYP2-HYB2)+2*PB*QB*(HXB2-DXP2)
  @ +2*QB*RB*(HZB2-DZP2)-2*PB*WB+2*RB*UB-PW2*PB*(HYB2-4*DYP2)
  @ +PDOW2*DZP2+2*PW2*PB-DP2*PB-PW2*QB*(DXP2-HXB2))
B(18)=ZAERO2+MW*G*COS(THETA)*COS(PHI+GAMA2)-MW*((2*PB*PB
CALL GAUSS(A,AA,B,X,24,24+1,24,24+1)

IF(K_EQ. 1)THEN
  !Perform 1st part of 2nd-order Range-Kutta integration
  UDOTB=X(1) !If(xn,un) = _DOTB
  VDOTB=X(2) !This is not a function of xn(time), only un(UBC)
  WDOTB=X(3)
  PDOTB=X(4)
  QDOTB=X(5)
  RDOTB=X(6)
  XDOT=UB*COS(THETA)*COS(PSI)+VB*(SIN(PHI)*SIN(THETA))*COS(PSI)
  @ -COS(PHI)*SIN(PSI)+VB*COS(PHI)*SIN(THETA)*COS(PSI)
  @ +SIN(PHI)*SIN(PSI)) !Calculate trajectory
  YDOT=UB*COS(THETA)*SIN(PSI)+VB*(SIN(PHI)*SIN(THETA))*SIN(PSI)
  @ +COS(PHI)*COS(PSI)+WB*(COS(PHI)*SIN(THETA))*SIN(PSI)
  @ -SIN(PHI)*COS(PSI))
  ZDOT=-UB*SIN(THETA)+VB*SIN(PHI)*COS(THETA)+WB*COS(PHI)
  @ *COS(THETA)
  UB=UDOTB*TDEL+UBC !Store k1+un in the temporary variables
  VB=VDOTB*TDEL+VBC
  lk1 = _DOTB*TDEL
  WB=WDOTB*TDEL+WBC
  PB=PDOTB*TDEL+PBC
  QB=QDOTB*TDEL+QBC
  RB=RDOTB*TDEL+RBC
  TEMPTHETA=THETA
  TEMPPHI=PHI
  THETA=Q*B*COS(TEMPPHI)-R*B*SIN(TEMPPHI))TDEL+THETAC
  PHI=(PB+QB*SIN(TEMPPHI))TAN(TEMPTHETA)
  @ +R*B*COS(TEMPPHI)*TAN(TEMPTHETA))TDEL+PHIC
  PSI=((Q*B*SIN(TEMPPHI)+R*B*COS(TEMPPHI)))/(1.0*COS(TEMPTHETA))
  @ *TDEL+PSIC
ELSE
!Perform 2nd part of 2nd-order Runge-Kutta integration
UDOTBC=X(1) !f(xn+h,un+k1) = _DOTBC
VDOTBC=X(2) !This is not a function of xn(time), only un(UBC)
WDOTBC=X(3)
PDOTBC=X(4)
QDOTBC=X(5)
RDOTBC=X(6)
UBC=(UDOTBC+UDOTB)/2.0*TDEL+UBC !k2=_DOTBC*TDEL
VBC=(VDOTBC+VDOTB)/2.0*TDEL+VBC !un+1 = un + (k1+k2)/2
WBC=(WDOTBC+WDOTB)/2.0*TDEL+WBC
PBC=(PDOTBC+PDOTB)/2.0*TDEL+PBC
QBC=(QDOTBC+QDOTB)/2.0*TDEL+QBC
RBC=(RDOTBC+RDOTB)/2.0*TDEL+RBC
TEMPTHETA=THETAC
TEMPHII=PHIC
THETAC=((QBC*COS(TEMPHII)-RBC*SIN(TEMPHII))
 @ +QB*COS(PHI)-RB*SIN(PHI))/2.0*TDEL+THETAC
PHIC=((PBC+QBC*SIN(TEMPHII))*TAN(TEMPTHETA)+RBC*COS(TEMPHII)
 @ *TAN(TEMPHII))+(PB+QB*SIN(PHI)*TAN(THETA)+RB*COS(PHI)
 @ +TAN(THETA)))/2.0*TDEL+PHIC
PSIC=((QBC*SIN(TEMPHII)+RBC*COS(TEMPHII))*1.0/COS(TEMPTHETA))
 @ +QB*SIN(PHI)+RB*COS(PHI))*(1.0/COS(THETA)))/2.0*TDEL
 @ +PSIC
XDOTC=UBC*COS(THETAC)*COS(PSIC)+VBC*SIN(PHI)*SIN(THETAC)
 @ *COS(PSIC)-COS(PHIC)*SIN(PSIC)+WBC*COS(PHI)
 @ +SIN(THETAC)*COS(PSIC)*SIN(PSIC)+SIN(PHI)*SIN(PSIC)) !Calculate trajectory
YDOTC=UBC*COS(THETAC)*SIN(PSIC)+VBC*SIN(PHI)*SIN(THETAC)
 @ *SIN(PSIC)+COS(PHIC)*COS(PSIC)+WBC*COS(PHI)
 @ +SIN(THETAC)*SIN(PSIC)-SIN(PHIC)*COS(PSIC)
ZDOTC=UBC*SIN(THETAC)+VBC*SIN(PHI)*COS(THETAC)
 @ +WBC*COS(PHI)*COS(THETAC)
XC=(XDOTC+XDOT)/2.0*TDEL+XC
YC=(YDOTC+YDOT)/2.0*TDEL+YC
ZC=(ZDOTC+ZDOT)/2.0*TDEL+ZC

WRITE(LU2,8500)T,UBC,UDOTBC,VBC,VDOTBC,WBC,WDOTBC,PBC*180/PI
 @ ,PHIC*180/PI,QBC*180/PI,THETAC*180/PI,RBC*180/PI
 @ ,PSIC*180/PI,XC,YC,ZC

!Calculate total values for the unknown state variables
UBT=UBC+UBT
VBT=VBC+VBT
WBT=WBC+WBT
THETAT=THETAC+THETAT
PHIT=PHIC+PHIT
PSIT=PSIC+PSIT
! Assign values to the temporary variables for the next time increment
UB = UBC
VB = VBC
WB = WBC
PB = PBC
QB = QBC
RB = RBC
THETA = THETAC
PHI = PHIC
PSI = PSIC

END IF

25 CONTINUE
10 CONTINUE

! Calculate average values for the unknown state variables
UBA = UBT/REAL(IEND)
VBA = VBT/REAL(IEND)
WBA = WBT/REAL(IEND)
THETAA = THETAT/REAL(IEND)
PHIA = PHIT/REAL(IEND)
PSIA = PSIT/REAL(IEND)
WRITE (6,9000) UBA, VBA, WBA, THETAA*180/PI, PHIA*180/PI, PSIA*180/PI
100 CONTINUE

125 FORMAT (12F10.2)
150 FORMAT (1X,'U-VELOCITY', 3X,'W-VELOCITY', 3X,'PITCH RATE', 3X,
@'PITCH ANGLE', 4X,'GAMA', 7X,'ALFAT')
200 FORMAT (2X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4,5X,F7.4,F7.4)
8500 FORMAT (16(F14.8))
9000 FORMAT ('AV.U-VEL=',F8.4,1X,'(m/s)  AV.V-VEL=',F8.4,1X,
@'(m/s)  AV.W-VEL=',F8.4,1X,'(m/s)  AV.THETA=',F8.4,1X,
@'(deg)  AV.PHI=',F8.4,1X,'(deg)  AV.PSI=',F8.4,1X,'(deg)')

STOP
END